A NON-LINEAR ANALYSIS FOR PLANE-STRAIN UNDRAINED PRESSUREMETER EXPANSION TESTS
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ABSTRACT
This paper presents the solution for the undrained expansion of a cylindrical cavity in a non-linear workhardening soil. The solution is obtained using an inverse hyperbolic sine law to represent the relation between applied radial pressure and shear strain induced at the cavity wall. As in the case of a linearly elastic perfectly plastic (Tresca) material, three parameters are required to implement the solution, the initial horizontal pressure $p_0$, the undrained shear strength $S_u$, and the maximum shear modulus $G_{max}$. The values of these parameters are estimated in a particular case by linear regression analysis. Comparisons are also made with solutions obtained by means of power law, simple hyperbolic and linearly elastic perfectly plastic stress-strain relationships.

RÉSUMÉ
Cet article présente la solution pour l’expansion non-drainée d’une cavité cylindrique dans un sol non linéaire. La solution est obtenue en utilisant une loi sinus hyperbolique inverse décrivant la relation entre la pression radiale et la déformation de cisaillement. Comme dans le cas d’un matériau de type Tresca, trois paramètres sont nécessaires, la pression horizontale initiale $p_0$, la résistance au cisaillement non-drainé $S_u$, le module de cisaillement à l’origine $G_{max}$. Ces paramètres sont estimés dans un cas particulier à l’aide de régressions linéaires. Des comparaisons sont aussi effectuées avec des solutions obtenues en utilisant une loi de puissance, une relation hyperbolique simple, et un modèle Tresa.

1. INTRODUCTION
When the preconsolidation pressure $\sigma'_p$ of soft natural clays, which is measured by means of constant strain rate (CRS) consolidation tests, is plotted as a function of the strain rate $\dot{\varepsilon}$ used, the resulting experimental relationship can be described by an inverse hyperbolic sine curve of the form (Silvestri et al. 1986):

$$\sigma'_p = \lambda + \mu \sinh^{-1}(\nu \dot{\varepsilon})$$

where $\lambda$, is the value of $\sigma'_p$ that would be measured in a CRS consolidation test performed at a strain rate $\dot{\varepsilon} = 0$, $\mu$ is a viscosity coefficient, and $\nu$ is a time coefficient. Such an equation is encountered when soil deformation is considered as a rate process (Mitchell 1993).

Curiously enough, when the applied total pressure $p$ in a self-boring pressuremeter (SBP) test in clay is plotted as a function of the current shear strain $\gamma$ induced at the cavity wall, the resulting relationship is found to be quite similar to that given by Eq.1, except that the strain rate $\dot{\varepsilon}$ is replaced with the current shear strain $\gamma$, that is:

$$p = \lambda + \mu \sinh^{-1}(\nu \gamma) \quad 0 \leq \gamma \leq 1$$

where, now, $\lambda$ is the value of the pressure $p$ (i.e., $\lambda$ represents the horizontal geostatic stress $p_0$, when the borehole is not disturbed prior to the performance of the test), and $\mu$ and $\nu$ are material parameters.

It will be shown in the next section that the use of Eq.2 results in a non-linear workhardening stress-strain curve for clay.

2. STRESS-STRAIN RESPONSE
From the work of Hill (1950), Ladanyi (1972), and Palmer (1972) it is found that the stress-strain curve of clay may be obtained from the experimental measurement of radial pressure and shear strain, by using the following relationship:

$$\tau = \gamma \frac{dp}{d\gamma}$$

where $\tau$ is the shear stress induced at the cavity wall, $\gamma$ is the corresponding current shear strain ($0 \leq \gamma \leq 1$), and $dp/d\gamma$ is the slope of the radial pressure-shear strain curve.

Introducing Eq.3 into Eq.2 yields

$$\tau = \frac{\mu \nu \gamma}{(1 + \nu^2 \gamma^2)^{1/2}} \quad 0 \leq \gamma \leq 1$$

where $\mu$ and $\nu$ are material parameters.

