Augmented slope stability analysis approach to determine setback distance based on target factor of safety criteria



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ABSTRACT

Determination of setback lines for developments along river valley and ravine slopes continues to be a challenge for engineers. In particular, two tasks include consideration of long term processes in the slope stability model, and selection of a reasonable slip surfaces for a non-critical factor of safety (FOS). The conventional approach, which uses long term soil parameters (zero or near-zero cohesion) directly in the slope stability model, may result in conservative setback distances due to unreasonable slip surfaces. In this paper, an augmented approach is presented, which uses present soil parameters (non-zero cohesion) in the model to determine reasonable trial slip surfaces. Subsequently, the impact of long term processes is considered on these slip surfaces to determine a long term, non-critical FOS, upon which optimized setback distances can be determined. In general, the augmented approach is an attempt by the authors to provide a starting point for further refinement of setback distance determination methods that are based on FOS criteria.

RÉSUMÉ

La détermination des marges de recul pour les développements le long des vallées fluviales et des pentes des rivières continue d'être un défi pour les ingénieurs. En particulier, deux tâches incluant la prise en compte des processus à long terme dans le modèle de stabilité de la pente, et la sélection d'une surface de glissement raisonnable pour un facteur de sécurité (FDS) non-critique. L'approche conventionnelle, qui utilise des paramètres du sol a long terme (cohésion nulle or quasi-nulle) directement dans le modèle de stabilité des sentes, peut donner lieu a des distances de recul conservatrices dues à des surfaces de glissement démesurées. Dans se document, une approche augmentée est présentée, qui utilise les paramètres de sol présents (cohésion non-nulle) dans le modèle pour déterminer des surfaces de glissement d'essai raisonnables. Par la suite, l'impact des processus a long terme est considéré sur ces surfaces de glissement pour déterminer, a long terme, un FDS non critique sur lequel des distances de retrait optimisées peuvent être déterminées. En général, l'approche augmentée est une tentative par les auteurs de fournir un point de départ pour affiner davantage les méthodes de détermination de la distance de recul qui sont basées sur les critères du FDS.

1 INTRODUCTION

1.1 Setback Lines for Developments near Slopes

For developments occurring in the vicinity of slopes, determination of safe setback distances from the top of bank (TOB) is an important undertaking that permits the following:

- Protection of developments from environmental hazards, such as slope instability or flooding.
- Minimizing adverse impacts of encroaching developments on sensitive slopes and valleys.
- Ensuring general public safety as well as preserving natural ecosystems and habitats.

In general, TOB properties are often highly desired, highly valued properties, with developments being constructed ever closer towards the TOB of slopes. As developers continually try to push the envelope to capitalize on panoramic views, setback line definition has become more critical to mitigate the adverse effects of potential slope instability on developments, and vice versa.

1.2 City of Edmonton Policy for Setback Distance Determination

For developments within the City of Edmonton (COE), criteria for setback line determination is outlined in Policy

C542 (the Policy), titled "Development Setbacks from River Valley/Ravine Crests". In the Policy, the setback line is defined as the Estimated Long Term Line of Stability (ELTLS).

The ELTLS is determined using slope stability analysis, based on a slip surface with a minimum long term factor of safety (FOS) of 1.30 or 1.50. In the context of the Policy, "long term" is clearly defined such that slope stability analysis must consider long term processes, which may lead to progressive reduction in the FOS, over the lifetime of the development. Such processes include valley rebound (Matheson and Thomson 1973), softening (Morgenstern, 1990), and changes in the groundwater table. As such, typical measures used to consider these processes in the slope stability analysis include the assumption of zero or near-zero cohesion for cohesive soils, and groundwater profiles that are above measured groundwater levels.

1.3 Limit Equilibrium Methods for Slope Stability Analysis

Historically, limit equilibrium (LE) methods such as Fellenius (1936), Bishop (1955), Janbu (1954), Spencer (1967), Morgenstern-Price (1965), and others have been used for slope stability analysis. Based on their work, LE methods for slope stability analysis have continued to progress.

For example, conventional LE methods have been enhanced with various optimization techniques to further minimize FOS values and to locate critical, non-circular slip surfaces (Celestino and Duncan 1981; Nguyen 1985; Chen and Shao 1988; Greco 1996; Malkawi et al. 2001). LE methods have also been combined with numerical methods, which have provided more accurate representations of the actual stresses and stress-strain behaviors within a soil mass (Fredlund & Scoular 1999). More recently, a critical stress field approach was proposed by Zhu (2001), and a dynamic programming approach, which built upon prior work by Baker (1980), was proposed by Pham & Fredlund (2003).

