LANDSLIDE DYNAMIC ANALYSIS IN 2D AND 3D
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ABSTRACT
Continuum dynamic modelling of landslide motion is a useful tool in landslide risk assessment and mitigation. Both 2D and 3D models are available. The latter do not require the input of a pre-determined path direction and width, more accurately account for energy losses caused by complex topography and account explicitly for lateral variations in intensity, but are not as efficient or as simple to use. In this paper, the main similarities and differences between the 2D model DAN and its 3D counterpart DAN3D are highlighted, with reference to a back-analysis of a historical rock avalanche in the Swiss Alps. These similarities and differences, which likely extend to other landslide continuum dynamic models, are important to consider when selecting a model for a given application and when interpreting the results.

RÉSUMÉ
La modélisation dynamique des mouvements de terrain à l’aide de la mécanique des milieux continus est un outil utile en matière d’évaluation des risques et de leur réduction. Il existe des modèles 2D ainsi que 3D. Ces derniers ne nécessitent pas d’information préalable concernant le chemin suivi par l’éboulement (direction et largeur), prennent mieux en compte les pertes d’énergie dues à la complexité de la topographie, et tiennent explicitement compte des variations latérales d’intensité, mais ils ne sont pas aussi efficaces et aussi simples d’utilisation que les 2D. Dans cet article, les principales similitudes et différences entre le modèle 2D DAN et son équivalent 3D, DAN3D, sont mises en avant, en s’appuyant notamment sur une analyse rétrospective d’un cas historique d’éboulement rocheux dans les Alpes suisses. Il est important de prendre en considération ces différences et points communs, qui peuvent probablement s’appliquer à d’autres modèles dynamiques de mouvements de terrain employant l’hypothèse de la mécanique des milieux continus, lors du choix de l’utilisation d’un modèle pour une application particulière et lors de l’interprétation des résultats.

1. INTRODUCTION
A number of continuum dynamic models have been developed for the simulation of extremely rapid landslide motion, and they can provide valuable quantitative information for use in landslide risk mapping and the design of countermeasures. Models capable of simulating motion along a 2D path or across a 3D surface are available. The 2D model DAN (Hungr 1995) includes features to account for the most important characteristics of rapid landslides, including the effects of internal strength, material entrainment and variations in rheology both within the slide mass and along the path. DAN is a calibration-based model, which means that the appropriate rheological parameters must be constrained by trial-and-error back-analysis of previous real landslides. The trials are judged in terms of their ability to simulate the bulk characteristics of a prototype event, including the total travel distance, the distribution of deposits and the velocities estimated along the path. A large number of case studies have been analyzed and a preliminary database of calibrated parameters has been created (e.g., Hungr et al., 2005).

A recently-developed 3D extension of this model, DAN3D (McDougall and Hungr 2004 and 2005), retains the key features of the original while addressing its inherent limitations; it does not require the input of a pre-determined path direction and width, more accurately accounts for energy losses that occur due to changes in direction and confinement, and accounts explicitly for lateral variations in intensity (e.g., flow depth and velocity). These additional capabilities come at the expense of some model transparency and computational efficiency. However, comparative analyses using both models with the same set of input parameters consistently demonstrate good correspondence, with important practical implications. First, these results suggest that DAN is a useful tool for preliminary calibration of the new model, and that the existing database of DAN-calibrated parameters remains a useful reference. This can improve analysis efficiency in individual cases, but may also reduce the amount of time and effort required to obtain a thorough general calibration of the new model. Second, they suggest that 3D capabilities may not always be necessary, particularly when the path direction and width of the landslide in question are known or predictable, as well as approximately constant. In such cases, faster and simpler 2D analyses may be sufficient.

Although these findings are based only on analyses using DAN and DAN3D, it is likely that they apply to other, similar landslide continuum dynamic models as well. The purpose of this paper is to highlight the main similarities and differences between DAN and DAN3D and, by extension, 2D and 3D models in general. A sample back-analysis of a historical rock avalanche is used to demonstrate the typical good correspondence between the two models and practical implications are discussed.
2. THE DYNAMIC MODELS DAN AND DAN3D

Landslides are extremely complex phenomena that are very challenging to model. Both DAN and DAN3D are based on a simplifying approach formalized by Hungr (1995) but used tacitly by many workers. Instead of accounting explicitly for the complex micro-mechanics, the landslide material is modelled as a hypothetical material that is governed by simple rheological relationships. As mentioned previously, the required rheological parameters are not measured but are calibrated through back-analysis, and therefore represent apparent, rather than actual, material properties.

