Stiffness and shear strength characteristics of unsaturated glass-beads: experimental observations

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ABSTRACT
Despite the significant growth in understanding the behavior of unsaturated materials, the behavior of wet granular materials or industrial bulk solids like unsaturated soils and glass-beads present many engineering challenges due to matric suction. From numerical modelling perspective, both finite element and discrete element methods have been used to assess the shear strength characteristics of unsaturated materials. However, such efforts must be complemented and validated with experimental studies. In this article, the unsaturated stiffness and shear characteristics of uniformly-sized glass-beads were studied using the conventional direct shear tests at different water contents. The outcome of study offers an insight in optimization of the numerical model parameters to best fit the laboratory test data. Studies suggest that the presence of unsaturated pores and weak suction stress can affect the stiffness and shear strength characteristics such as, finite shear modulus, internal friction, dilation angle which should be accounted in both numerical and theoretical constitutive modeling.

RÉSUMÉ

Bien que les matériaux non saturés fassent l'objet d'une attention croissante en recherche, le comportement des matériaux granulaires humides ou des granulats industriels comme les sols non-saturés et les billes de verre présente de nombreux défis d'ingénierie à cause de la succion matricielle. Du point de vue de la modélisation numérique, la caractérisation de la résistance au cisaillement des matériaux non-saturés est basée sur la méthode des éléments finis et discrets. Dans cet article sont étudiées la rigidité non saturée et les propriétés de cisaillement de billes en verre de taille uniforme par le biais d'essais de cisaillement direct à différentes teneurs en eau. Les résultats de l'étude permettent l'optimisation des paramètres de modèles de comportement. On remarque que la présence d'air dans les pores et de contraintes de succion ont un effet sur la rigidité et les paramètres en cisaillement tels que le module de cisaillement, l'angle de frottement interne et l'angle de dilatance. Cette dépendance à la succion doit être incluse dans les modèles théoriques et numériques.

1 INTRODUCTION

The complex behaviour of unsaturated materials is often influenced by the matric suction arising from the interaction between the gas and liquid phases and pressure deficiency in the voids which cannot be easily predicted using classical soil mechanics theories. A comprehensive understanding of unsaturated materials backed by theoretical and numerical studies is required for any safety and design analysis in most geotechnical problems. As a result, a strong need to properly describe the strength of unsaturated materials arises which can be acquired through experimentation.

Incomplete description of complex stress state in unsaturated condition at bulk scale has urged the researchers to focus on micro scale studies to find how shear strength and volume change characteristics are affected by microscopic inter-particle interaction and matric suction. Extensive studies have been done using discrete element modelling in this regard (Gladkyy and Schwarze, 2014). However, such efforts would remain impractical and cannot represent the overall behavior of unsaturated materials if they are not examined experimentally.

At Université de Sherbrooke, this issue is investigated by defining a wider research project at which the hydro-mechanical behavior of unsaturated granular material is evaluated through experimental approaches and numerical modeling. It is required to compare the obtained results from numerical micro-mechanical simulation with macroscopic hydro-mechanical response of granular materials under the influence of suction to calibrate the numerical model parameters. The use of idealized materials such as glass-beads provides the possibility to isolate shear strength response of a specimen regardless of its particle shape and size distribution effects. This paper aims to present some insights regarding the effect of matric suction on the stiffness and shear strength characteristics of uniformly-sized glass-beads which were acquired by experimental studies. To do so, a series of conventional direct shear tests under different mechanical loads was performed on uniformly-sized glass-beads specimens with two different bead sizes at different water contents. In order to understand the state of suction at tested specimens, the water retention curves (WRCs) were obtained using negative water column technique (NWCT) and then the matric suction was back-calculated with the corresponding water content of each specimen. It must be pointed out that due to the volumetrically inert behavior of glass-beads, the assessment of volumetric behavior of is considered out of the scope and only the stiffness and shear strength characteristics were evaluated.