Examination of this equation shows that when $\gamma$ is very large (i.e., $\gamma = 1$), the shear stress is given by

$$\tau = \frac{\mu \nu \gamma}{(1 + \nu^2)^{1/2}}$$
Since $v >> 1$ for most soft clays, as indicated below, then $\tau = \mu$ when $\gamma = 1$. Thus, the material parameter $\mu$ represents the undrained shear strength of the clay, $S_u$.

Differentiation of Eq.4 with respect to $\gamma$ allows to determine the tangent shear modulus $G$:

$$\frac{d\tau}{d\gamma} = G = \frac{\mu v}{\left(1 + v^2 I_r^2\right)^{3/2}}$$  \[6\]

Substituting $\gamma = 0$ into this equation permits to find the maximum shear modulus $G_{\text{max}}$:

$$\frac{d\tau}{d\gamma}_{\gamma=0} = G_{\text{max}} = \mu v$$  \[7\]

The second material parameter $v$ is thus equal to:

$$v = \frac{G_{\text{max}}}{\mu} = \frac{G_{\text{max}}}{S_u} = I_r$$  \[8\]

where $I_r$ is the rigidity index of the clay.

Because $\mu = S_u$ and $v = I_r$, from above, Eq.2 may also be written as:

$$p = p_0 + S_u \sinh^{-1}(I_r \gamma)$$  \[9\]

The limit pressure $p_{\text{limit}}$ is found by putting $\gamma = 1$ in this equation leading to:

$$p = p_0 + S_u \sinh^{-1}(I_r)$$  \[10\]

In addition, as the rigidity index $I_r$, which is equal to $v$, ranges between a minimum value of about 50 to a maximum value of 150 for soft clays, it follows that $I_r >> 1$. As a consequence, when a SBP test is pursued to strains of even as low as 10%, the inverse hyperbolic sine term in Eq.9 may be approximated by an exponential function of the form (Mitchell 1993):

$$\sinh^{-1}(I_r \gamma) \approx \ln(2I_r \gamma)$$  \[11\]

yielding

$$p = p_0 + S_u \ln(2I_r \gamma)$$  \[12\]

which shows that the undrained shear strength can be obtained from the gradient (i.e., slope) of a plot of total applied pressure versus the natural logarithm of the current shear strain induced at the cavity wall.

Since $\mu = S_u$ and $v = I_r$ as before, the stress-strain curve of Eq.4 becomes:

$$\tau = \frac{G_{\text{max}}}{{\left[1+(I_r \gamma)^2\right]}^{3/2}}$$  \[13\]

The curve described by this equation represents a non-linear workhardening constitutive relationship, as shown in Fig.1.

Figure 1. Non-linear workhardening stress-strain curve.

3. APPLICATION

3.1 Inverse Hyperbolic Sine Law

The relationships given by Eqs.9 and 13 were applied to sel-boring (SBP) test data reported by Bolton and Whittle (1999). The results are compared with those obtained by these authors using a power law function. Further comparisons are carried out by assuming a simple hyperbolic stress-strain curve as well as a linearly elastic perfectly plastic response.

The experimental radial pressure versus shear strain curve is shown in Fig.2. The data was obtained by scanning and digitizing the original results of Bolton and Whittle (1999). In order to get the best fit to the data, the value of the undrained shear strength was first determined from the slope, at large strain, of the linear portion of the $p$-$\log \gamma$ relationship of Fig.2. This yielded $S_u = 178$ kPa, as also found by Bolton and Whittle (1999). Secondly, Eq.9 was transformed into:

$$\sinh{\left(\frac{p - p_0}{S_u}\right)} = I_r \gamma$$  \[14\]

Linear least-squares analyses were then carried out by assuming different values for $p_0$. Eq.14 was thus approximated by the straight line relationship:
\[ \hat{y}_i = \hat{a}_i + \hat{b} x_i \]  

[15]

\[ \tau (kPa) = \frac{54219 \gamma}{\left[ 1 + (304.6 \gamma)^{\frac{1}{2}} \right]^2} \]  

[16]

Examination of the curve described by this equation, which is shown in Fig.3, indicates that the shear stress \( \tau \) reaches a horizontal asymptote for large strain. As for the values of the limit pressure \( p_{\text{limit}} \), calculated on the basis of Eq.10, Table 1 shows that these are quite insensitive to the choice of the parameters \( \hat{a}_i \) and \( \hat{b}_i \). Indeed, the limit pressure varies from 1610.1 kPa to 1611.5 kPa.