Both models are based on continuum numerical solutions of the depth-averaged, Lagrangian equations of motion for this so-called “equivalent fluid” (Hungr 1995). In the 3D model, the depth-averaged momentum balance is solved in two directions, which are tangent to the local sliding surface and orthogonal to each other. Using a local, right-handed, Cartesian coordinate system \((x,y,z)\), in which \(z\) is aligned with the local bed-normal direction and \(x\) is aligned with the local direction of motion, the \(x\) (longitudinal) and \(y\) (lateral) momentum balance equations used in the current version of DAN3D are, respectively:

\[
\rho \frac{D \bar{v}_x}{Dt} = \rho g_x - k_x \sigma_{z(x,b)} \frac{\partial h}{\partial x} + \tau_{x(x,b)} - \rho \bar{v}_x E_x
\]

\[
\rho \frac{D \bar{v}_y}{Dt} = \rho g_y - k_y \sigma_{z(x,b)} \frac{\partial h}{\partial y}
\]

where \(\rho\) is the material bulk density (assumed to be spatially and temporally constant), \(h\) is the bed-normal flow depth, \(v_x\) and \(v_y\) are flow velocities (overbars denote depth-averaged values), \(g_x\) and \(g_y\) are bed-parallel components of the gravitational acceleration vector, \(\sigma_{z(x,b)}\) is the total bed-normal stress at the base of the flow, \(k_x\) and \(k_y\) are stress coefficients (where \(k_x = \sigma_x/\sigma_z\) and \(k_y = \sigma_y/\sigma_z\)), \(\tau_{x(x,b)}\) is the basal shear stress (always negative in this reference system), \(E_x\) is the positive “erosion velocity” (Takahashi 1991) and \(t\) is time.

The terms on the left side of Equations [1] and [2] represent local depth-averaged accelerations of a moving reference column of material (multiplied by the mass of the column per unit basal area). The first two terms on the right side represent gravity and normal stresses, respectively. The third and fourth terms on the right side of Equation [1] represent the basal shear stress and the momentum flux due to entrainment of path material, respectively. These terms do not appear in Equation [2] because of the chosen orientation of the reference coordinate system. Transverse shear stress terms do not appear in either equation because, in the current version of DAN3D, it is assumed that the lateral direction corresponds with one of the 3D principal stresses.

In DAN3D, Equations [1] and [2] are discretized using a numerical method adapted from Smoothed Particle Hydrodynamics (e.g., Monaghan 1992) in which mass balance is satisfied by interpolation, based on the known positions of a collection of moving reference masses. These reference masses, or “smoothed particles”, are each fixed to a reference column and advected with the flow according to updated velocities obtained by numerical integration of Equations [1] and [2].

In the 2D model DAN, the depth-averaged momentum balance is solved in only one direction, which is tangent to the local user-specified 2D path. This approach is equivalent to neglecting lateral bed and free surface gradients in DAN3D (as well as transverse shear stresses) and aligning \(x\) with the downslope direction, which in this case corresponds with the assumed direction of motion. With these assumptions, Equation [1] also represents the momentum balance equation used in DAN, where \(g_x = g \sin \alpha\) and \(\alpha\) is the inclination of the local path from horizontal. In contrast to DAN3D, DAN uses a traditional Lagrangian finite difference method, in which mass balance is maintained between pairs of moving reference slices, to discretize Equation [1] (Figure 1).

Both models approximate the total bed-normal stress at the base of the flow using:

\[
\sigma_{z(x,b)} = \rho h \left( g \cos \alpha + \frac{\bar{v}_z^2}{R} \right)
\]

where \(R\) is the bed-normal radius of curvature of the path in the direction of motion. Both models also increment the internal stresses in proportion to the estimated internal
strains, with the resulting stress coefficients limited by a frictional internal yield criterion, based on classical Rankine earth pressure theory (Savage and Hutter 1989). The stress redistribution algorithm used in DAN3D gives priority to the direction of motion, which is the same assumption used implicitly in DAN and similar 2D models.

