A study on the efficiency of the hardening soil model for soft clay

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ABSTRACT
The stress resistant and deformation characteristics of fine-grained soils, especially soft clays are remarkably influenced by the soil softness. Therefore, it is important to utilize a model which can simulate the effects of this phenomenon accurately. A constitutive model must be able to balance among the number of parameters such as the process of parameters determination and the simplicity of the computational calculations. In the present research, the performance of the hardening model for soft soils is investigated. The main advantage of utilizing this model is reproduction of nonlinear behavior via mobilized shear strength and mobilized dilation parameters. The model has been calibrated by incorporating soft Bangkok clay data which have been derived from a CD triaxial test performed by previous researches. Due to extensive deformations of the soft clay, a function has been introduced in the calibration process to fit the model with the test data. For common engineering practices, where the problem is mainly solved by applying the shear stress, employing the mentioned function, leads to acceptable results.

Résume
Les caractéristiques de résistance aux contraintes et de déformation des sols à grains fins, en particulier les argiles molles, sont grandement influencées par la raideur du sol. Par conséquent, il est important d'utiliser un modèle qui peut simuler les effets de ce phénomène avec précision. Un modèle constitutif doit être capable d'équilibrer le nombre de paramètres tels que le processus de détermination des paramètres et la simplicité des calculs informatiques. Dans la présente recherche, la performance du modèle de durcissement pour les sols mous est étudiée. Le principal avantage de l'utilisation de ce modèle est la reproduction du comportement non linéaire par la force de cisaillement mobilisée et les paramètres de dilatation mobilisés. Le modèle a été calibré en incorporant des données d'argile molle de Bangkok qui ont été produites à partir d'essais triaxial CD effectué lors de recherches antérieures. En raison des déformations importantes de largile molle, une fonction a été introduite dans le processus d'étalonnage pour adapter le modèle aux données d'essai. Pour les pratiques d'ingénierie courantes, où le problème est principalement résolu en appliquant la contrainte de cisaillement, l'utilisation de la fonction mentionnée conduit à des résultats acceptables.

1 INTRODUCTION

Many constitutive models have been introduced to analyze soil behavior which can be simple or complex depending on the number and process of determining their parameters. More complex models usually lead to introducing more parameters, which determining them is often possible by carrying out complex soil mechanics experiments. On the other hand, in simple models, although fewer parameters are used, the accuracy of modeling is reduced.

Different constitutive models predict the soil deformation due to the applied stresses in different accuracies. To the current date, many constitutive models have been introduced from the simplest model (Hook Law) to the most complex ones (multi-surface constitutive model with kinetic hardening). The number of the parameters of simpler constitutive models are less which speed up the modeling calculations but can't predict the real behavior of the soil accurately in most problems. On the other hand, complex constitutive models analyze the soil behavior more precise, but are less practical in engineering problems due to the multicity of their parameters (Brinkgreve, et al., 2005).

Another disadvantage of complex constitutive models is that their parameters can't be obtained from common soil mechanics tests and special experiments must be carried out (Liu, et al., 2013). It is worth mentioning that despite introducing the most complex models, there is no constitutive model, which can accurately express the soil behavior in all conditions (Kok Sien Ti , et al., 2009)

There are various constitutive models to analyze the static behavior of soils but generally, they can be divided to five major sub-groups. (Surarak, 2010)

1- Elastic
2- Elastic-Perfect Plastic
3- Single-surface Strain Hardening Elasto-plastic
4- Two-surface Strain Hardening Elasto-plastic
5- Multi-surface Strain Hardening Elasto-plastic

The elastic models present the simplest stress-strain relation with only one stiffness parameter, which comply with the elasticity relations. In this group, Duncan, et al. (1970) presented a non-linear elastic constitutive model which leads to better results compared to the linear elastic model (Brinkgreve, et al., 2005).

The elastic-perfect plastic model, i.e. the Mohr-Coulomb model (Coulomb, 1773), is used in many usual engineering practices. In this group, although the formulation is simple, the results are acceptable.

