Anisotropic behaviour of a sand predicted by a modified NorSand soil model

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ABSTRACT
Anisotropy of sands is easily observed through fixed principal stress direction shear tests using the hollow cylinder apparatus. The numerical modelling of such tests using an isotropic soil model would inevitably yield unsatisfactory behaviour predictions. In this study, the isotropic NorSand soil model was modified to account for soil anisotropy using the anisotropic critical state theory. This modified model was used to predict the behaviour of a Japanese clean sand (Toyoura sand) for various fixed principal stress direction shear tests. The predicted sand behaviour was satisfactory and compared favorably with laboratory results obtained by other authors. The characteristic contractive behaviour of sands loaded almost perpendicularly to their consolidation direction was well predicted by the modified anisotropic NorSand soil model.

RÉSUMÉ
L'anisotropie des sables est aisément observable grâce aux essais de cisaillement à direction des contraintes principales fixe effectués sur l'appareil de cisaillement sur cylindre creux. L'utilisation de lois de comportement isotropes pour la modélisation numérique de ce genre d'essais mène inévitablement à des résultats de prédiction de comportement insatisfaisants. Pour cette étude, la loi de comportement isotrope NorSand a été modifiée à l'aide de la théorie de l'état critique anisotrope pour prendre en compte l'anisotropie des sols. Cette version modifiée de la loi de comportement a été utilisée pour prédire le comportement d'un sable uniforme japonais (le sable Toyoura) pour différents essais de cisaillement à direction des contraintes principales fixe. Les prédictions de comportement obtenues se comparent de manière satisfaisante à des résultats d'essais de laboratoire analogues effectués par d'autres auteurs.

1 INTRODUCTION

Within the field of numerical modelling of soils, isotropy is a rather usual hypothesis. To consider identical soil properties in every direction is simpler in terms of modelling procedure but also requires much less model parameters to be defined. In their basic versions, both simple and complex constitutive laws are isotropic (Mohr-Coulomb, CamClay, NorSand, UBCSAND, etc.). However, real soils are anisotropic, and some geotechnical designs may require considering such anisotropy.

The modification of an isotropic soil model (NorSand) to take anisotropy into account is presented in this article. The modified anisotropic model is used to predict the behaviour of a Japanese clean sand under fixed principal stress direction shearing.

2 ANISOTROPY

2.1 Forms of anisotropy

There are two main forms of anisotropy: inherent anisotropy and induced anisotropy (Casagrande & Carillo, 1944). Inherent anisotropy is an intrinsic material characteristic caused by the shape of soil particles. This form of anisotropy is independent of strain history (Arthur & Menzies, 1972) and is more prominent when particle shapes are highly anisotropic. Hence it is expected that clay-like elongated particles will exhibit stronger inherent anisotropy than sand-like round particles.

Induced anisotropy is related to strain (or loading) history (Arthur *et al.*, 1977). Consider two identical spherical particle assemblages. Subject the first to vertical loading and the other to horizontal loading. Particles in both assemblages will rearrange according to the loading direction they are affected by, forming preferential grain arrangement patterns that are stiffer in that specific direction. After this initial directional deformation, if both assemblages were to be loaded vertically, they would exhibit very contrasting behaviour. The assemblage that was previously loaded vertically would be stiffer than its counterpart that was loaded horizontally because of the anisotropy induced by their respective previous loading direction. Vertically reloading the vertically consolidated assemblage solicitates its grain arrangement patterns favorably, hence the stiffer response. Exactly the opposite situation arises in the case of the horizontally consolidated assemblage loaded vertically. Its grain arrangement patterns were not built to withstand such a loading direction, yielding a softer and possibly contractive response (see Li & Dafalias, 2012, for a detailled demontration on induced anisotropy).

2.2 Anisotropic behaviour of sands

To fully appreciate the anisotropic behaviour of sands, one must load samples following various loading directions. As stated before, it is expected that a sand loaded in the same direction it has previously been consolidated will be stiffer than the very same sand would be if loaded perpendicularly to its consolidation direction. The hollow cylinder torsional shear apparatus is a most useful tool to study such anisotropic behaviour, as it allows for the loading direction to be independently controlled during shearing. This apparatus was used by many authors during the last 35 years to showcase the importance of anisotropy on sand behaviour.