Similar methods are also used to simulate volume changes during entrainment. In both models, the depth of erodible bed material is specified by the user and entrainment can proceed until the available material is exhausted. In DAN, the entrainment rate is adjusted automatically by the model to ensure that the material is exhausted only after the entire landslide has passed a given location. Such a method, which requires a predefined path width, is not possible in the 3D model. Instead, a volume growth rate must be specified by the user, but can be adjusted manually to produce behaviour similar to the 2D model. Rheology changes, typically coinciding with entrainment, can be implemented in either model, both of which allow the selection of a variety of rheological relationships governing $\tau_{\text{BCD}}$. 

Finally, both models output the simulated distribution of intensity along the landslide path, information that is required for risk assessment. In DAN3D, the resulting distribution is spatial, rather than simply linear. This additional information is useful, but relatively more difficult to process and present.

In summary, despite their inherent differences as 2D and 3D models, DAN and DAN3D are actually very similar. The following back-analysis is an example of the resulting typical good correspondence between the two models.

3. BACK-ANALYSIS OF THE SIX DES EAUX FROIDES ROCK AVALANCHE

3.1 Event description

On May 30, 1946, a magnitude 4.4 earthquake triggered a rock avalanche in the Andins Valley in Valais, Switzerland (CREALP 2001). The event initiated as a slide in limestone on the south face of an eastern arête of Six des Eaux Froides. Normal faults dipping at about 40 to 43º to the south formed the main failure surface between elevations 2300 and 2700 m (CREALP 2001).

The slide mass overrode talus on the source slope and impacted Lake Luchet and its surrounding marshy areas on the valley floor. Rapid undrained loading (e.g., Sassa 1985) and entrainment of this wet and weaker path material probably contributed to the relatively high mobility of the event, which had a fahrböschung of only 16º (CREALP 2001).

A ridge in the valley, oriented perpendicular to the initial direction of motion, deflected the flow both westward (up-valley) and eastward (down-valley). The eastern distal flow travelled about 1.5 km from the toe of the source slope and formed a relatively shallow, muddy deposit (CREALP 2001). Secondary rock falls and slides may have contributed to the deeper and coarser proximal deposit, although the timing and total duration of the event are unknown (CREALP 2001). An oblique aerial view of the landslide area is shown in Figure 2.

![Figure 2. Oblique aerial view of the Six des Eaux Froides rock avalanche (looking northwest). Photograph courtesy of Raphaël Mayoraz.](image_url)

3.2 Analysis methodology

Pre and post-event digital elevation models (DEMs) were provided. The source depths were approximated by subtracting the post from the pre-event DEM, filtering negative results, and isolating the probable main failure zone. The resulting source volume of approximately 4.2 Mm$^3$ was bulked 20% to account for fragmentation, giving a total slide volume of approximately 5 Mm$^3$. The initial conditions are shown in Figure 3.

The event was back-analyzed first using the 2D model DAN. To simulate the division of flow up and down the valley, two separate analyses were performed along path profiles corresponding to the two main streamlines of motion, denoted by (a) and (b) in Figure 3. In similar proportion to the real event, slide volumes of 2 Mm$^3$ and 3 Mm$^3$ were used in cases (a) and (b), respectively. Path widths in each case were estimated from post-event aerial photographs.

The following Voellmy (1955) basal resistance relationship was used:

$$ r_{\text{BCD}} = -\left(\sigma_{\text{BCD}} f + \frac{\rho g y}{\zeta}\right) $$

where $f$ is the friction coefficient and $\zeta$ is the so-called turbulence parameter. The first term on the right side of Equation [4] accounts for any frictional component of resistance, while the second term, which has the same form as the Chézy equation for turbulent resistance,
accounts empirically for any possible velocity-dependence.

The two Voellmy parameters were systematically adjusted until both simulations approximately reproduced the observed: 1) total travel distance, and 2) distribution of deposits. Better constraint of the input variables would have been possible with independent flow duration and velocity estimates, but none were available. Nevertheless, the need to satisfy the above two calibration criteria in both cases provided reasonably tight constraint.

