Time-dependent ground surface subsidence due to room and pillar coal mining

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ABSTRACT
Long term room and pillar coal mine subsidence has been a challenge to predict. Ground surface subsidence can take place in tens of years in areas of abandoned mines where partial extraction was achieved. Long term or residual subsidence is caused by gradual increase of pillar stress due to diminishing of coal pillar strength and/or pillar dimensions with time as result of weathering, spalling and other factors. A Monte Carlo simulation method is proposed to address the reliability of subsidence prediction with time using subsidence monitoring data collected from different mines worldwide.

RÉSUMÉ
La prediction de subsidence à long terme des chambres et piliers a toujours été un défi scientifique. Le subsidence de la surface de terre peut avoir lieu dans des dizaines d’années dans les secteurs des mines abandonnées où une extraction partielle a été réalisée. Le subsidence à long terme, ou résiduel, est provoqué par l’augmentation progressive de l’effort de pilier provoqué par la diminution de la force de pilier de charbon et/ou des dimensions de pilier avec le temps, résultant de facteurs de désagrégation, de délitescence ou des autres facteurs. Ici, Une méthode de simulation du Monte-Carlo est proposee pour adresser la fiabilité de la prévision de subsidence avec le temps en utilisant des données de surveillance des subsidences rassemblées de mines différentes dans le monde entier.

1 INTRODUCTION
Subsidence is a predicted consequence of underground mining, and can be small and localized or extend over a large surface area. Subsidence can occur immediately or be delayed for many years. Subsidence is a function of many interrelated factors including, for example, mine geometry, coal mechanical properties, overburden depth and percentage of hard rock, existence of weak seams and parting, and rate of coal and parting deterioration (Hartman et al. 1992).

Researchers have inspected and monitored abandoned mines and observed residual subsidence several years to decades after completing room and pillar mine-work. For instance, Ivey (1978) observed subsidence in Colorado 73 years following mine abandonment. Gray et al. (1977) and Bauer and Haunt (1981) concluded that residual subsidence occurred over a period of decades and up to 100 years in Pennsylvania and Illinois, respectively. Also, Van der Merwe (1993) found in South Africa that residual subsidence occurred over several decades, up to 50 years. He predicted a pillar deterioration rate as a function of a mine height. Tsang et al. (1996) conducted a survey of room and pillar mines in the eastern Appalachian region and found that some of the coal seams contain one or more mudstone or claystone layers with variable thicknesses. They inspected coal pillars as old as 100 years and reported several kinds of coal and parting weathering with varying degrees of severity.

Biswas et al. (1999) determined the time-dependant strength of coal pillars in the field using a borehole penetrometer. They tested coal pillars developed 5, 15 and 50 years before the time of their study. A nonlinear regression was performed on in-situ collected strength of coal and parting as dependent variable and time and depth as independent variables. Biswas et al. recommended the formulas below to predict the reduction of coal and parting strength.

\[
\% \text{ parting strength} = 100 (1.01 - e^{-0.5D}) - 0.45t \quad [1]
\]

\[
\% \text{ coal strength} = 100 (1.01 - e^{-3.5D}) - 13t \quad [2]
\]
in which D is depth into rib in feet (1 m = 3.28 ft) and t is time in years after mining. In these equations, the strength is defined as a percent of the original strength near a pillar core.

In this study, a Monte Carlo simulation method is proposed to address the reliability of subsidence prediction with time and the corresponding probability of failure of a pillar. The method is a practical tool to assess the potential risk of room and pillar coal mining residual subsidence.

2 RELIABILITY ANALYSIS
A subsidence reliability index and the corresponding probability of pillar failure are determined using pillar load and resistance. To accomplish this, a limit state equation is developed that incorporates and relates together the variables that affect pillar stability. The parameters of load and resistance are considered as random variables. The limit state equation is (Allen et al. 2005):

\[
g = R - Q \quad [3]
\]
in which g = the random variable representing the safety margin; R = the random variable representing resistance; and Q = the random variable representing load. Figure 1
shows \( g \) (the difference between \( R \) and \( Q \)) and the probability of failure of pillars, \( P_f \). In Figure 1, the load and resistance are normally distributed and failure is represented by the zone where the load and resistance distributions overlap. The hatched area under the curve equals the probability of failure of pillars (\( P_f \)). \( P_f \) is typically represented by the reliability index, \( \beta \), which represents the number of standard deviations of the mean of \( R - Q \) to the right of the origin.