Typical laboratory results of fixed loading direction hollow cylinder torsional shear tests are shown in Figure 1, where $\bar{\sigma}$ is the effective stress, ε is the deformation, the subscripts 1, 2 and 3 indicate major, intermediate and minor principal directions respectively, α is the loading direction with respect to the vertical axis, b is the intermediate principal stress ratio, e is the void ratio, D_r is the relative density and $\bar{\sigma}_m$ is the mean effective stress. As can be seen on the figure, as loading direction increases from $15°$ to $75°$, the response gets softer (from very dilative to very contractive). Such stress-strain curves (Figure 1a) and stress paths (Figure 1b) are characteristic examples of anisotropic sand responses. Multiple authors have reported very similar behaviours for other sands (Shibuya, 1985; Uthayakumar & Vaid, 1998; Yoshimine *et al.*, 1998). A detailed analysis of such shear tests is presented in Section 4 and 5.

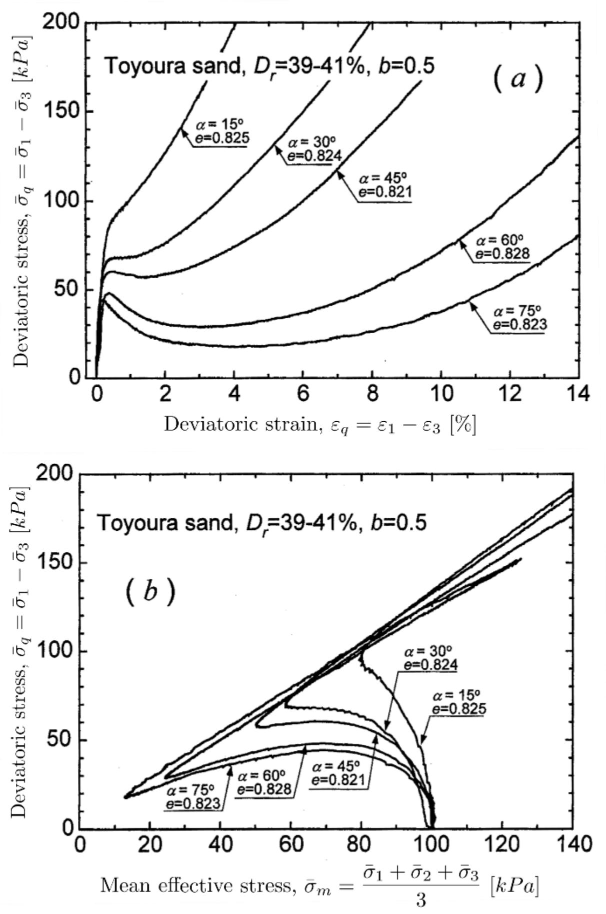

Figure 1. Anisotropic behaviour of Toyoura sand (modified from Yoshimine *et al.*, 1998)

Real-life loading cases where the loading direction is not parallel to the consolidation direction are plenty (i.e. slope stability, earthquake motions, water waves effect, etc.). Most geotechnical loading cases in fact consist of situations where the loading direction continually rotates along the shearing plane considered (Uthayakumar & Vaid, 1998). In such situations, not considering the softening effect of anisotropy would most probably lead to unsafe design.

3 NUMERICAL MODELLING OF ANISOTROPIC SAND BEHAVIOUR

There exist many anisotropic constitutive laws anyone can choose from to model soil behaviour. However, there are also many very capable isotropic soil models that could benefit from having the ability to model anisotropic soil behaviour. The latter is the core subject of this paper: the NorSand soil model was modified to consider anisotropy using the anisotropic critical state theory (Li & Dafalias, 2012).

3.1 The anisotropic critical state theory

The development of discrete element modelling (DEM) over the last two decades has allowed for substantial advancements in our comprehension of granular deformation and most notably fabric evolution through shearing processes. DEM provides a way to take a closer look into complex phenomena that are otherwise extremely difficult to measure in laboratory, such as the achievement of critical state conditions of deformation for example. Based on the DEM work of Li & Li (2009) on fabric evolution, Li & Dafalias (2012) proposed an extension for the critical state theory (Roscoe *et al.*, 1958) to account for anisotropic soil behaviour: the anisotropic critical state theory (ACST).

