Numerical simulations to assess the stress state in backfilled stopes with inclined walls

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ABSTRACT
Adding backfill in mine stopes improves ground stability and reduces the volume of solid wastes disposed on the surface. To ensure the safe use of backfill, practical tools are required to assess the stress state in the stopes and on support structures. Previous work has shown that these stresses are influenced by an arching effect related to the stress transfer between the soft fill and stiff rock mass. However, in inclined stopes, the shear stresses developing along the walls are not the same on each side of the opening, so the arching effect is typically less well developed than in vertical stopes. In this article, 2D and 3D numerical simulations results are presented to illustrate the interaction between the backfill and rock mass in stopes with inclined walls. The influence of strength parameters of the backfill and of stope geometry is examined and discussed.

1 INTRODUCTION
Underground mining involves the creation of large excavations to extract the ore. These stopes are often backfilled.

Rock fill, hydraulic fill and cemented paste backfill are the most popular filling technologies currently used in mines (e.g. Hassani and Archibald, 1998; Liston, 2014). Such backfills are generally much less stiff than the surrounding rock mass. After deposition in the underground stope, the self weight settlement of the fill material generates shear stresses along the rough walls of the opening, which act to transfer some of the overburden pressure to the rock mass (e.g. Askew et al., 1978; Aubertin et al., 2003; Li et al., 2003, 2005). Analytical and numerical solutions can be used for estimating the stresses in backfilled stopes. The majority of existing solutions have been developed for vertical openings (e.g. Aubertin et al., 2003; Li et al., 2005; Li and Aubertin, 2008, 2009b; Pirapakaran, 2008). In reality, most stopes have inclined walls. Previous studies have shown that in these latter cases, the stresses developing along the hanging wall and footwall are different (Li et al., 2007; Li and Aubertin, 2009a; Jahanbakhshzadeh, 2015).

In this article, the authors investigate the stresses in vertical and inclined stopes. New 2D and 3D numerical simulations of backfilled stopes with inclined walls are presented and discussed.

2 NUMERICAL SIMULATIONS
Numerical analyses are very useful and powerful to assess the response of underground openings. These have been commonly used to evaluate the behavior of backfilled stopes (e.g. Aubertin et al., 2003; Li et al., 2003, 2005; Pirapakaran and Sivakugan, 2006; Falaknaz et al., 2015a, 2015b). Additional simulations have been conducted in this investigation to evaluate the stresses in stopes with walls having different inclination angles, considering various sizes and backfill properties (under a dry condition).

Figure 1 shows the conceptual model of an inclined backfill stope, which is based on previously conducted 2D simulations (from Li and Aubertin 2009a) using the code FLAC (Itasca, 2002). This is a finite difference program that uses a Lagrangian calculation scheme and a mixed discretization zoning technique. This program can...
simulate geotechnical problems with several stages, including sequential excavation and filling of mine stopes.

Figure 1. Conceptual model for the numerical simulations of inclined backfilled stopes (not to scale)

Figure 2 shows a typical model with the mesh used for the 2D numerical simulations (i.e. \( L \gg B \)).

For both 2D and 3D simulations, the elements are smaller inside and near the stope to obtain more accurate results. The number of elements depends on the stope geometry and model size. As the global size of the model can affect the simulations results, the external boundaries have been placed far enough to avoid any influence on the stresses obtained inside and near the walls of the backfilled stope. This assessment was performed for each simulation for the various cases identified in Table 1, with different stope sizes, walls inclination angle and backfill properties. For instance, the model constructed for the stope with walls inclined at \( \beta = 80^\circ \), with \( B = 6 \) m and \( H = 45 \) m (Case 0, in 2D simulations illustrated in Figure 2), the boundaries have been placed 227 m from the bottom and top of the stope in the vertical direction and 250 m from the center of inclined stope in the horizontal direction. It was verified that increasing the size of this model would have no effect on the simulation results.

The boundary conditions applied to the numerical models are also shown in Figure 2. Horizontal displacements are prevented on both sides and the vertical displacements are prevented at the base of the model.

Gravity is applied as the initial condition. The excavation is then created in one step. After elastic convergence of the walls, filling of the stope is performed in 6 steps (layers) for 2D simulations and in 4 steps for 3D simulations, to minimize the effect of adding the fill too quickly (Li and Aubertin, 2009a).

The stope is filled to a final height of 44.5 m, with 0.5 m of void space left at the top of the stope (i.e. total height of the opening \( H = 45 \) m).

In these simulations, the inclination of the stope walls \( \beta \) varies from 90° (vertical) to 60° with respect to the horizontal axis (for the different cases).