Figure 1. Probability of failure and reliability index

2.1 Development of Limit State Function

The limit state function (\( g \)) was developed using \( Q \) and \( R \) (Allen et al. 2005), as follows:

\[
Q = Q_n \times \lambda_Q \tag{4}
\]

\[
R = R_n \times \lambda_R \tag{5}
\]

in which \( \lambda_Q \) is the load bias (measured load / predicted load = \( Q_m / Q_p \)) and \( \lambda_R \) is the resistance bias, (measured resistance / predicted resistance = \( R_m / R_p \)). \( Q_n \) is the nominal load (predicted pillar vertical stress, \( S_v \), as a function of a pillar tributary area) and is determined as follows:

\[
S_v = \frac{\gamma H (W + B)(L + B)}{WL} \tag{6}
\]

in which \( \gamma \) is the overburden unit weight, \( H \) is a seam depth, \( W \) is a pillar width, \( L \) is a pillar length and \( B \) is an entry width. \( R_n \) is the nominal resistance (predicted pillar strength, \( S_p \)), according to, for example, Bieniawski (1992):

\[
S_p = S_1 \left(0.64 + 0.36 \frac{W}{h}\right) \tag{7}
\]

in which \( S_1 \) is an in-situ seam strength and \( h \) is a seam height. The statistical properties (mean, \( \mu \), and standard deviation, \( \text{Stdev} \)) of \( \lambda_Q \) and \( \lambda_R \) represent \( \mu \) and \( \text{Stdev} \) of \( Q \) and \( R \), respectively.

Gale (1999) reported in-situ pillar stress measurements at different mine conditions and geometry and their corresponding predicted stresses. Statistical properties of \( \lambda_Q (Q_m / Q_p) \) are shown in Table 1. Figure 2 indicates agreement between measured and predicted pillar stresses except for few scattered points.

Measured in-situ strength values of coal pillars (\( R_m \)) with rock contact (no parting), and clay contact (parting) were reported by Mark et al. (1988) and Maleki (1992) for U.S. mines and Gale (1999) for Australian and U.K. mines. Predicted strength of coal pillars (\( S_p = R_p \)) was determined using Equation 7 adopting \( S_1 = 6.5 \text{ MPa} \) (Gale 1999). Statistical properties of \( \lambda_R (R_m / R_p) \) are summarized in Table 1. Figure 3 compares measured and predicted coal pillar strength values. Figure 3 indicates that typically \( R_m \) is less than \( R_p \) for coal pillars with parting. However, it depicts that \( R_m \) is greater than \( R_p \) for coal pillars without parting.

Normal and lognormal statistical distributions are typically used in a reliability analysis of geotechnical data. Using a Monte Carlo analysis requires predetermination of the statistical distribution of the data. The Shapiro-Wilk test for normality (W test) was applied on the statistical data of \( \lambda_Q \) and \( \lambda_R \) (representing \( Q \) and \( R \)) to check the goodness of fit of each data set to a normal distribution (Shapiro and Wilk, 1965). The test statistic \( W \) ranges from \( 0 < W \leq 1 \), where \( 0 \) indicates no normality and \( 1 \) indicates normality. The W tests were performed using Lumenaut (2007) software, Version 3.4.21, which is compatible with Microsoft Excel. The normality W test results are shown in Table 1. The strength data are well represented by a normal distribution (\( W > 0.9 \)). The load data are moderately represented by a normal distribution (\( W \sim 0.5 \)). A lognormal distribution was fitted to the load data and also, resulted in a moderate \( W \) of approximately \( 0.6 \). A normal distribution was used for the Monte Carlo simulation analysis of the load and the resistance.

Figure 2. Comparison between measured and predicted coal pillar stress values (measured data from Gale 1999)
Table 1. Statistical properties of load bias and resistance bias.