2 TESTING PROCEDURE

2.1 Tested materials

To study the unsaturated shear strength of granular materials, two sizes of uniformly-sized glass-beads were studied. The general-purpose glass-beads were made by Whitehouse Scientific Ltd, England with the average specific gravity, G_s, the Young modulus, E, and Poisson's ratio,ν of 2.46 gr/cm^3, 6.89×10^4MPa and 0.21 respectively. The sizes of chosen glass-beads were 1.25mm -1.40mm and 0.25mm - 0.30mm.

2.2 Water retention curve (WRC)

To obtain the WRC of glass-beads, a modified Tempe cell setup that was made at Université de Sherbrooke (Maleksaeedi, Nuth, and Chekired,2016) was used. For each glass-beads size, two tests were performed and the readings were averaged. The errors in measurements were less than 5%. The matric suction was applied using negative water column techniques during desaturation of glass-beads. The drying water retention curves were obtained under zero net vertical stress starting from fully saturation condition. The during hydraulic retention tests, the void ratios were assumed constant and equal to 0.60 which was achieved by compaction effort. Figure 1 shows WRCs of tested glass-beads. The data were fitted with empirical water retention model proposed by Van Genuchten (1980) as in equation 1. The air entry value (AEV) of these glass-beads were around 0.35 kPa and 2.50 kPa.

$$S_r = S_{r,r} + \frac{(1-S_{r,r})}{\left[1+\left(\frac{(u_a-u_w)}{\alpha}\right)^n\right]^m} \qquad \text{Eq.1}$$

where n, m and α are empirical fitting parameters related to pore size distribution and air entry value (AEV). $S_{r,r}$ is the residual degree of saturation and $(u_a - u_w)$ corresponds to the matric suction. Table 1 shows the fitting parameters of WRCs used in this study.

Table 1. Fitting parameter of WRCs of glass-beads

Diameter (mm)	$S_{r,r}$	α	n	m
1.25-1.40	0.22	1.83	3.92	140.5
0.25-0.30	0.06	5.50	7.38	301.0

2.3 Unsaturated direct shear test

To obtain unsaturated stiffness and shear strength characteristics, the conventional direct shear apparatus was used to measure the shear modulus at finite strain level and shear strength parameters at critical state by performing water content- controlled tests. Overall, more than 50 tests were performed to improve the reliability of the results.

For unsaturated tests, the glass-beads batches were separately prepared by mixing distilled water corresponding to the target water contents of 8%,10%, 12%, 15%, 18%, and 20%. The samples were then placed in a sealed plastic container to reach water equilibrium. Then, the unsaturated glass-beads were added to the shear cell in three layers while compacting each layer to the densest arrangement possible in order to achieve the target void ratio of 0.60. The remainder of glass-beads in the batch was then used to measure the water content before performing the test.

The specimens corresponding to the target water contents were then sheared by constant displacement rate of 0.0002 mm/s at the nominal vertical stresses of 50 kPa, 150 kPa and 300 kPa. The displacement rate was assumed be sufficiently low enough to prevent any sudden change in pore water pressure. Evidently, during compaction process and shearing, certain amount of water could be evaporated or drained due to compaction and shearing energies which were considered as the error of using this approach. After each unsaturated test, the water content of specimens was measured. The induced errors during shearing tests varied from 1% to 8% as the water content increased. In this manner, the unsaturated shear strength of each size of glass-beads were measured at five different target water contents. The average of water contents at the beginning and the end of each test was then associated with the corresponding matric suction using the SWRCs.