Unfortunately, these models are not recommended for soft soils, because the volumetric strain and stress path can't be truly modeled. (Rowe, et al., 1983); (Addenbrooke, et al., 1997); (Karakus, et al., 2005)

The single-surface isotropic strain hardening constitutive models were the first models which could truly predict the behaviors of soft soils. The most elementary model of this section is the Cam-Clay constitutive model (Roscoe, et al., 1968). This model consists of an expandable ellipse which distinguishes the elastic and plastic behavior. Many models have been introduced by changing or correcting a part of the preliminary Cam-Clay model. (Karakus, et al., 2003); (Finno, et al., 1985); (Morena, et al., 2017)

The two-surface isotropic strain or kinematic hardening constitutive models have been introduced to consider the effect of the shear stress and principle stress together. The most prominent model of this group is "the hardening soil" model (Schanz, et al., 1999) which has been improved by (Vermeer, et al., 1984) model.

The Hardening Soil Model (Schanz, et al., 1999) can predict the soil settlements and deformations more accurately, especially in excavation practices (Benson Hsiung, et al., 2014). The model presented in this section has a high efficiency in modelling soil behavior. Therefore, many commercial softwares such as Plaxis, FLAC, and Abaqus support this model.

Multi-surface kinematic hardening constitutive models are the most recent models which can capture more complex soil behavior. In this category, the bubble model and the three-surface kinematic hardening model can be mentioned (Al Tabbaa, et al., 1989); (Wood, 1995); (Atkinson, et al., 1991). Complexity and multiplicity of the parameters of these models are the main limitation of them (Surarak, 2010).

Selecting a constitutive model must be done by establishing a proper balance between the considered stress paths for modeling, the number of parameters, the process of defining the parameters, and the simplicity of the computational calculations (Kok Sien Ti, et al., 2009).

In this study, the applicability of the hardening soil model for soft soils is investigated. An elastoplastic hardening relation has been employed where the internal friction and dilation angle of soil are evaluated by a function of the plastic shear strain. The following section expresses the formulation and hardening rules of the model.

2 CONSTITUTIVE MODEL EXPLANATION

2.1 Dependency of Stiffness to the Confining Stress

The stiffness dependency with the confining stress can be seen in almost all geotechnical materials. The following relationship, initially proposed by Duncan, et al. (1970), has been used to define this dependency.

$$E = K_e P_a \left(\frac{\sigma'_3}{P_a}\right)^n \qquad [1]$$

where
E = Young modulus
P_a = the atmospheric pressure
σ'_3= the effective confining stress
K_e & n = constants

This relationship has been widely utilized by many researchers for different types of materials (Escuder, et al., 2005).

2.2 Shear Yield Function

In theory, this model is very similar to the Mohr-Coulomb Model except that in this model the friction angle, cohesion, dilation angle, and the tensional strength change with the increase of plastic shear strain. The shear yield function of the model is as

$$f = \sigma'_1 - \sigma'_3 N_{\varphi_m} + 2c_m\sqrt{N_{\varphi_m}} \qquad [2]$$

$$N_{\varphi_m} = \frac{1+\sin\varphi_m}{1-\sin\varphi_m} \qquad [3]$$

where
φ_m = mobilized friction angle
c_m = mobilized cohesion
σ'_1= the effective principle stress
σ'_3= the effective confining stress
The hardening function given by Vermeer, et al. (1984), has been used to estimate the hardening behavior of soil, which is presented as

$$\sin\varphi_m = 2\,\frac{\sqrt{\varepsilon_s{}^p\varepsilon^f}}{\varepsilon_s{}^p+\varepsilon^f}\sin\varphi^p \qquad \varepsilon_s{}^p < \varepsilon^f \qquad [4]$$

$$\sin\varphi_m = \sin\varphi^p \qquad \varepsilon_s{}^p > \varepsilon^f \qquad [5]$$