A preliminary DAN3D simulation using the entire 5 Mm$^3$ slide volume and the same calibrated Voellmy parameters was then performed. In both the DAN and DAN3D cases, an internal friction angle of 35º was used to limit the internal stresses. This value is considered appropriate for the dry, granular rock avalanche material that comprised the bulk of the main mass, although neither model is very sensitive to this parameter within a reasonable range. Volume changes, which were likely small relative to the source volume, were neglected.

The results of the two DAN analyses are shown in Figure 4. The best simulations of the total travel distance and distribution of deposits in cases (a) and (b) were obtained using a friction coefficient of $f = 0.13$ and a turbulence parameter of $\xi = 450$ m/s$^2$. These values are very close to the values of $f = 0.1$ and $\xi = 500$ m/s$^2$ recommended by Hungr and Evans (1996) on the basis of DAN back-analyses of 23 other rock avalanches. In case (b), the main proximal deposit and the long and relatively thin distal deposit were reproduced, in approximately the same proportion as estimated by CREALP (2001). The total duration of case (b) was just over 150 s, with frontal velocities up to 87 m/s recorded near the toe of the source slope. Both of these values are within reason for a long-runout rock avalanche of this size.

3.3 Analysis results

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The results of the preliminary DAN3D simulation using the same input parameters ($f = 0.13$ and $\xi = 450$ m/s$^2$) are shown in Figure 5. Good correspondence between these results, the real event and the preceding DAN results is evident. DAN3D correctly reproduced the relatively thick proximal deposits and division of flow caused by the ridge in the valley. The subsequent down-valley, distal flow was also simulated, although it did not come completely to rest within 150s (the flow front was still moving at about 8 m/s at this time). The reason for this is that the slope gradients in the distal path remained higher than the specified friction coefficient, in contrast to the flatter distal path used in DAN case (b), which did not follow the direction of steepest descent. This sensitivity to slope gradient is an important characteristic of the Voellmy model and other basal resistance relationships that include a frictional term.

The simulated runup against the ridge was also too high (some overtopping occurred, which was not observed in the real event). This could have been caused by inaccuracy of the input sliding surface, as smoothing of topographic details occurred during processing of the DEMs and may have reduced the steepness and/or height of the ridge. It is also possible that the simulated velocities were too high, perhaps in part because the basal resistance was too low. But besides this discrepancy, a reasonable simulation of the extent of the impact area and the distribution of material within it was produced, in good agreement with the calibrated DAN results.
4. DISCUSSION

It is important to emphasize that no attempt has been made to optimize the input based on the preliminary DAN3D results. As part of a more detailed back-analysis using DAN3D, the observed runup could be used as an additional calibration criterion. In this case, a slight decrease in the specified turbulence parameter may reduce the simulated velocities enough to prevent overtopping of the ridge. Alternatively, a slight increase in the friction coefficient may also reduce the simulated runup and, at the same time, allow deposition to occur in the distal path. It may also be possible to improve the simulation by accounting explicitly for changes in flow character caused by different path materials, most notably the valley floor deposits. For example, a change from purely frictional to Voellmy resistance may be appropriate as the flow transitions from dry to wet and coarse to fine after the onset of entrainment (e.g., Hungr and Evans 2004).

In practice, significant efficiency can be gained by taking advantage of the correspondence between the two models and using them together in a complementary manner. Preliminary input parameters can be selected from the existing database of calibrated values. DAN3D can then be used to estimate the path direction and width, and DAN can in turn be used to rapidly adjust the resistance parameters. This added efficiency is equally important for trial-and-error back-analysis and parametric forward-analysis, both of which require multiple model runs.

5. CONCLUSION

DAN and DAN3D are based on the same theory, use similar numerical solution methods and are capable of producing very similar results, as demonstrated in the case study presented in this paper. As such, their relative strengths must be considered when determining which model is more appropriate for a given application. DAN is more efficient, while DAN3D does not require the input of a pre-determined path direction and width. In many cases, both of these strengths can be exploited by using the two models together in a complementary manner.

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