According to the classical critical state theory, there are two conditions to be met for critical state to be reached: the volumetric strain rate $\dot{\varepsilon}_v$ must be zero and the rate of volumetric strain rate $\ddot{\varepsilon}_v$ must also be zero while the soil is continuously sheared (Jefferies & Been, 2015). The anisotropic critical state theory proposes to add a third requirement for critical state conditions to be met: in simple terms, the fabric of the soil must be aligned with the loading direction. At critical state, if loading direction is suddenly changed, critical state requirements are no longer satisfied because the soil's fabric is no longer aligned with the loading during. The soil must necessarily undergo additional volumetric deformations to reach its critical state again.

The anisotropic critical state theory introduces the anisotropy parameter ψ_A (see Figure 2) that shifts the position of the critical state line (CSL) to account for anisotropy. The newly positioned critical state line is called the dilatancy state line (DSL) and is used to measure the dilatancy state parameter ζ, acting in lieu of the usual state parameter ψ (Been & Jefferies, 1985) to dictate contractive or dilative behaviour (see Equation 1). The anisotropy parameter varies according to fabric arrangement and loading direction: the stronger the effect of anisotropy, the larger the anisotropy parameter. As the critical state of deformation is reached, the anisotropy parameter falls to zero, resulting in the coincidence of the dilatancy state line and the critical state line.

Specifics of the anisotropic critical state theory are thoroughly explained in Li & Dafalias (2012).

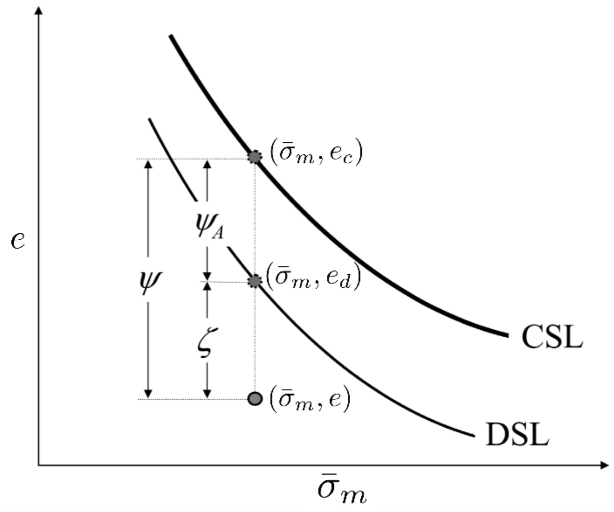

Figure 2. Anisotropic critical state parameters (modified from Li & Dafalias, 2012)

$$\zeta = \psi - \psi_A \qquad [1]$$

3.2 NorSand

NorSand (Jefferies, 1993) is a constitutive law within the broad family of critical state soil models. It uses the same bullet shaped yield surface as the original CamClay model (Roscoe & Schofield, 1963). It is anchored on the state parameter ψ to accurately predict soil (but mostly sand) behaviour. It has been extensively used to predict triaxial tests results (Jefferies & Been, 1992).

NorSand's formulation is isotropic, meaning that principal stress direction α has no effect on the simulation results it yields. However, the model was recently modified by its authors to account for principal stress rotation $\dot{\alpha}$ (the change in principal stress direction) in an attempt to improve simulation results of cyclic simple shear tests (Jefferies *et al.*, 2015). However, it is important to note that while the rotation of principal stresses has an effect of NorSand's yield surface, the actual direction of principal stresses has no effect on any component of the model.

In-depth presentation of NorSand is available in Jefferies & Been (2015).

3.3 Modelling Procedure

To demonstrate the usefulness of integrating the anisotropic critical state theory into NorSand, numerical modelling results of undrained fixed principal stress direction shear tests (constant loading direction) are presented in this article. The modelling was performed using the software MATLAB R2016b. The modelled tests replicate undrained hollow cylinder torsional shear tests such as those presented in Figure 1. These tests are performed at constant intermediate principal stress coefficient b (see Equation 2), constant principal stress direction α (see Equation 3, where τ_{xy} is the shear stress) and constant mean total stress σ_m. All tests modelled in this study used a b value of $0{,}5$ and a mean total stress of $100\ kPa$. Principal stress directions were varied from $15°$ to $75°$.

$$b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} \qquad [2]$$

$$\sin 2\alpha = \frac{2\tau_{xy}}{\sigma_1 - \sigma_3} \qquad [3]$$

An example of test boundary conditions is shown in Figure 3 and 4 for a principal stress direction of $60°$. In Figure 3, as deviatoric strains accumulate during shearing, both α and b remain constant throughout the modelled test. Analogously in Figure 4, the total mean stress (σ_m) remains constant throughout the test while pore-water pressure (u) varies during shearing, producing proportional fluctuations of mean effective stress ($\bar{\sigma}_m$).