Figure 2. Radial pressure-logarithm of current shear strain relationship.

where \( \hat{y}_i \) is the estimated mean of sinh \( [(p - p_0) / S_u] \) for each \( x_i = \gamma_i \), \( \hat{a}_i \) is the intercept, and \( \hat{b}_i \) is the slope (i.e., the estimated mean of \( l_i \)).

It was found that the values of the parameter \( p_0 \) reported in the first three rows of Table 1 resulted in a very satisfactory fit to the data. Examination of the entries in this table indicates that the choice of \( p_0 = 460 \) kPa yields a slightly better fit, because it gives the highest value for the coefficient of correlation \( r \) (i.e., \( r = 0.99916 \)). In this case, the estimated mean value of the rigidity index \( I_r \) equals 320.3.

However, as it would be expected that sinh \( [(p - p_0) / S_u] \) in Eq.14 should be zero when \( \gamma = 0 \), implying a straight line approximation passing through the origin (i.e., \( \hat{a}_i = 0 \) in Eq.15), additional analyses were performed by forcing the regression line to pass through the origin. The results which are reported in the last four rows of Table 1 indicate that the choice of \( p_0 = 450 \) kPa yields, this time, a slightly better fit than the other values of \( p_0 \). The solutions given in Table 1 are, however, practically equivalent in that they all give a very good fit to the data, for the whole range of shear strains (i.e., \( \gamma \leq 0.1353 \)). This notwithstanding, it was found that the initial part of the radial pressure versus shear strain relationship of Fig.2 (i.e., for \( \gamma \leq 0.002 \)) was better approximated by using \( p_0 = 470 \) kPa with either \( \hat{a}_i = 0 \) and \( \hat{b}_i = 1 \) (i.e., \( r = 0.99760 \)) or \( \hat{a}_i = 0.0170 \) and \( \hat{b}_i = 302.7 \) (i.e., \( r = 0.99908 \)). The curve shown in Fig.2 corresponds to Eq.9 with \( S_u = 178 \) kPa, \( p_0 = 470 \) Kpa, \( \hat{a}_i = 0 \), and \( \hat{b}_i = l_r = 304.6 \). As these parameters imply \( G_{\text{max}} = S_u l_r = 54219 \) kPa (54.2 MPa), the corresponding stress-strain curve is:

\[ \tau = \eta \beta \gamma^\beta \quad \gamma \leq \gamma_y \]  

[17a]

and

\[ \tau = S_u \quad \gamma > \gamma_y \]  

[17b]

where \( \eta \) and \( \beta \) are material parameters, with \( \beta < 1 \) and \( \gamma_y = (S_u / \eta \beta)^\frac{1}{\beta} \) is the shear strain at yield. It should be also noted that complete solutions for the expansion of both spherical and cylindrical cavities, based exactly on this same assumption, were published by Ladanyi and Johnson (1974), and Ladanyi (1975). The power law function in Eq.17 describes a non-linear elastic perfectly plastic (Tresca) model. The corresponding radial pressure versus shear strain curve is:

\[ p = p_0 + \eta \beta \gamma^\beta \quad \gamma \leq \gamma_y \]  

[18a]

and

3.2 Power Law Representation

The radial pressure versus shear strain relationship of Fig.2 was approximated using the power law of Bolton and Whittle (1999). These authors proposed that the stress-strain curve be represented by:

Figure 3. Stress-strain curves.
\[ p = p_0 + S_u \left[ \frac{1}{\beta} + \ln \left( \frac{\gamma}{\gamma_y} \right) \right] \quad \gamma > \gamma_y \] \quad \text{[18b]}

In addition, the limit pressure is found by putting \( \gamma = 1 \) into Eq.18b.