The rock mass is considered homogeneous, isotropic and linearly elastic, with the following basic properties: \( E_r = 30 \) GPa (Young's modulus); \( \nu_r = 0.3 \) (Poisson's ratio) and \( \gamma_r = 27 \) kN/m\(^3\) (unit weight).

The backfill obeys an elasto-plastic law with the Coulomb yield criterion. Its properties are described by the values of \( E \) (Young's modulus), \( \nu \) (Poisson's ratio) and \( \gamma \) (unit weight), with the internal (constant volume) friction angle \( \phi' \); cohesion \( c' = 0 \) kPa and dilatation angle \( \psi' = 0^\circ \) (i.e. with a non-associated flow rule). As there is no water pressure in these cases, the effective and total stresses (and related parameters) are the same.

Table 1 gives the geotechnical and geometric parameters used for the different simulations; the backfill properties are mainly based on data taken from Belem et al. (2000, 2004), Belem and Benzaazoua (2008) and El Mkadmi (2012).

Tableau 1. Parameters used for the different simulations with FLAC and FLAC 3D, including stope size and material properties (with \( H = 45 \) m; unit weight \( \gamma = 18 \) kN/m\(^3\), Young’s modulus \( E = 300 \) MPa)

<table>
<thead>
<tr>
<th>Backfill properties and stope geometry</th>
<th>Case 0</th>
<th>Case I</th>
<th>Case II</th>
<th>Case III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson’s ratio ( \nu' )</td>
<td>0.33</td>
<td>0.33</td>
<td>0.26</td>
<td>0.33</td>
</tr>
<tr>
<td>Internal friction angle ( \phi' ) ( (^\circ) )</td>
<td>30</td>
<td>30</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>Stope width ( B ) (m)</td>
<td>6</td>
<td>20</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Inclination angle ( \beta ) ( (^\circ) )</td>
<td>60, 70, 80, 90</td>
<td>60, 90</td>
<td>60, 90</td>
<td>60</td>
</tr>
<tr>
<td>Stope length ( L ) (m)</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>12</td>
</tr>
</tbody>
</table>

\( \nu' \approx \phi' \) (see eq. 1)
The following relationship was applied to relate the value of Poisson’s ratio ($\nu$) and the internal friction angle $\phi'$ of the backfill:

$$\nu = \frac{1 - \sin \phi'}{2 - \sin \phi'}$$  \[1\]

Equation 1 is used for consistency, so a single value is obtained for the earth pressure coefficient at rest ($K_0$), based either on Jaky’s equation ($K_0 = 1 - \sin \phi'$) or on the relationship with the Poisson’s ratio ($K_0 = \frac{1}{2} - \frac{\nu}{1 + \nu}$), which are thus considered equivalent (e.g. McCarthy, 2007; Falaknaz, 2014). This approach differs somewhat from the one adopted in previous numerical analyses, where $\phi'$ and $\nu$ were considered independent from each other; it also leads to somewhat different results in some cases.

The simulations results are first used to illustrate the effect of stope geometry and backfill properties on the stress distribution. The simulations are the discussed.

3 SIMULATIONS RESULTS

Various cases have been analyzed. The main simulations results are presented in the following for cases where a single parameter is varied from one model to the other.

3.1 Effect of stope inclination

The first series of 2D simulations illustrates the influence of stope walls inclination $\beta$; the two walls are parallel in all cases. Case 0 (Table 1) gives the characteristics of a stope with $B = 6$ m and $H = 45$ m, with $\beta$ varying from 90° (vertical) to 60° (for $\phi' = 30°$).

Figure 3 shows iso-contours of the vertical stress within the backfill for $\beta = 90°$ (Fig. 3a) and 60° (Fig. 3b). It is seen that for the vertical stope (left), there is a significant stress transfer from the backfill to the rock mass abutments (associated with arching); the vertical stress are thus significantly lower than the overburden pressures, especially at depth.

Figures 3b show the iso-contours of the vertical stress for the inclined stope (with $\beta = 60°$). It can be seen that the stresses are not uniformly distributed along the width of the stope. The vertical stress is typically lower near the hanging wall than at the center and near the footwall. The difference in the stresses along the two walls tends to vary with depth.

Figure 4 shows the stresses along the width of the stope at mid-height ($H/2$) for an inclination angle $\beta$ that varies from 90° (vertical) to 60°. As the walls become more inclined, the vertical stresses (Fig. 4a) within the backfill tend to decrease close to the hanging wall and, to a much lesser extent, toward the footwall. The horizontal stresses (Fig. 4b) also change with the inclination angle, particularly near the footwall where they significantly decrease with angle $\beta$.