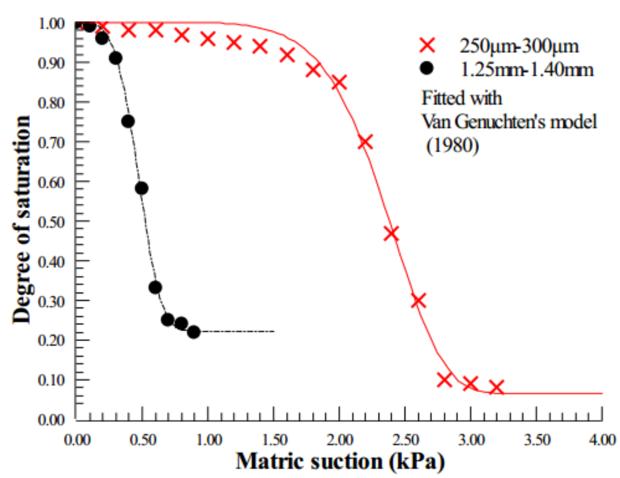

Figure 1. Water retention curves of glass-beads under zero net stress

3 RESULTS AND DISCUSSIONS

3.1 Shear stress-shear strain curves of unsaturated glass-beads

Figure 2a and 2b show the main results from water content-controlled direct shear tests in terms of shear stress-shear strain curves for glass-beads of 1.25mm-1.40mm and 0.25mm-0.30mm specimens respectively. As one of the limitations of direct shear test, due to non-uniformity of stress and strain in shear box, the results are often misinterpreted. The interpretation of the direct shear box is often on the basis of assuming that the deformation of a specimen conforms of ideal simple shear test (Potts and Zdravković, 2001). Numerical studies of Potts et al. (1987) indicated that the stress-strain behavior in simple shear tests results is dependent on the volume change characteristics and the initial stress, along with the shear strength characteristics. They also stated that the predicted average behavior of non-strain-softening soil in the direct shear box is very similar to that of ideal simple shear. Meanwhile, Kirby (1998) suggested that for direct shear tests in constant vertical stress condition, due to symmetry of the shear box, lateral stresses are at rest condition. However, since the principal stresses are unknown, in this study, it is assumed that the minor principal stress follows the equation 2 where it can be obtained knowing the shear stress at the given strain level.

$$\sigma_3' = \frac{1}{2}(1+K_0)\sigma_v - \sqrt{\frac{\sigma_v^2(1-K_0)^2}{4} + \tau_h^2} \qquad \text{Eq. 2}$$

Where K_0 is the lateral coefficient at rest condition, σ_v, is the applied vertical stress and τ_h is the shear stress at a given strain level. This method is borrowed from direct simple test interpretation in which it is assumed that the soil specimen is subjected to pure shear (Kang et al. 2015). Evidently, such assumption induces errors in calculation since the specimens in direct shear tests do not undergo pure shearing.

As in Figure 2a and 2b, it is evident that there was a slight variation in shear stress as the water contents varied. The observation herein is consistent with experimental observations available in literature (Escario and Saez, 1986, Vanapalli et al. 1996; Maleksaeedi et al, 2017) where a slight variation in the friction angle were observed. Such behavior is attributed to the presence of capillary and surface tension in local scale (micro scale). Glass-beads were then pulled together causing the global scale (bulk scale) shear strength to be changed. Although both apparent cohesion and friction angles were changed by suction, one must consider the fact that materials such as glass-beads essentially have very low changes in shear strength compared to real fine soils such as silts and clays. Yet, such observations are equally important when it comes to calibration the numerical models.

Figure 3. shows the variation of apparent cohesion with matric suction. As it can be seen, the finer glass-beads experienced higher apparent cohesion compared to the coarser one. For both glass-beads, the apparent cohesion increased with a decreasing rate as suction increased until reaching a peak value which is close to the AEV and beyond that, the apparent cohesion began to decrease. Numerous researchers reported a linear increase in apparent cohesion before AEV followed by a nonlinear variation beyond AEV (Ridley 1995; Vanapalli and Lane 2002). Yet, since in this study the effect of suction on shear strength was studied indirectly by varying the water content, evaluating the linearity of apparent cohesion before AEV could not be observed.

Lu and Likos (2013) stated that the drained cohesion and apparent cohesion are interlinked through the suction stress concept which is defined as an isotropic tensile stress originated from physico-chemical forces such as van der Waals attraction, double layer repulsion , and capillary attraction. Using this definition, apparent cohesion depends on saturation and material type that physcially behaves as the mobilized suction stress and correspondingly, to the effective stress. For granular materials, the mobilised suction stress decreases to a minimum value near AEV and then its effect of unsaturated shear strengh diminishes as the volume of water bridges between partciles decreases.