where
φ^p = ultimate friction angle
$\varepsilon_s{}^p$ = plastic shear strain
ε^f = ultimate plastic shear strain respectively
In this model the flow rule is unassociated and the potential function is as following

$$Q^s = \sigma'_1 - \sigma'_3\,\frac{1-\sin\psi_m}{1+\sin\psi_m} \qquad [6]$$

where
ψ_m = the mobilized dilation angle
Instead of introducing the mobilized dilation angle as a function to the plastic shear strain Rowe (1963) has presented it as a function of the mobilized friction angle as

$$\sin\psi_m = \frac{\sin\varphi_m - \sin\varphi_{cv}}{1-\sin\varphi_{cv}\sin\varphi_m} \qquad [7]$$

$$\sin\varphi_{cv} = \frac{\sin\varphi_p - \sin\psi_p}{1-\sin\varphi_p\sin\psi_p} \qquad [8]$$

where
ψ_p = the ultimate dilation angle
φ_{cv} = the friction angle in the constant volume (ultimate state).

It is evident that in the presented relations as the plastic shear strain (mobilized friction) increases, the dilation angle starts from a negative value, reaches zero at $\varphi_m=\varphi_{cv}$ and increases until it reaches to its ultimate value, ψ_p. It is worth mentioning that the effect of pore water pressure has not been studied and the drained parameters of soil are considered.

3 STUDIED TESTS

3.1 Bangkok Subsoils

Bangkok is located in the flood plain and delta of Chao Phraya River in Thailand. The lower plain represents various sediments and eight aquifers. Bangkok clay is one of the most well-known sedimentary soils which has been extensively employed to investigate the behavior of soft soil experimentally (Chaudhry, 1975). Based on numerous test results, the critical state theories has been primarily verified for normally and over consolidated clays. The selected soil sample in this paper is chosen from the Bangkok Aquifer (BK). Bangkok soil is mainly divided to three major types:

1- Weathered Clay
2- Soft Clay
3- Hard Clay

Weathered clay is located in the surface and has a maximum depth of 3 meters. Following, soft clay is located to the depth of 12 m and hard clay is located to the depth of 35 meters. In this study the characteristics of soft Bangkok clay has been employed to investigate the efficiency of the hardening model for soft soils. Table 1 shows the index properties of soft Bangkok clay (Surarak, et al., 2012).

Table 1. Soft Bangkok clay characteristics

Characteristics	Value
Moisture Content (%)	122 - 130
Porosity ratio	3.11-3.64
Sand (%)	4.0
Silt (%)	31.7
Clay (%)	64.3
Specific gravity	2.75
Liquid limit (%)	118±1
Plastic limit (%)	43±0.5
Dry density (kg/cm^2)	16.5
Color	Sludgy grey
Saturation (%)	98±2

3.2 Test Results

In the present research a set of drained triaxial test results were utilized to determine the strength properties for the Hardening model. Undisturbed samples were driven from the depth of 6 meters and the consolidated drained (CD) triaxial test was performed in five confining pressures (138, 207, 276, 345, 414 kPa). The effective friction angle and cohesion obtained were 23.6° and zero respectively (Chaney, 1978). Figures 1 and 2 show the results of the CD triaxial test.