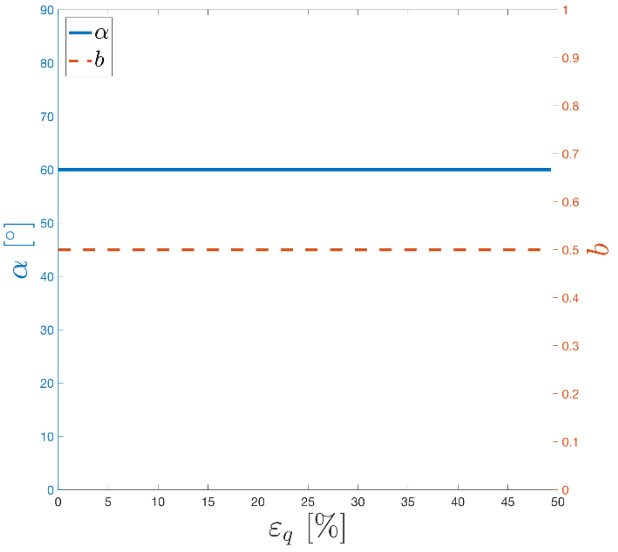

Figure 3. Principal stress direction (α) and intermediate principal stress coefficient b during modelling procedure

3.4 Toyoura sand

Results of numerical modelling of fixed principal stress direction shear tests (using the original and the modified anisotropic NorSand models are compared in Section 4. Behaviour predictions are made for Toyoura sand, a

Japanese clean sand. NorSand's model parameters for this sand are presented in Table 1. Likewise, the anisotropic critical state parameters for this sand are presented in Table 2.

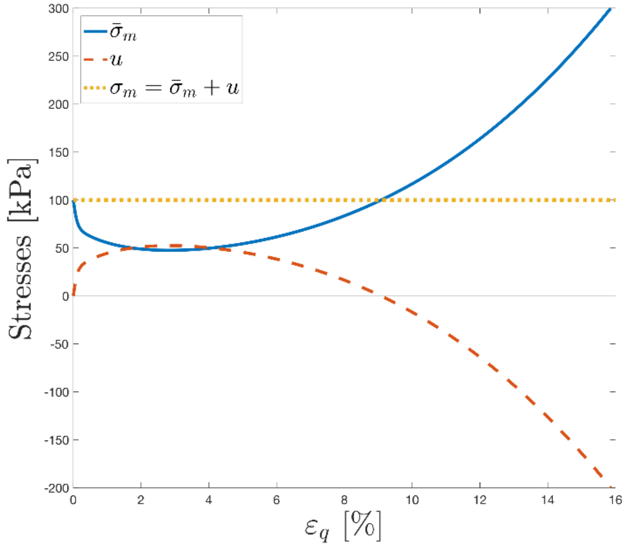

Figure 4. Total and effective mean stresses and pore-water pressure during modelling procedure

Table 1. NorSand parameters for Toyoura sand (from Ghafghazi & Shuttle, 2008)

NorSand parameters	Toyoura sand
Critical state line	
Γ	0.983
λ_e	0.019
Plasticity	
M_{tc}	1.28
χ_{tc}	4.4
N	0.41
H	400[1]
Elasticity	
$Ir = G/\bar{\sigma}_m$	$878\left(\frac{2.17-e}{1+e}\right)^2\left(\frac{\bar{\sigma}_m}{100}\right)^{-0.47}$
ν	0.2

[1] The hardening modulus was adjusted to best fit the laboratory results of Yoshimine *et al.* (1998)

Table 2. Anisotropic critical state theory (ACST) parameters for Toyoura sand (from Li & Dafalias, 2012)

ACST parameters	Toyoura sand
c	5.7
r	1
χ	0
e_A	0.094
F_{in}	0.6

4 MODELLING RESULTS

Both the original (isotropic) NorSand and the modified (anisotropic) NorSand models were used to predict fixed principal stress direction shear tests. Five different loading directions were modelled for each NorSand version. Each simulation was carried out using a void ratio of 0.828 and an initial mean effective stress of $100\ kPa$.The resulting stress-strain responses and stress paths are shown in Figure 5 and Figure 6 respectively (where $\bar{\sigma}_q$ and ε_q are the deviatoric stress and strain respectively).