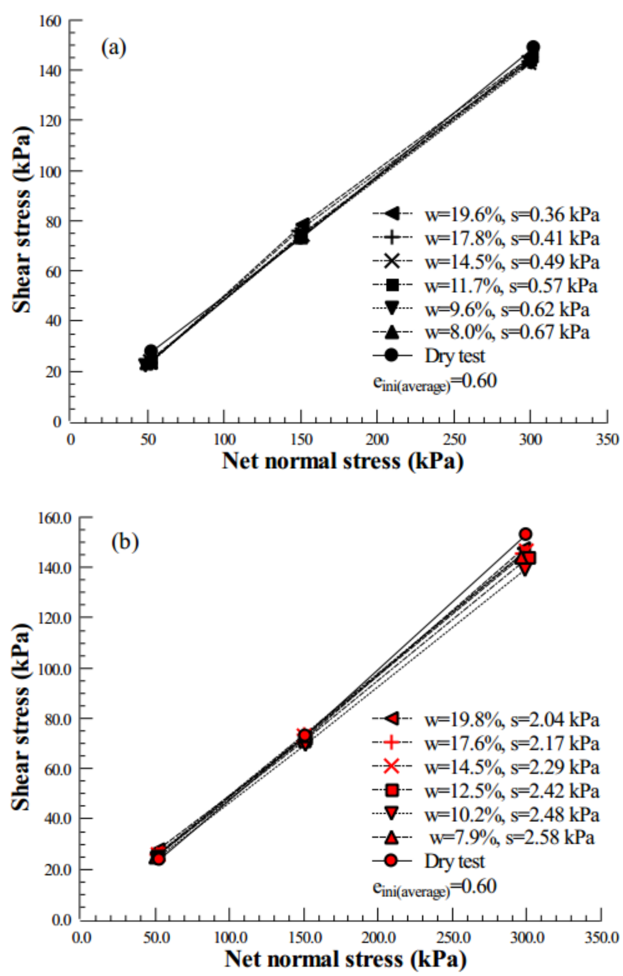

Figure 2. shear stress-shear strain curve of glass-beads with diameters of (a) 1.25mm-1.40mm (b) 0.25mm-0.40mm

3.1 Variation of friction angle with suction

Figure 4a and 4b illustrate the variations of the friction angle of each glass-beads with the degree of saturation in both peak and critical state. The results are generally in agreement of other experimental observations available in literature (Vanapalli et al., 1996; Chen et al. 2013) where a slight variation in the friction angle were observed. However, the authors could not find any research regarding the evolution of friction angle with suction since such slight changes in friction angles are often accounted as error in performing the tests. For tested glass-beads, despite minimal variations, as the amount of water increased in pores, the friction angle decreased gradually in both states. It is due to the development of water film around particles and the lubrication of particle-particle friction which results in the reduction of friction angle (Zou and Brusewitz 2001).

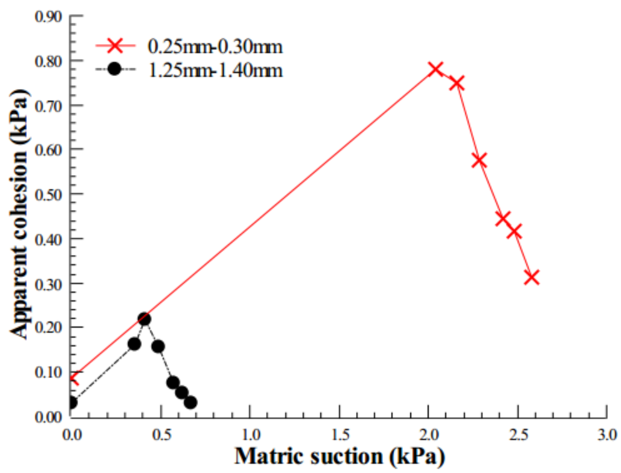

Figure 3. Variation of apparent cohesion with matric suction for both glass-beads.