3.2 Effect of stope width B

Previous simulations results have shown that the stope width $B$ can greatly influence the stresses in vertical and inclined stopes (e.g. Li et al., 2003, 2007; Li and Aubertin, 2009a). Simulations have been conducted here to assess more specifically the effect of this factor, for the additional cases considered in this investigation. Again, the model size was adjusted to avoid significant influence on the simulation results; the external boundaries are thus placed far enough from the opening (with $B = 20$ m).

Figure 5 shows the vertical stress iso-contours in plane strain for a stope width $B = 20$ m (Case I, Table 1), for $\beta = 90°$ and 60°; this figure can be compared with Figure 3 for $B = 6$ m (Case 0) to assess the influence of width $B$. Figure 5a indicates that for this relatively wide vertical stope, the transfer of the stresses from the backfill
to the rock mass is less pronounced than for $B = 6\, m$ (Figure 3a), leading to a weaker arching effect.

The influence of inclination and width is seen in Figure 5b, for $\beta = 60^\circ$ (compared to Fig. 3b). It is observed that the vertical stresses are lower near the hanging wall than along the center line and near the footwall. When the inclined stope width is 20 m (Case 1), the vertical stresses along the hanging wall and footwall are larger than those obtained for $B = 6\, m$ (Fig. 3b).

These results show that an increase of the stope width can significantly raise the stress magnitude in the stope. These also show that the distribution along the width varies with $B$.

These results confirm that the arching effect is less pronounced when the stope width is larger. When the value of $B$ increases, the stresses tend to approach those due to overburden pressure (i.e. $\sigma_v = \gamma h$ and $\sigma_h = K_0 \sigma_v$). This tendency is well predicted by analytical solutions developed for vertical stopes (Aubertin et al., 2003; Li et al., 2003, 2005), but not as well for inclined stopes (Jahanbakhshzadeh, 2015).

The 2D simulations results shown here also indicate that when the stope walls are inclined, the stresses evolve somewhat differently along the hanging wall, central line (CL) and footwall when the width is changed. For instance, near the base of the inclined stope (i.e. depth $h = 44.2$) with $\beta = 70^\circ$, $\sigma_v = 0.198\, MPa$ along the CL for $B = 6\, m$ and $\sigma_v = 0.496\, MPa$ for $B = 20\, m$ (increase of about 150%); along the hanging wall, $\sigma_v = 0.060\, MPa$ for $B = 6\, m$ and $\sigma_v = 0.122\, MPa$ for $B = 20\, m$ (increase of about 100%); along the footwall, $\sigma_v = 0.141\, MPa$ for $B = 6\, m$ and $\sigma_v = 0.338\, MPa$ for $B = 20\, m$ (increase of about 140%).

### 3.3 Effect of internal friction angle $\phi'$ of the backfill

The next 3 figures show simulations results that illustrate the influence of the (constant volume, or ultimate) internal friction angle $\phi'$ of the backfill on the vertical and horizontal stresses, for $B = 6\, m$ and $\beta = 60^\circ$ and $90^\circ$ (see Table 1). Two values are considered: $\phi' = 30^\circ$ (and $\upsilon = 0.33$), as in the previous cases (Cases 0 and 1; see solid lines), and $\phi' = 40^\circ$ ($\upsilon = 0.26$) for Case II (dashed lines).

The results are presented along the CL (Figure 6), FW (Figure 7), and HW (Figure 8).

The results in Figure 6 indicate that the vertical stresses along the CL are not affected much by the variation of the internal friction angle $\phi'$ (accompanied by a change in the Poisson’s ratio $\upsilon$) for the range considered here. The Figure also shows that the horizontal stresses tend to decrease along the CL when the friction angle is increased. For instance, when the friction angle goes from $30^\circ$ to $40^\circ$ in the inclined stope with $\beta = 60^\circ$, the value of $\sigma_h$ decreases by up to $20\%$ (i.e. from 0.0784 to 0.0621 MPa) in the backfill near mid-height of the stope (Fig. 6).

A similar trend can be observed along the footwall of the backfilled stope (Figure 7). As the friction angle $\phi'$ increases from $30^\circ$ to $40^\circ$, the horizontal stresses decrease significantly. For instance, the horizontal stresses in the backfill with an inclination angle $\beta = 60^\circ$ decrease by about $25\%$ (from 0.061 to 0.045 MPa) near mid-height. Again, the vertical stresses (Fig. 7) change marginally with $\phi'$.

Along the hanging wall, the horizontal stresses (Fig. 8) in the backfilled stopes with $\phi' = 40^\circ$ are smaller by about $20\%$ near mid-height than those obtained for the internal friction angle $30^\circ$, for vertical ($\beta = 90^\circ$) and inclined stopes ($\beta = 60^\circ$). An increase of the friction angle does not affect the vertical stresses as much along the HW.
Figure 7. Stresses distributions along the FW for various internal friction angle $\phi'$ of the backfill for stopes with various inclination angles $\beta$ (90° and 60°); (Case II, Table 1).