The values of the parameters \( \beta \) and \( \eta \) were determined by Bolton and Whittle (1999) using unload/reload loops. On the basis of the last two loops reported by these investigators, which resulted, respectively, in \( \ln \eta_1 = 8.326 \) and \( \beta_1 = 0.5758 \), with \( r = 0.99955 \) and \( \ln \eta_2 = 8.3505 \) and \( \beta_2 = 0.573 \), with \( r = 0.99960 \), the corresponding average values of \( \ln \eta \) and \( \beta \) are: \( \ln \eta = 8.3383 \) or \( \eta = 4181 \) kPa and \( \beta = 0.5744 \). As \( \gamma_y = (S_u/\eta)^0 \) from Eq.17, the shear strain at yield is equal to 0.01078 for \( S_u = 178 \) kPa, \( \eta = 4181 \) kPa, and \( \beta = 0.5744 \). However, Bolton and Whittle (1999) retained \( \beta = 0.57 \) and \( \gamma_y = 0.0086 \), corresponding to \( \eta = 4698 \) kPa. In addition, these authors found that \( p_0 = 449 \) kPa, represented a satisfactory geostatic stress value. Eq.18 was, therefore, used with \( p_0 = 449 \) kPa, \( S_u = 178 \) kPa, and \( \beta = 0.5744 \). This curve is also shown in Fig.2. Comparison between the numerical relationship based upon Eq.18 with that obtained using the inverse hyperbolic sine indicates that the two are quite similar.

The corresponding stress-strain curve, based upon Eq.17, is reported in Fig.3. Comparison between the curves shown in this figure indicates that the strain-stress relationship obtained by means of the inverse hyperbolic sine law is stiffer than the one based upon the power law, in the range 0.001 \( \leq \gamma \leq 0.007 \).

Furthermore, in order to determine whether the material parameters \( \beta \) and \( \eta \) could be also obtained from the initial portion of the pressure-expansion curve (i.e., for \( \gamma_y \leq 0.01087 \)), without having to use the unload/reload loops, Eq.18 was first transformed into:

\[ \ln(p_0 - p) = \ln \eta + \beta \ln \gamma \] \quad \text{[19]}

and then approximated by the regression line of Eq.15, where \( \hat{\gamma} \) is now the estimated mean of \( \ln (p-p_0) \) for each value of \( x_i = \ln \eta \), \( \hat{\beta} \) in \( \ln \eta \) is the intercept, and \( \beta = \hat{\beta} \) is the slope. Regression analyses were again performed by assuming different values for \( p_0 \). The results are reported in Table 2. In this table are also given the values of \( \gamma_y = (S_u/\eta)^0 \). Examination of the various entries indicates that the choice of \( p_0 = 420 \) kPa constitutes the best fit to the data with \( \hat{\beta} = \ln \eta = 8.3688 \) or \( \eta = 4308 \) kPa, \( \hat{\beta} = 0.535 \), and \( r = 0.99996 \). Recalling that the results retained by Bolton and Whittle (1999) were \( \eta = 4698 \) kPa, \( \beta = 0.57 \), \( \gamma_y = 0.0086 \) with \( r = 0.9996 \), it appears that these correspond to a choice of \( p_0 \) close to 430 kPa in Table 2.

As for the values of the limit pressure \( p_{\text{limit}} \), determined on the basis of Eq.18b for \( \gamma = 1 \), Table 2 indicated that these vary between 1602.6 kPa and 1610.1 kPa.