Two possible explanations for the increase in friction angle of glass-beads with water content are the testing performance and the compaction procedure. For instance, the tilting of the top half of shear box or the top cap during shearing process might be a possible source affecting the failure mechanism and increasing the friction angle (Amirpour et al. 2015). Comparing Figure 4a and 4b shows that the changes in friction angle with degree of saturation is more pronounced in peak state rather than critical state. The friction angle is not principally a material constant since it is dependent on void ratio, fabric, interlocking effects, and the state of stress particularly in peak state. The results herein signify the fact that implicitly setting the friction angle as a suction-independent parameter might affect the possible failure surface as stated by Delage (2002).

3.2 Variation of maximum dilation angle with suction

The shear strength of granular materials such as gravels, sands, artificial sands and glass-beads depends on the confining effective stress, friction angle and dilatancy. The dilatancy depicts the ability of soils to change in volume during shearing process. For simplicity, this can be viewed as the ability of granular particles to slide upward (positive) or downward (negative) over each other during shear. It has been well-established that an increase in the normal effective stress results in a decreasing expansion potential or value of dilation for the soil, in other words, as normal stress increases, the dilation angle decreases.

Figure 5a and 5b confirm these assumptions as they show the variation of maximum dilation angle with degree of saturation. In both figures, the overall trends of dilation changes depended on the applied vertical stresses (Yang and Li 2004; Hossain and Yin, 2010). Large normal effective stresses which depend on the nature of soil and particle arrangement, suppress the riding up tendencies of the grains, thus restraining dilation.

Although in Figure 5 no precise pattern was observed between the dilation angle and degree of saturation, it indicates the notion that dilation angle can be equally affected by matric suction as well as confining or normal stresses. Possible explanation for such pattern is the frictional resistance between the soil and side walls of shear cell (Hossain and Yin, 2010).

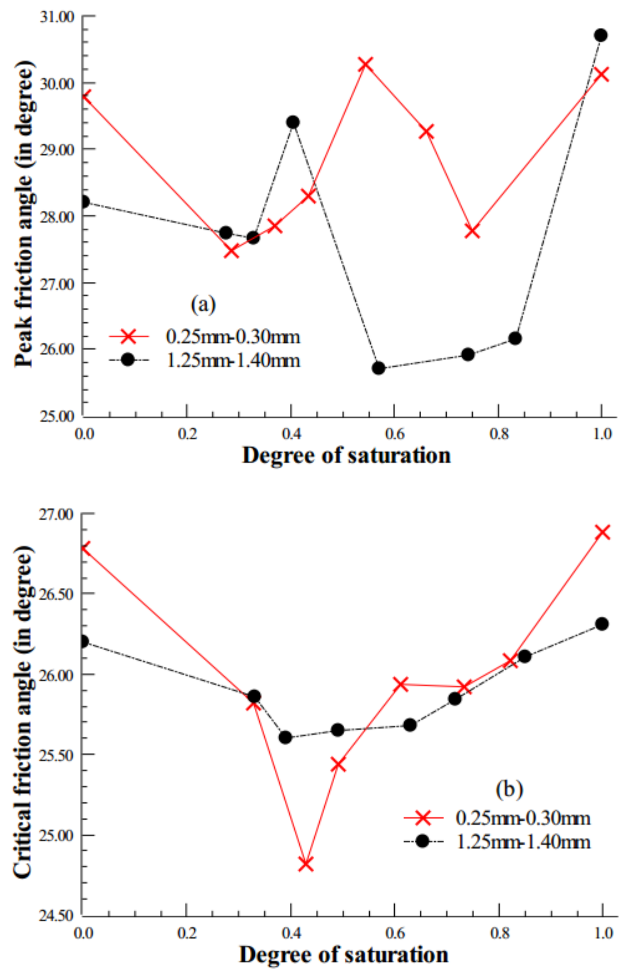

Figure 4. Variation of friction angle with degree of saturation for glass-beads in (a) peak state (b) critical state

In total, it was observed that the presence of matric suction due to the changes in water contents creates the suction stress which in return pulls the particles together. As a result, particles cannot freely move and merely try to move on or over each other when sheared.