These trends are in good agreement with the results obtained by Li and Aubertin (2009a) for inclined stopes and from Falaknaz et al. (2015b) for isolated vertical stopes.

3.4 Effect of stope length $L$

In reality, mine stopes have a limited length along horizontal plane. Hence, the stress state in stopes is often influenced by the four walls. Previous work has indicated that the stope length $L$ can greatly influence the stresses in stopes (e.g. Li et al., 2005; Li and Aubertin, 2009b).

Results from 3D simulations obtained with the finite difference program FLAC-3D are presented to illustrate the effect of length $L$ on the stresses in an inclined backfilled stope.

Figure 9 presents the iso-contours of the vertical stresses ($\sigma_v$) in the backfill stope with $\beta = 60^\circ$ for case III (Table 1, $B = 6$ m, $L = 12$ m, $H = 44.5$ m); the results are shown for the vertical plane of symmetry (center of the stope) that is parallel to the $x$-$z$ plane. It can be seen that the vertical stresses are not uniformly distributed along the width of the inclined stope.

This figure can be compared with Figure 3b for the 2D simulation (Case 0) to illustrate the effect of the length $L$ (third dimension). It can be seen that the vertical stresses along the CL, FW and HW are smaller for the 3D case than for the 2D simulation. For example, along the hanging wall, the stresses obtained from the 3D simulation are smaller by about 30% near mid-height of the inclined stope (with $\beta = 60^\circ$) than those obtained from the 2D numerical simulations. The results shown here (and others not presented) also confirm that the stresses transferred to the four walls exceed those along the two walls obtained in 2D.

Simulations results for different lengths $L$ (from 12 to 60m), with different inclination angles (not shown here) further indicate that a decrease in stope length reduces the vertical stresses (details given in Jahanbakhshzadeh, 2015). Hence, the arching effect is more pronounced when the stope length decreases. This aspect should be taken into account when $L$ is relatively small.
As stated above, the backfill properties used in the numerical simulations are based on the relationship between the values of $\phi^*$ and $\nu$ (expressed by Equation 1). The results obtained here indicate that the horizontal and vertical stresses are usually somewhat smaller than those obtained based on the independent values for $\phi^*$ and $\nu$ (obtained by Li and Aubertin, 2009a). More details and additional results related to this aspect are included in Jahanbakhshzadeh (2015).

Analytical solutions can also be used to assess the stresses in backfills stopes. Most analytical solutions have been developed to evaluate the stresses in vertical stopes (in 2D and 3D) (e.g. Aubertin et al., 2003; Li et al., 2005; Li and Aubertin, 2008, 2009b; Pirapakaran, 2008). A few solutions have also been proposed for the stress state inside inclined stopes (e.g. James et al., 2004; Blight, 2010; Caceres et al., 2006; Singh et al., 2009, 2011; Ting et al., 2011). Numerical results have been compared with the results given by such analytical solutions (results presented in Jahanbakhshzadeh, 2015). This comparison indicates that the applicability of the latter solutions is limited because they typically neglect the difference in the stress state along the central line, hanging wall and footwall.

Finally, many other factors, not considered here, can also influence the stress state in stopes, such as generation and dissipation of pore-water pressures (Li and Aubertin, 2009b, 2010; EL Mkadmi, 2012; EL Mkadmi et al., 2014) and excavation of neighboring stopes (Falaknaz et al., 2015a, 2015b); the interested readers are referred to these complementary investigations.

5 CONCLUSION

This article presents some of the results of a numerical investigation that illustrates the effect of the geometry and material properties on the stresses in vertical and inclined backfilled stopes. The results obtained with FLAC indicate that the stress transfer and arching effect in the stopes depend on many parameters, including the stope width and length, the inclination angle of the walls, and the internal friction angle of the backfill.

The results specifically show how the stress distribution in the opening varies with the stope inclination angle $\beta$. The simulations indicate that a decrease of the stope inclination angle $\beta$ (from 90° to 60°) leads to a significant decrease in the vertical stress along the hanging wall. However, the horizontal stress along the hanging wall can increase when $\beta$ is decreased.

The numerical simulations also indicate that the fill material properties have influence the stresses in inclined backfilled stopes. An increase of the fill internal friction angle $\phi^*$ (from 30° to 40°) leads to a decrease of the horizontal stresses along the hanging wall.

The code FLAC 3D also was used to evaluate the effect of the length $L$ on the stresses. It has been shown that the stresses are affected by the presence of the four walls when $L$ is not too large. The results in 2D and 3D can then be quite different.

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