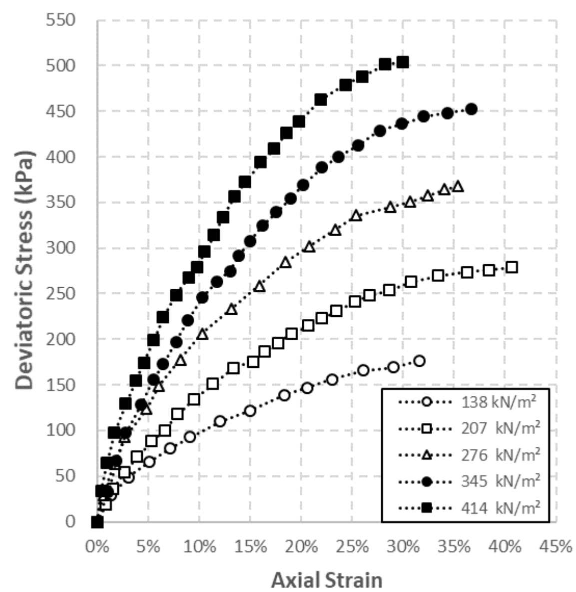

Figure 1. Stress-strain - CD Triaxial test - Bangkok soft clay

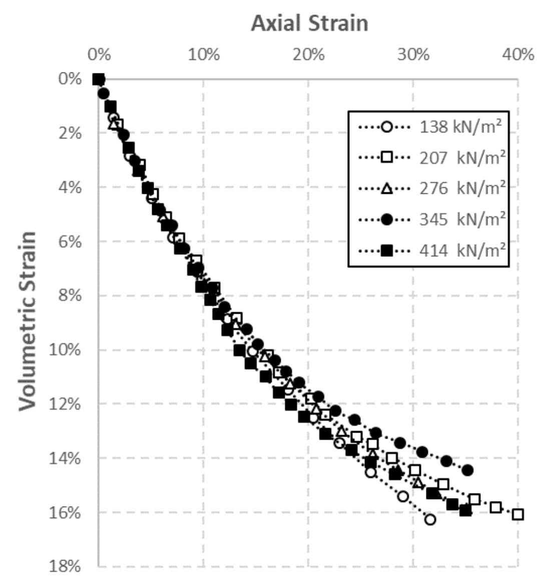

Figure 2. Volumetric strain vs. axial strain - CD Triaxial test - Bangkok soft clay

4 CALIBRATION PROCEDURE

The shear stiffness and the shear bulk modulus, namely *G* & *K*, are the most fundamental parameters for the shear yield surface. As mentioned previously, the Young modulus is defined as a function of the confining pressure. In the present study, the values of n and K_e are obtained according to the proposed method by Duncan, et al. (1970) as 0.9975 and 19.7161 respectively. It can be noticed that the value of n is close to unity which is in accordance with soft soil behavior. The shear stiffness and shear bulk modulus can be calculated with regard to the elasticity relations and Poisson ratio of 0.2 assumed for soft materials.

The ultimate shear parameters, have a key role in the shear behavior of materials. These parameters are calculated based on the CD triaxial experimental results as 0 kPa and 23.6º for the ultimate cohesion (c^p) and ultimate shear friction (φ^p), respectively.

To calculate the ultimate plastic shear strain, $\varepsilon_s{}^p$ the following steps have been employed.

1- Calculating the elastic axial strain

$$\varepsilon_1^e = \frac{\sigma_1 - 2\upsilon\sigma_3}{E} \quad [9]$$

2- Calculating the plastic axial strain

$$\varepsilon_1{}^p = \varepsilon_1 - \varepsilon_1{}^e \quad [10]$$

3- Calculating the elastic volumetric strain

$$\varepsilon_v{}^e = \frac{(1-2\upsilon)(\sigma_1+2\sigma_3)}{E} \quad [11]$$

4- Calculating the plastic confining strain (triaxial test)

$$\varepsilon_v = \varepsilon_v{}^e + \varepsilon_v{}^p \quad [12]$$

$$\varepsilon_v{}^p = \varepsilon_1{}^p + 2\varepsilon_3{}^p \quad [13]$$

5- Finally, Calculating of the plastic shear strain

$$\varepsilon_s{}^P = \tfrac{2}{3}(\varepsilon_1^p - \varepsilon_3^p) \quad [14]$$

Where

ε_1 = the total axial strain

ε_3 = the total lateral strain

ε_s = the total shear strain

ε_v = the total volumetric strain

e and p superscripts represent the elastic and plastic portion of the total strain.