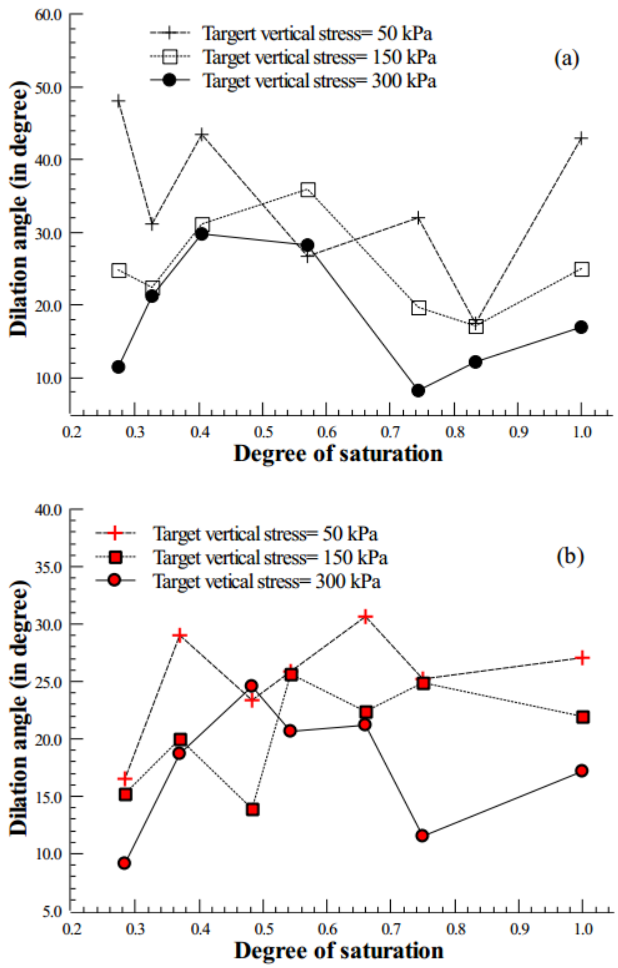

Figure 5. Variation of maximum dilation angle with degree of saturation for (a) 1.25mm-1.40mm (b) 0.25mm-0.40mm

3.3 Variation of shear modulus with suction at finite strain level

It is a well-known fact that soils often undergo nonlinear increase in shear stress as shear strain develops and subsequently, it follows a gradual descending trend until shear stress reaches a peak or critical state value. When the shear strain level or amplitude is very small, i.e. less than 0.001%, the variation of shear stress-shear strain can be considered linear. In this range of deformation, the soil structure does not undergo significant change and only particles oscillate due to the elastic wave propagation caused by vibration (Dong, Lu, and McCartney 2017). The shear modulus at strain level is commonly known as small strain shear modulus, G_0 which can be measured using various techniques including bender element method or P-RAT element technique (Youn, Choo, and Kim, 2008; Karray, Ben Romdhan, and Hussien 2015).

As shear strain increases beyond cyclic strain threshold, the progressive stress redistribution at paticle's contacts results in particle rearrangement which correspondingly induces permanent deformations under static or cyclic loading. For most soils, this threshold is considered to be 1% of strain level, commonly known as finite strain level. Under cyclic loading, the finite strain shear modulus can be obtained using resonant column tehcnique (Khosravani and McCartney 2011). Under static loading however, the secant shear modulus at 1% of strain level is often measured using common triaxial tests or direct simple shear test (ASTM D6528-17). For direct shear test on the other hand, due to non-uniformity of stress and strain, it is often suggested not to be used for stfifness measurement. In this paper, the authors suggest that such method of thinking depends on the chosen strain level for calculation of shear modulus. This hypothesis is supported with the studies of Lings and Dietz (2004) and Amirpour et al. (2017) on direct shear results, stating that within the deformation zone across the specimen centre, there is good uniformity of strain and the occurred deformation approximate the simple shear.