In general, these optimization techniques focus on locating and evaluating the slip surface with the critical (local minimum) FOS. However, determination of setback distance is typically based on a target, non-critical FOS. Consequently, there can be numerous slip surfaces which all mathematically yield the target FOS. As such, it may not be possible to create an optimization technique for a non-critical, target FOS, as such a slip surface does not represent a local minimum in the FOS value.

1.4 Uncertainty and Risk for Slope Stability Analysis

Deterministic, conventional LE methods for slope stability analysis, based on inferred input data, cannot address uncertainties due to soil parameters, groundwater, spatial variability, etc. Therefore, LE methods cannot produce a single FOS that represents the variation of conditions for a particular slope stability model. For example, a larger FOS for a particular model may not imply a smaller risk of instability, as larger uncertainties may exist for that model.

Therefore, an approach based on conventional LE methods require a considerable level of judgement regarding selection of input data by the geotechnical engineer, who is typically faced with limited input data. Consequently, conservative assumptions regarding soil parameters and groundwater conditions are typically used in the model.

One way to consider uncertainty in the definition of the model is to apply a probabilistic method, which analyzes the impact of variable parameters or spatial variability on the FOS (Yong et al. 1977; El-Ramly et al. 2002).

1.5 Objectives and Scope

This paper presents an augmented approach for setback distance determination which builds on existing concepts of LE methods. The augmented approach attempts to reduce the conservatism and judgement inherent in the conventional approach, while being sensible enough for use in practice.

In general, the augmented approach is an attempt by the authors to provide a starting point for further refinement of LE methods with respect to setback distance determination. The authors hope that this paper may initiate further discussion within the geotechnical community in this regard, and open the door to other possible methods that may be used define reasonable setback distances based on FOS criteria.

2 CONVENTIONAL APPROACH FOR SETBACK DISTANCE DETERMINATION

2.1 FOS of Slopes Based on LE Methods

Historically, the FOS of a slope has been defined using deterministic, conventional LE methods as follows:

- A slip surface, with assumed position and shape, is defined in the slope stability model.
- The soil mass above the slip surface is then discretized into slices.
- For each slice, forces tending to move soil down the slope (driving forces) and available soil strength to resist the movement (resisting forces) are determined.
- These forces are then summed across all slices, with the FOS defined as the ratio of total available resisting force to total driving force.

Selection of input data (cohesion, friction angle, unit weight, pore pressure, etc.) has a significant impact on the FOS of a slope.

For the assumed slip surface, a FOS equal to 1.0 (unity) implies that the slope is marginally stable; in contrast, a FOS greater than unity implies that the slope is stable to a certain degree.

When movements initially occur, it can be deduced that equilibrium between the driving forces and resisting forces has just been achieved. However, the actual FOS of a stable slope cannot be conclusively determined, only that it is greater than unity (Vick, 1950).

2.2 Setback Distance Determination Based on Non-Critical FOS

As mentioned in Section 1.3, determination of setback distance based on a target, non-critical FOS does not have a singular solution. This is because many slip surfaces, with similar FOS values, can be readily obtained using conventional LE methods. To illustrate, Figure 1 shows a range of such slip surfaces for an arbitrary slope.



Figure 1. Typical model showing multiple slip surfaces, which all mathematically yield a FOS of 1.50

As illustrated, determining the setback line based on a slip surface which meets the target FOS is rather ambiguous, since there is considerable variation in the position and shape of slip surfaces (radius, entry point, exit point) which meet the target FOS.

As no single setback line is produced, and with the number of possible slip surfaces being quite numerous for some models, engineers may have to use a subjective approach, based on their judgement and experience, to determine setback distance.

2.3 Safety Map and Modeling of Long Term Conditions for Slope Stability Analysis

As discussed, the determination of a setback line based on a target, non-critical FOS is rather ambiguous, since there is no singular solution and no available optimization technique.

One way of addressing this issue is to employ a "safety map" approach using LE methods. A safety map groups slip surfaces with similar FOS values in order to create envelopes that represent the resulting FOS ranges. Figure 2 shows a typical safety map for an arbitrary slope.



Figure 2. Typical safety map, showing different colored envelopes for arbitrary FOS ranges

It should be noted that the envelopes themselves do not represent actual slip surfaces; rather, they are an amalgamation of slip surfaces within certain FOS ranges.