The test results indicate that the ultimate axial plastic strain for different confining pressures are almost the same and equivalent to 40%, Therefore, based on Equations 9 to14 the ultimate plastic shear strain for all the confining stresses can be calculated as $\varepsilon_s{}^f$=29.3%

The parameters discussed, are sufficient to calibrate the stress behavior of the soil but to calibrate the volumetric strain, the ultimate dilation angle (ψ^p), which is a non-negative parameter, is required. Since no dilation is observed in the CD Triaxial test (Figure 2) the minimum value of the dilation angle (ψ^p =0) is chosen. Table 2 summarizes the parameters of this model, their meaning and the values assigned to them for this study.

Table 2. Parameters of the Hardening soil model for soft Bangkok clay

Parameter	Description	Value
n	Stiffness power constant	0.9975
K_e	Stiffness linear constant	19.1761
c_p	Ultimate cohesion	0
φ_p	Ultimate friction angle	23.6°
ψ_p	Ultimate dilation angle	0°
$\varepsilon_s{}^f$	Ultimate plastic shear strain	29.3%
v	Poisson ratio	0.2

Figures 3 and 4 show the predicted model base on the parameters discussed above versus the experimental data.

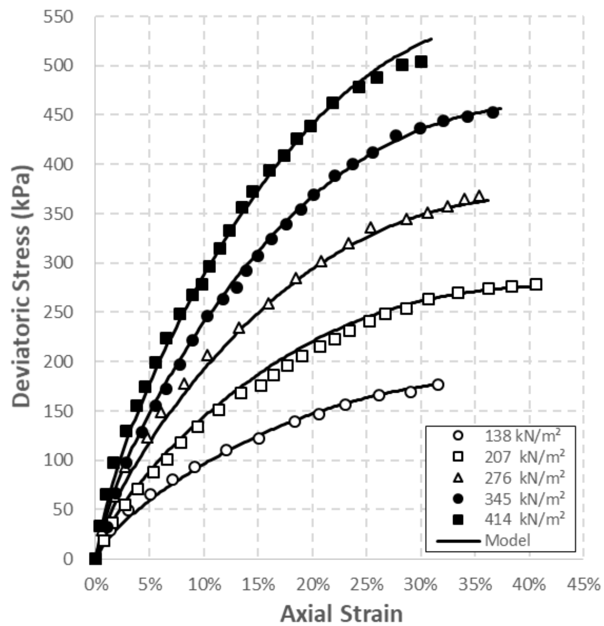

Figure 3. The adjustment of the deviatoric stress in the CD Triaxial test using the hardening model

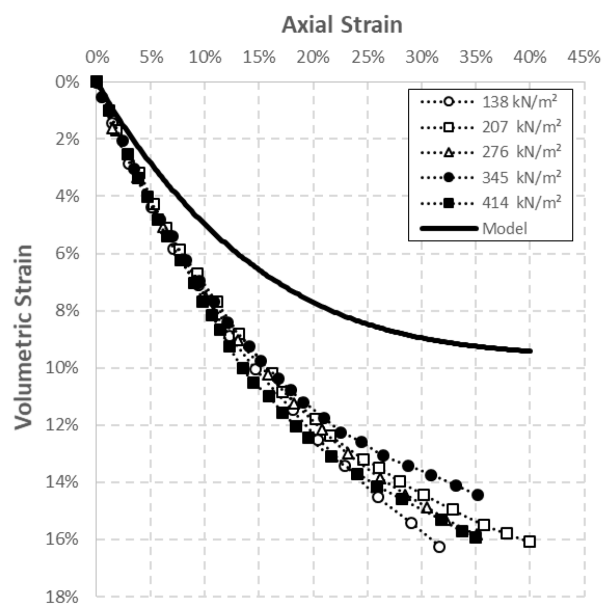

Figure 4. The maladjustment of the volumetric strains in the CD Triaxial test using the hardening model