In general, shear modulus changes significantly depending on soil physical properties including the soil type, plasticity index, initial density, the soil volume change characteristics such as void ratio, overconsolidation ratio (OCR), mechanical loading condition such as loading cycles and frequencies, and hydraulic retention characteristics such as soil degree of saturation and matric suction. Figure 6a and 7a show the variation of finite strain shear modulus with vertical stress at different water contents obtained with conventional direct shear tests. The overal convergence of results supports the hypothesis that at 1% strain level, the uniformity of shear bond was gradually developing while at smaller strain levels, i.e, less than 1%, the disparity of results was higher. The results also revealed that the shear modulus is stress dependent as well as suction dependent. This is also shown in Figure 6b and 7b for both unsaturated glass-beads. Figure 6b and 7b suggest that at low confinement, the variation of suction does not affect the stiffness of specimen since the unsaturated glass-beads essentially experience low suction stress. Yet, as the confinement effect increases, the contribution of suction to stiffness increases. This is probably attributed to the changes in local (micro scale) distribution of inter-particle forces. The results in Figure 6 and 7 show good agreements with the findings of Dong et al. (2016).

Dong et al. (2016) presented a unified model for small strain shear modulus of unsaturated soils within the suction stress characteristics framework developed by Lu and Likos (2006) as in Equation 3.

$$G_0 = G'(S_e) \times G''(\sigma') = A\left(\frac{1}{S_e}\right)^{\beta}\left[\frac{(p-u_a)-\sigma_s}{p_{ref}}+1\right]^{\gamma} \quad \text{Eq.3}$$

where $G'(S_e)$ represents the contribution of suction to the small strain shear modulus while $G''(\sigma')$ shows the changes in shear modulus due to mechanical loading. In equation 3, S_e is the effective degree of saturation, p_{ref} is the reference pressure, $(p-u_a)$ is the mean net stress, σ_s is the suction stress while A, β, and γ are fitting parameters. Assuming that at very small strains, both secant and tangent shear moduli follow the same pattern, equation 1 and 3 can be used to assess the unsaturated shear modulus at finite strain level.

Although a more thorough statistical analysis is required for assessing the obtained results, in this paper, studies were performed on $G'(S_e)$ and $G''(\sigma')$ separately. The results indicated that there is a high dependency between these two components in a sense that β and γ are stress- and suction- dependent respectively. Obviously, further research is required to support the observation made in this study.

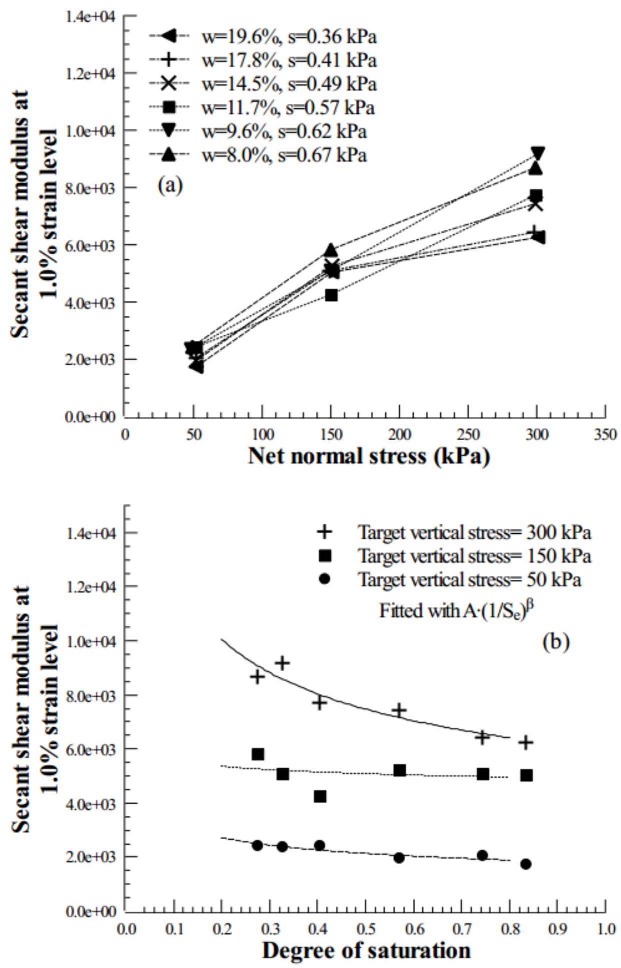

Figure 6. Variation of finite secant shear modulus at strain level of 1% for glass-beads 1.25mm-1.40mm (a) with net normal stress (b) degree of saturation