As mentioned Section 1.2, slope stability analysis should consider long term processes, which may lead to progressive reduction in the FOS, over the lifetime of any development. As such, zero or near-zero cohesion for cohesive soils is typically assumed in the slope stability model.

When cohesion is set to zero or near-zero in the model, the resulting FOS envelopes on the safety map tend to be governed by relatively flat (large radius) slip surfaces. Consequently, setback distances can become conservative if governed by these slip surfaces. For example, Figure 2 shows an arbitrary slope (10 m height, 2.5H:1.0V slope) where cohesion was set to zero. It is apparent that the relatively flat slip surface shown governs the resulting safety map.

While setting zero or near-zero cohesion in the model may be justified to consider long term processes, such processes may not reduce a cohesive slope to an effectively cohesionless condition within the lifespan of a residential development, which is typically defined as 50 to 100 years.

2.4 Summary

Based on what has been discussed, and in the context of this paper, the conventional approach for setback line determination consists of the following:

1. Input slope geometry into the slope stability model, based on survey data.

- 2. Input soil stratigraphy, based on field drilling and laboratory testing results.
- 3. Input long term soil parameters (zero or nearzero cohesion) and groundwater profiles (raised groundwater levels).
- 4. Run slope stability analysis and create a safety map for the model to view FOS envelopes.
- 5. Determine setback distance based on target FOS and extent of target FOS envelope.

As discussed, inputting long term soil parameters (zero or near-zero cohesion) directly into the model can result in relatively flat slip surfaces. As such, setback distances have the potential to be conservative.

3 AUGMENTED APPROACH FOR SETBACK DISTANCE DETERMINATION

3.1 Overview

The proposed augmented approach tries to reduce the conservatism and judgement that may be applied in the conventional approach. In general, the augmented approach is an iterative process consisting of two parts, which are described in the following sections.

3.2 Part 1: Determination Slip Surfaces Based on Present Conditions

Part 1 of the augmented approach attempts to reduce the variation in slip surfaces that correspond to a non-critical, target FOS. This is achieved by inputting present soil parameters (non-zero cohesion) and groundwater profiles (measured groundwater levels) directly into the model. In particular, inputting non-zero cohesion helps to ensure that unreasonably flat slip surfaces do not govern setback distance determination. Part 1 of the augmented approach consists of the following steps:

- 1. Input slope geometry and soil stratigraphy into the slope stability model (same as conventional approach).
- 2. Input present soil parameters (non-zero cohesion).
- 3. Input present groundwater profiles (measured groundwater levels).
- 4. Run slope stability analysis to verify assumptions and general validity of the model by reviewing the critical slip surfaces. For example, if the slope is observed to be relatively stable, then deep slip surfaces with FOS less than unity would not be a valid model.
- Create a safety map for the model and define a single point for slip surface entry. Place the entry point at the extent of the FOS envelope that corresponds to the target FOS (see Figure 4).
- 6. Run the slope stability analysis for the entry point, and determine the slip surface with the governing FOS (minimum FOS for that entry point).

3.3 Part 2: Consideration of Long Term Processes in Slope Stability Analysis

Part 2 of the augmented approach considers how long term processes will impact the FOS of the governing slip surface determined in Part 1. This is achieved by inputting long term soil parameters (zero or near-zero cohesion) and groundwater profiles (raised groundwater levels) and recalculating the FOS for the governing slip surface determined in Part 1. Part 2 of the proposed approach consists of the following steps:

- 1. For the governing slip surface, input long term soil parameters and groundwater profiles and determine the resulting long term FOS.
- 2. Compare the long term FOS to the target FOS. If the long term FOS is not adequate, move the entry point further away from the TOB.
- Rerun slope stability analysis for the new entry point and determine a new governing slip surface; then determine the resulting long term FOS. Repeat this process until the long term FOS meets the target FOS.
- 3.4 Comparison between Conventional Approach and Augmented Approach

As discussed, the conventional approach considers long term soil parameters (zero or near-zero cohesion) and groundwater profiles (raised groundwater levels) directly in the model. For the conventional approach, the setback distance would be determined based on the safety map. Figure 3 shows the resulting safety map for an arbitrary slope with a FOS envelope of 1.50 and below, with long term soil parameters inputted directly into the model. Based on the safety map, it is evident that the upper edge of the envelope is governed by relatively flat slip surfaces.