Due to large volumetric strains in soft soils, as it can be seen in Figure 4, the volumetric strain has not been significantly calibrated and the relations introduced in the Hardening Model are not efficient. The maladjustment cannot be resolved by modifying other parameters of the model, since it effects the stress-strain graph. Therefore, to overcome this deficiency, the authors have introduced a constant, *P*, to modify equation 7 as following

$$\sin\psi_m = P.\frac{\sin\varphi_m - \sin\varphi_{cv}}{1 - \sin\varphi_{cv}\sin\varphi_m} \quad [15]$$

where *P* is greater than one for soft soils. Generally, the effect of coefficient *P* on the estimation of the volumetric strain can be seen in Figure 5.

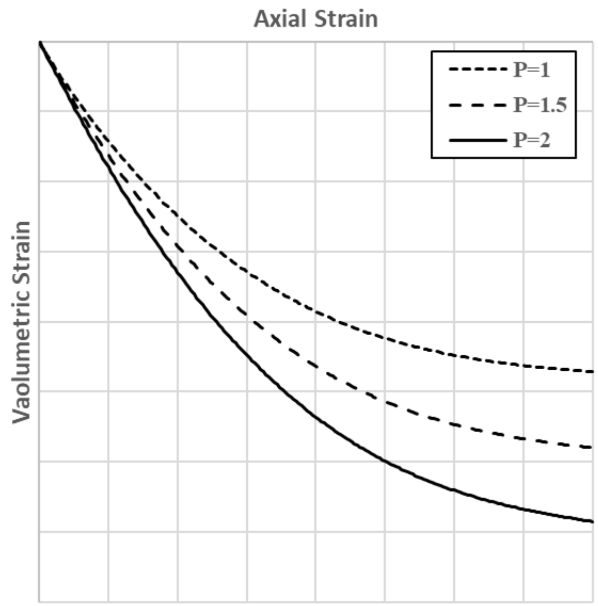

Figure 5. The effect of coefficient *P* on the evaluation of volumetric strains

The value of P has been determined by back analysis which is equal to 2.45 and the outcome graph is illustrated in Figure 6. It must be mentioned that this modification has no effect on the stress-strain graph (Figure 3).

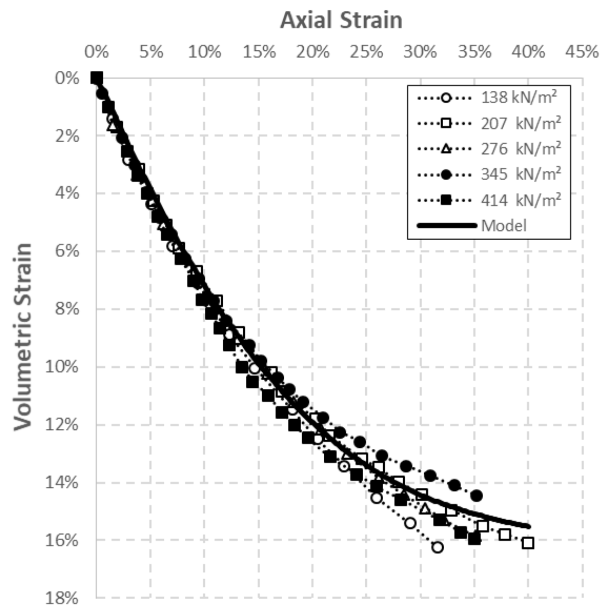

Figure 6. The adjustment of the volumetric strain by using the correction coefficient *P*

5 CONCLUSION

In this study, the efficiency of the hardening soil model for Bangkok soft clay was reinvestigated. The data selected for the calibration process was a CD triaxial test derived from previous researches. The applicability of the model has been investigated. For common engineering practices where the problem is mainly solved by applying the shear stress, using the parameter *P* to predict the soil deformation, leads to acceptable results and although numerous parameters are not required in the model, the results are quite precise. Another advantage of this model is that it can be calibrated with common soil mechanics tests.

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