<table>
<thead>
<tr>
<th>Bias for</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>COV</th>
<th>W statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>1.12</td>
<td>0.51</td>
<td>0.45</td>
<td>0.47</td>
</tr>
<tr>
<td>Strength - parting</td>
<td>0.55</td>
<td>0.12</td>
<td>0.22</td>
<td>0.97</td>
</tr>
<tr>
<td>Strength - no parting</td>
<td>1.20</td>
<td>0.25</td>
<td>0.21</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Figure 3. Comparison between measured and predicted pillar strength (measured data from Mark et al. 1988, Maleki 1992 and Gale 1999)

2.2 Monte Carlo analysis

Monte Carlo simulation was used to determine a pillar design reliability index and the related probability of failure with time. The analysis was performed for coal pillars with and without parting. An average mine geometry was adopted in the analysis using the following parameters: an overburden thickness (H) = 200 m, a mine height (h) = 2.5 m, and an entry width (B) = 6 m. The pillar width (W = L for square pillars) varied between 13 m and 37.5 m corresponding to a W/h ratio between 5.2 and 15 and a safety factor at the time of pillar development between 1.5 and 5.8. The percent of parting and coal strength reduction at t = 0, 25, 50, 75 and 100 years were estimated using Equations 1 and 2.

Cain (1999) recommended to maintain mine stability that the final pillar safety factor (strength) should be at least 70% of the original pillar safety factor (strength). Figure 4 shows the reduction of coal and parting strength with time. Figure 4 indicates that the coal strength was 88% of the original core strength after 100 years of mining. Therefore, the Monte Carlo analyses were conducted for coal pillars without parting at the above five time intervals. Figure 4 shows that the parting strength is reduced to 78% of the original strength after 50 years of mining. The parting strengths were 67% and 56% of the original strength after 75 and 100 years of mining, respectively. Therefore, the stability of coal pillar with parting was considered unsatisfactory after 75 years of mining. The Monte Carlo analyses were conducted for coal pillar with parting at 0, 25 and 50 years after mining. Cain’s strength reduction criterion was also used to determine an effective pillar width with time. Strength reduction was determined from a pillar edge to its core.

At a depth D into rib and at a time t, the strength becomes 70% of its original value. In this case, a pillar width is reduced by 2D. Table 2 summarizes the reduction of pillar width with time for pillars with and without parting.

Table 2. Reduction of pillar width with time.

<table>
<thead>
<tr>
<th>Time after mining (years)</th>
<th>Reduction in pillar width (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pillar with parting</td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>25</td>
<td>2.00</td>
</tr>
<tr>
<td>50</td>
<td>2.99</td>
</tr>
<tr>
<td>75</td>
<td>W1</td>
</tr>
<tr>
<td>100</td>
<td>W2</td>
</tr>
</tbody>
</table>

1 Pillar is unstable due to strength reduction to 67% of the original core strength.
2 Pillar is unstable due to strength reduction to 56% of the original core strength.

For each Monte Carlo run using Lumenaut (2007) software, 10,000 iterations (i.e., g’s) are generated. The iteration results are summarized in the form of a histogram and corresponding intervals are summarized in a table. A spreadsheet, including the actual 10,000 iterations, is also generated as an output. The g values are then sorted and ranked in an ascending order. The cumulative probability at each g, P(g), is calculated as (g rank/(10,000+1)). Then the standardized normal value (z) of a P(g) is calculated using the NORMSINV function in Microsoft Excel where $z = \text{NORMSINV}(P(g))$. The reliability index, $\beta$, is equal to -z at g = 0. The probability of failure, $P_f$, is equal to the number of gs < 0 divided by 10001.

For example, Figure 5 shows the Monte Carlo analysis results for two coal pillars, one with parting and the other without, where W/h = 6.4 at t = 0 years (immediately after mine development). The design safety factor for both pillars was 1.5. The coal pillar without parting has a reliability index, $\beta = 2.52$ and a corresponding probability of failure, $P_f = 0.6%$. However, the coal pillar with parting has $\beta = -0.02$ and $P_f = 51%$. The above results indicate that the pillar without parting is stable after mining with a...
relatively low probability of failure. However, the results indicate that the pillar with parting is likely to be unstable and larger pillar width should be used to ensure proper pillar performance.