### 3.3 Simple Hyperbolic Representation

In order to determine whether a simple hyperbolic stress-strain curve of the form:

\[ \tau = \frac{G_{\text{max}}\gamma}{1 + I_{r}\gamma} \] \quad \text{[20]}

could also be successfully used, this equation was substituted in Eq.2 to obtain the radial pressure versus shear strain curve. This resulted in:

\[ p = p_0 + S_u \ln(1 + I_{r}\gamma) \] \quad \text{[21]}

For \( \gamma \) very large, \( I_{r} \gg 1 \), and Eq.21 may be approximated by:

\[ p = p_0 + S_u \ln(I_{r}\gamma) \] \quad \text{[22]}

indicating, once again, that the undrained shear strength can be obtained from the slope, at large strain, of a plot of total pressure versus the natural logarithm of the shear strain. In order to carry out linear regression analyses, Eq.21 was first transformed into:

\[ e^{\left(\frac{p-p_0}{S_u}\right)} - 1 = I_{r}\gamma \] \quad \text{[23]}

and then approximated by the regression line of Eq.15, where this time \( \hat{\gamma} \) is the estimated mean value of \( e^{\left(\frac{p-p_0}{S_u}\right)} - 1 \) for each value of \( x_i = \gamma_i \), \( \hat{\alpha} \) is the intercept, and \( \beta = \hat{\beta} \) is the slope. Results of these regression analyses are summarized in Table 3 for different values of \( p_0 \). Although the calculated values of the coefficient of correlation \( r \) reported in this table are quite high, it was found that none of the regression equations could satisfactorily approximate the radial pressure versus shear strain relationship for shear strain \( \gamma \leq 0.01316 \). This is due to the presence of the negative \( \hat{\alpha} \) term. As a consequence, additional regression analyses were performed by forcing the regression line to pass through the origin (i.e., \( \hat{\alpha} = 0 \)). The results are also reported in Table 3. Examination of the entries shown indicates that the choice of \( p_0 = 450 \) kPa constitutes the best fit to the data \( (r = 0.99793) \), yielding \( \hat{\beta} = I_{r} = 669.5 \). These parameters (i.e., \( p_0 = 450 \) kPa, \( I_{r} = 669.5 \), \( S_u = 178 \) kPa and \( G_{\text{max}} = I_{r} S_u = 119171 \) kPa) were then introduced into Eqs.20 and 21 to plot both the stress-strain curve and the radial pressure versus shear relationship, as shown in Figs.2 and 3. Examination of the
predicted values of the total pressure \( p \) in Fig.2 indicates that they overestimate the actual trend at low moderate shear strain (i.e., for \( \gamma \leq 0.03 \)). Concerning the hyperbolic stress-strain curve plotted in Fig.3, it appears that it is initially much stiffer than both the power law and inverse hyperbolic sine relationships.

As an additional point concerning the stress-strain curves reported in Fig.3, discrete data points, obtained from a direct application of Palmer's approach (i.e., Eq.2), were scaled from the results reported by Bolton and Whittle (1999) and are also included in Fig.3. It appears that the latter points follow quite closely the power law approximation.

As for the limit pressure \( p_{\text{lim}} \), it was determined by putting \( \gamma = 1 \) into Eq.21 and the results are also reported in Table 3. Examination of the various entries shows that it varies in a very small range, from 1608.3 kPa to 1609.6 kPa. Further, comparison between Eqs.21 and 12 shows that for the limit pressure to be unique, the rigidity index obtained from hyperbolic model should be approximately equal to twice the value determined on the basis of the inverse hyperbolic approach. This is perfectly borne out by the results presented in Table 1 and 3.

3.4 Linear Elastic Perfectly Plastic Response

Finally, in order to examine the possibility that a simple linearly elastic perfectly plastic (Tresca) stress-strain relationship of the form:

\[
\tau = G \gamma, \quad \gamma \leq \gamma_y = S_u / G \quad [24a]
\]

and

\[
\tau = S_u, \quad \gamma > \gamma_y = S_u / G \quad [24b]
\]

might be appropriate for the clay, these equations were substituted in Eq.2 to obtain the well-known pressure-expansion relationships (Gibson and Anderson, 1961):

\[
p = p_0 + G \gamma, \quad \gamma \leq \gamma_y = S_u / G \quad [25a]
\]

and

\[
p = p_0 + S_u \left( 1 + \ln \frac{\gamma}{\gamma_y} \right), \quad \gamma > \gamma_y \quad [25b]
\]

Again, the limit pressure may be found by putting \( \gamma = 1 \) into Eq.25b.