Since today's understanding of the microscopic scale response of soil is mainly based on numerical simulation of macroscopic scale, these studies must be verified with real tests results using idealized materials such as glass-beads. This shows the importance of the current study findings for numerical simulation. In case of discrete element modelling (DEM), the obtained results can be used to verify how matric suction can influence the inter-particle contact laws and how it can correspondingly change the stiffness and shear strength characteristics as suggested by Jiang and Shen (2013). Furthemore, in macroscopic scale, the obtained experimental results provides the necessary information for verifying the constitutive formulations developped for finite-element modelling (FEM). Finite element algorithms for evaluating the unsaturated soils must be formulated in a way that they cover different aspects of soils behavior. Understanding how suction can influence the shear strength parameters and which numerical model parameters are more affected by the presence of matric suction are key questions that must be answered in order to develop a comprehensive approach in design and evaluation of unsaturated soils.

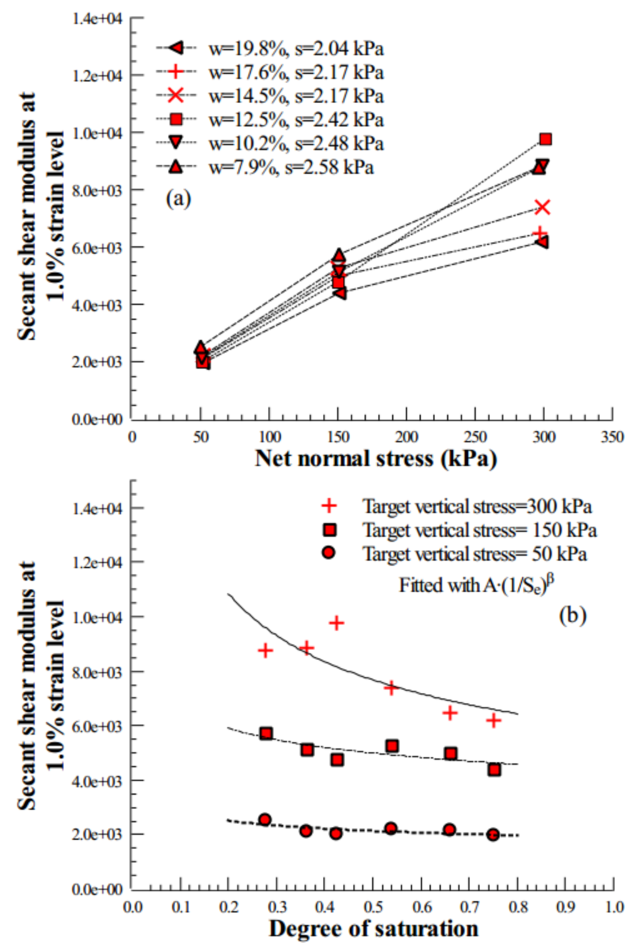

Figure 7. Variation of finite secant shear modulus at strain level of 1% for glass-beads 0.25mm-0.40mm (a) with net normal stress (b) degree of saturation

4 CONCLUSION

This paper presents some insights regarding how matric suction can affect the stiffness and shear strength characteristics of unsaturated uniformly sized glass-beads. A series of conventional direct shear tests was performed on unsaturated glass-beads in water content-controlled condition along with water retention tests under zero net normal stress.

Regarding shear strength parameters, the results indicated that matric suction affects both internal friction angle and maximum dilation angle. The variation of internal friction is more pronounced at peak state compared to critical state which means that considering peak state friction angle independent of suction might possible failure surface.

In case of stiffness characteristics, the results suggest that direct shear test can provide meaningful description of stiffness parameters at finite strain level although further studies are required to confirm this hypothesis. It was also found that matric suction has an impact on secant shear modulus at 1% strain level. The contribution of suction to stiffness depends on the applied confinement to specimen.

The results herein can be used as a benchmark for numerical simulations for both discrete element and finite element modelling.

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