Figure 3. Typical safety map, showing FOS envelope of 1.50 and below based on long term conditions



Figure 4. Typical safety map, showing FOS envelope of 1.50 and below based on present conditions.

In contrast, using the augmented approach, Figure 4 shows the resulting safety map for the same slope with a FOS envelope of 1.50 and below, with present soil parameters inputted directly into the model.

Compared to Figure 3, the FOS envelope shown in Figure 4 indicates a general absence of relatively flat slip surfaces, resulting in the FOS envelope being tighter.

Whereas the conventional approach considers long term reduction in soil parameters and fluctuations in groundwater profile directly in the model, the augmented approach only considers this for the governing slip surfaces, which were generated based on present conditions.

While the augmented approach is, in many ways, an extension of the conventional approach, setback distances determined using the augmented approach are anticipated be equal to, or less than, the setback distances determined using the conventional approach. Example case studies demonstrating this are presented in Section 4.

3.5 Limitations of Augmented Approach

As the augmented approach is of conventional LE methods, limitations of the augmented approach reflect these methods, including the following:

- The validity of the slope stability model is completely dependent on reliability of subsurface input data. Based on observed conditions, the validity of the model can only be verified in a general sense. As Vick (1950) states, 'models may be corroborated by data to varying degrees, but this is always a matter of interpretation, not proof.'
- The model assumes that a well-defined slip surface forms instantly. As such, failure of a slope due to progressive formation of a slip surface cannot be considered in the model.
- The model assumes that a sliding mass behaves as a block. Therefore, the model cannot consider internal shearing and deformation of the sliding mass.
- In certain cases, the critical failure mode of the model may change between present and long term conditions. In such cases, all critical failure modes must be considered in the analysis.

It should be noted that the augmented approach does not specifically depend on all assumptions inherent with LE methods. For instance, stress-strain behavior of soil could be considered by first utilizing a finite element analysis to generate a numerical model of in-situ conditions. Then, chosen slip surfaces can be evaluated through the finite-element model using LE methods, as per Fredlund and Scoular (1999).

4 EXAMPLE CASE STUDIES

The following examples illustrate how the augmented approach may be used in practice. The two slopes

presented are located on the banks of the Edmonton River Valley, adjacent to the North Saskatchewan River. As per typical practice, the slopes were modelled based on survey data, field drilling and laboratory testing results, and measured groundwater levels. For the two examples considered, the corresponding models determined for slope stability analysis are shown in Figure 5 and Figure 6.

Analysis of the models was carried out in GeoStudio's SLOPE/W slope stability software, using the Morgenstern-Price method. As a proof of concept, only cohesion values were changed for the models. Other soil parameters (friction angle, unit weight, etc.) and groundwater profiles were not changed. In addition, only rotational failure modes were considered for the models.

4.1 Setback Distances Based on Conventional Approach

For the conventional approach, cohesion values selected for the models are shown in Table 1 and Table 2. Based on the conventional approach, resulting setback distances for the examples are shown on Table 3 and Table 4.

Table 1. Conventional approach, cohesion values for model in Example 1

Soil	Cohesion (kPa)
Fill	0
Silt and Clay	0
Clay Till	0
Sand	0
Clay Shale	25

Table 2. Conventional approach, cohesion values for model in Example 2

Soil	Cohesion (kPa)
Clay	0
Silt	0
Clay Till	0
Sand	0
Saskatchewan Sand & Gravel	0
Clay Shale	25

Table 3. Conventional approach, resulting setback distances for Example 1

Slip Surface	Minimum FOS	Setback Distance (m)
Circular	1.30	6.4
Circular	1.50	9.8

Table 4. Conventional approach, resulting setback distances for Example 2

Slip Surface	Minimum FOS	Setback Distance (m)
Circular	1.30	12.8
Circular	1.50	22.0

4.2 Setback Distances Based on Augmented Approach

For the augmented approach, cohesion values selected for the models are shown on Table 5 and Table 6. For the long term FOS, cohesion values selected for the governing slip surfaces are shown in Table 7 and Table 8.