Figure 6 indicates increase of $\beta$ with increase of W/h. Also, it depicts decrease of $\beta$ by increasing mine age. Figure 6 shows $\beta$ less than 2 for W/h up to 10 at t = 0. Figure 7 illustrates decreasing $P_f$ with increasing W/h. However, it shows a significant increase of $P_f$ with time due to degradation of parting strength. Figure 7 shows a probability of failure between approximately 10% and 90% for W/h less than 10. For W/h = 15, $\beta$ is equal to 2.36 at the time of mining and gradually decreased to 1.63 at $t = 50$ years. Also, the corresponding $P_f$ for W/h = 15 varies between approximately 1% at the time of mining and 5% at $t = 50$ years. Therefore, based on the reliability analysis results where parting exists, pillar dimensions should be relatively large (W/h > 10) to ensure pillar and mine stability and to reduce potential subsidence many years after mining.

Similarly, Monte Carlo analyses were conducted for coal pillars without parting. A pillar width to a mine height ratio (W/h) varied between 5.2 and 15 and the effect of time on the reduction of pillar strength was considered at $t = 0, 25, 50, 75$ and 100 years after mining. Figure 8 shows the resulting reliability indices of pillar stability for the above range of W/h and time intervals. A reliability index increases with increasing W/h and decreases with increasing exposure time of coal. Figure 9 depicts the probability of failure of pillars as a function of W/h and time after mining. The probability of failure dramatically decreases by increasing pillar dimensions. For example, $P_f$ at $t = 0$ decreases from almost 8% (W/h = 5.2) to 0.6% (W/h = 6.4).

For W/h = 5.2 (design safety factor = 1.5), $\beta$ is equal to 1.41 at $t = 0$, and after 100 years of mining, $\beta$ is equal to 0.86. The corresponding values of $P_f$ are almost 8% and 20% at $t = 0$ and 100 years, respectively. Due to the relatively high probability of failure and small $\beta$ values, it is recommended to have pillars with W/h greater than 5.2 to ensure pillar and mine stability many years after mining. For W/h = 6.4 (design safety factor = 2), $\beta$ is equal to 2.52 at $t = 0$ and reduces to 2.05 at $t = 75$ years and then 1.88 at $t = 100$ years. The corresponding probability of failure varies between almost 0.6% at $t = 0$ and 3% at $t = 100$ years. The stability and performance of coal pillars with W/h = 6.4 is generally acceptable over a period of 100 years after mining. For W/h ≥ 8, the reliability indices are relatively high ($\beta > 3.4$ at $t = 100$ years) and the probability of failure is relatively low ($P_f \leq 0.001\%$ at $t = 100$ years).

3 CONCLUSIONS

Long term room and pillar coal mine subsidence is assessed in this study. The long term stability of a coal pillar is a function of mine geometry, pillar dimensions, existence of parting, degradation of pillar strength in years after mining, pillar stress bias and pillar strength bias. The reliability of pillar stability with time and the corresponding probability of failure were determined adopting the above factors in a Monte Carlo simulation model. An average mine geometry and coal properties were adopted in the analysis. A pillar width to a mine height varied between 5.2, minimum safety factor = 1.5, and 15. The reliability of long term pillar stability was evaluated at 0, 25, 50, 75 and 100 years after mining.
The analysis results were summarized in charts correlating pillar stability reliability index and corresponding probability of failure with W/h and time after mining. The charts can be used as a guideline to select minimum pillar dimensions to ensure pillar long term stability at a target reliability index, $\beta$ (typically $\beta > 2$). For the selected mine geometry and coal properties in this study, W/h for a coal pillar with parting is recommended to be greater than 10 to ensure mine stability over approximately 50 years after mining.

Coal pillars with parting are likely to become unstable after 50 years of mining. For coal pillars without parting, it is recommended to have a pillar with W/h $> 6.4$ to ensure mine stability over approximately 100 years after mining. The proposed assessment reliability method can be used in assessing the long term stability of a mine using representative geometry and material properties.
REFERENCES