Instead of performing two separate regression analyses, that is, one for elastic phase of deformation and the other for the plastic response, it was deemed preferable to obtain by iteration a good fit to the experimental pressure-expansion curve. This was done by adjusting the values of the parameters \( p_0 \) and \( G_{\text{max}} \).

Two such solutions are reported in Table 4 and are compared in Fig.4 with the experimental pressure-expansion curve. Examination of the data shown in this figure indicates that the solution which corresponds to \( p_0 = 496 \) kPa, \( S_u = 178 \) kPa, \( G_{\text{max}} = 34563 \) kPa, and \( \gamma_y = 0.00515 \), compares well with the experimental results. In addition, the limit pressure \( p_{\text{lim}} \) which was calculated on the basis of Eq.25b for \( \gamma = 1 \) varies between 1610.1 kPa and 1611.8 kPa.

A stress-strain curve based upon the solution just mentioned is also shown in Fig.3. Examination of the various relationships reported in this figure indicates that although the simple linearly elastic perfectly plastic solution is much softer than the previously obtained relationships, it nevertheless gives a good fit to the experimental pressure-expansion curve of Fig.4.

4. DISCUSSION

It appears at first sight that the inverse hyperbolic sine solution obtained in this study and that of Bolton and Whittle (1999) are quite similar. Indeed, linear regression analyses yielded almost identical values for the coefficient of correlation.

There is, however, a slight divergence that arises between the two approaches. While the value of \( G_{\text{max}} \) is finite in the present approach as found from Eq.7, that determined by using the power law representation is infinite. Indeed, differentiation of Eq.17a gives:
which, when evaluated at $\gamma = 0$, leads to $G_{\text{max}} = \infty$, since $\beta < 1$. Such a particular behaviour of the power law representation at the origin is thought to be inappropriate for clay.

As for the simple hyperbolic stress-strain curve, it is shown that it is much stiffer than the inverse hyperbolic sine law. In addition, it is indicated that although the linearly elastic perfectly plastic criterion fits reasonably well the experimental pressure-expansion relationship, it nevertheless fails to represent the non-linear stress-strain response of the material at small strains.

As a final point worth of discussion, it appears from the results shown in Fig.2 and 3, that while any of the curve-fitted relationships is more or less adequate to represent the experimental pressure-expansion data, the derived stress-strain curves are however all quite different. This represents a formidable task for the geotechnical engineer because the derived stress-strain curve is dependent upon an assumed relationship for the pressure-expansion curve. It is thus impossible to make an objective assumption, even if one makes use of statistical methods. The difficulty is linked to the fact that the stress-strain curve is obtained from the differentiation of the pressure-expansion relationship. However, the reverse problem, that is, the task of obtaining the pressure-expansion curve from a known stress-strain relationship, is much simpler, because of the integration procedure.

5. CONCLUSIONS

This technical paper presents a method to obtain the stress-strain curve of clay from undrained plane-strain pressuremeter tests. The experimental radial pressure versus shear strain curve is approximated by an inverse hyperbolic sine function. The resulting stress-strain curve can be described as a non-linear workhardening soil model, having a finite modulus at the origin.

Because the stress-strain curve was obtained using Palmer’s approach, it was not necessary to separate the soil response into elastic and plastic components. Compared to the stress-strain curve based upon a power law representation, that obtained in this study was stiffer.

It was also found that a simple hyperbolic stress-strain curve resulted in a much stiffer response compared to both the inverse hyperbolic sine law and the power law representations. As for the simple linearly elastic perfectly plastic (Tresca) response, it failed to capture the pronounced non-linear stress-strain behaviour at small strains.

6. ACKNOWLEDGMENTS

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7. REFERENCES


Table 1: Results of regression analyses on inverse hyperbolic sine law.

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*Regression line forced to pass through the origin.

Table 2: Results of regression analyses on power law approximation.

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Table 3: Results of regression analyses on simple hyperbolic law approximation

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Table 4: Results of linearly elastic perfectly plastic response approximation

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