Table 5. Augmented approach, cohesion values for model in Example 1

Soil	Cohesion (kPa)
Fill	0
Silt and Clay*	5
Clay Till*	10
Sand	0
Clay Shale	25

*cohesion values different from conventional approach

Table 6. Augmented approach, cohesion values for model in Example 2

Soil	Cohesion (kPa)
Clay	0
Silt*	5
Clay Till*	10
Sand	0
Saskatchewan Sand & Gravel	0
Clay Shale	25

*cohesion values different from conventional approach

Table 7. Augmented approach, cohesion values for governing slip surfaces in Example 1

Soil	Cohesion (kPa)
Silt and Clay	0
Clay Till	0

Table 8. Augmented approach, cohesion values for governing slip surfaces in Example 2

Soil	Cohesion (kPa)
Silt	0
Clay Till	0







Figure 6. Second case study model (Example 2)

As shown, cohesion values selected to determine the long term FOS for the governing slip surfaces are equal to cohesion values selected for the conventional approach. As such, the long term FOS determined will reflect similar long term conditions considered in the conventional approach, but with different slip surfaces

For each example, the entry point was progressively moved further away from the TOB, until the long term FOS met the target FOS. Based on the augmented approach, setback distances determined for the examples are shown on Table 9 and Table 10.

Compared to the conventional approach, resulting setback distance reductions using the augmented approach are shown on Table 11 and Table 12.

For the two examples considered, the corresponding FOS envelopes (conventional approach) and slip surfaces (augmented approach) used for setback distance determination are shown in Figure 7 and Figure 8.

Table 9. Augmented approach, setback distances determined for Example 1

Slip Surface	Minimum FOS	Setback Distance (m)
Circular	1.30	5.9
Circular	1.50	8.1

Table 10. Augmented approach, setback distances determined for Example 2

Slip Surface	Minimum FOS	Setback Distance (m)
Circular	1.30	12.2
Circular	1.50	20.9



Figure 7. FOS envelopes and slip surfaces for Example 1



Figure 8. FOS envelopes and slip surfaces for Example 2

Table	11.	Augmented	approach	, setba	ick dis	tance
reducti	ons d	etermined for	Example 1			

Minimum FOS	Setback (m)	Distance	Reduction	Setback Distance Reduction (%)
1.30	0.5			7.8
1.50	1.7			17.3

Table 12. Augmented approach, setback distance reductions determined for Example 2

Minimum FOS	Setback (m)	Distance	Reduction	Setback Distance Reduction (%)
1.30	0.6			4.7
1.50	1.1			5.0

4.3 Discussion of Example Case Studies and Setback Distance Reductions

Compared to the conventional approach, resulting setback distance reductions using the augmented approach ranged from 4.7% to 17.3%.

For a given slip surface, the available resisting force can be characterized by the Mohr-Coulomb failure criterion. As such, it can be understood that cohesion would have a greater impact on shallower slip surfaces, where effective stresses would be relatively low. Conversely, cohesion would have a lesser impact on deeper slip surfaces, where effective stresses would be relatively high.

Depending on the time frame being considered for long term conditions, it may be more appropriate to reduce cohesion closer to the face of the slope only, rather than the entire depth of the slope. However, the latter allows possible retrogression of the TOB over time to be considered.

As mentioned, only cohesion values were changed for the models as a proof of concept. However, friction angle values can also be included, such that reductions over time can be considered for strain weakening soils. For example, friction angle values closer peak can be used for present conditions, while friction angle values closer to softened or residual can be used for long term conditions.

In general, the results show that the augmented approach has the potential to provide a reduction in setback distances, while still considering the effects of long term processes.

5 POTENTIAL VARIATIONS TO AUGMENTED APPROACH

The examples presented within this paper tailor the augmented approach to determine the ELTLS. However, the inherent flexibility of the augmented approach also allows application of probabilistic methods to consider uncertainty and spatial variability of soil parameters and groundwater profiles. For example, using Monte-Carlo analysis, the FOS distribution for a governing slip surface could be determined, producing a setback distance based on reliability criteria.

In general, inclusion of Monte-Carlo analysis in the augmented approach would allow quantification of uncertainty and risk when determining the ELTLS.

6 CONCLUSIONS

For developments within the City of Edmonton, setback line determination is based on the ELTLS, for which long term processes must be considered.

Research in this area has focused on locating and evaluating the slip surface with the critical FOS. However, determination of setback distance based on a target, noncritical FOS, tends to yield numerous slip surfaces.

The augmented approach presented in this paper attempts to develop a framework to define a setback distance based on a target non-critical FOS, while considering the effect of long term processes on the FOS of a selected slip surface.

In general, augmented approach is an extension of conventional methods, and is an attempt by the authors to provide a starting point for further refinement of LE methods with respect to setback distance determination. The authors hope that this paper may open the door to other possible methods that may be used define reasonable setback distances based on FOS criteria.

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