The original NorSand model being isotropic, the loading direction has no influence on the calculated stress-strain response and stress path. Each simulated loading direction yields the same behaviour resulting in each plotted curve (the dashed lines) being superposed on top of the previous ones. Only one curve is thus visible on both figures for the original NorSand modelling. The original model predicts the sand would behave as a fairly dense sand, exhibiting dilatancy both in terms of deformation and pore-water pressure generation.

The modified anisotropic NorSand model yields very interesting modelling results, showcasing the loading direction dependency of anisotropic soils (such as shown in Figure 1). As the modelled loading direction increases from 15° to 75°, the resulting predicted behaviour displays stronger contraction. This is very apparent in Figure 5 where a decrease of shear resistance is evident for greater loading directions (60° and 75°), characteristic of looser contractive sands.

The computed stress paths of Figure 6 also highlight the dependency of the anisotropic sand behaviour to the imposed loading direction. As the loading direction gets closer to the horizontal direction, the amount of generated pore-water pressure increases dramatically (keeping in mind these fixed principal stress direction shear tests have a constant total mean stress boundary condition).

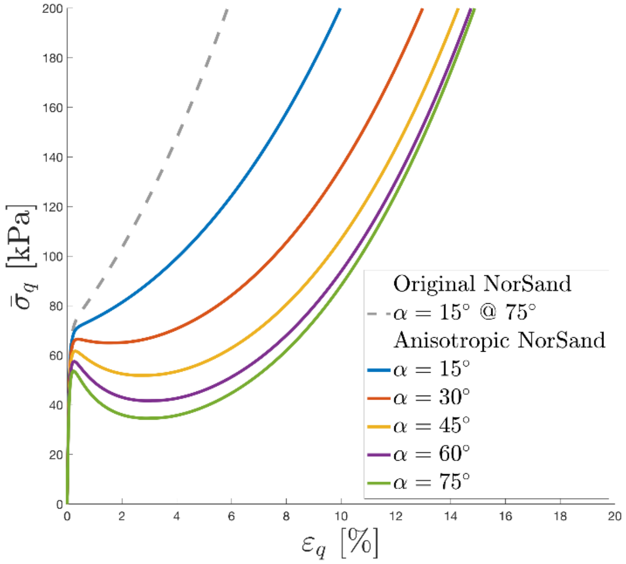

Figure 5. Original vs Anisotropic NorSand: Stress-strain response for five principal stress directions

The anisotropic NorSand model yields contractive behaviour for higher loading angles specifically because of the implementation of the anisotropic critical state theory in the model. As discussed earlier, the loading direction imposed during a shear test influences the anisotropy parameter ψ_A, which in turn modifies the dilatancy state parameter ζ. An example of this is provided in Figure 7 for a loading direction α of $60°$ (note that large deviatoric strains were modelled for the sake of discussion). At the beginning of shearing, the soil's state parameter ψ is -0.073. Therefore, a dense-like response is expected (just as displayed by the original isotropic NorSand in Figure 5 and 6). But to account for a different loading direction ($60°$) than the consolidation direction (vertical consolidation, $\alpha = 0°$, in this case), the anisotropic parameter ψ_A is subtracted from the state parameter to yield the dilatancy parameter ζ used in the calculations (see Equation 1). The resulting dilatancy parameter is positive (0.034), thus correctly yielding a contractive behaviour. As deviatoric strains progress in Figure 7, the soil's fabric aligns with the loading direction, reducing the prominence of anisotropy. In the process, the anisotropy parameter decreases and the difference between the dilatancy parameter and the state parameter also decreases. Ultimately, when critical state is to be reached, the anisotropy parameter will approach zero, bringing the critical state line and the dilatancy state line (see Figure 2) to converge.

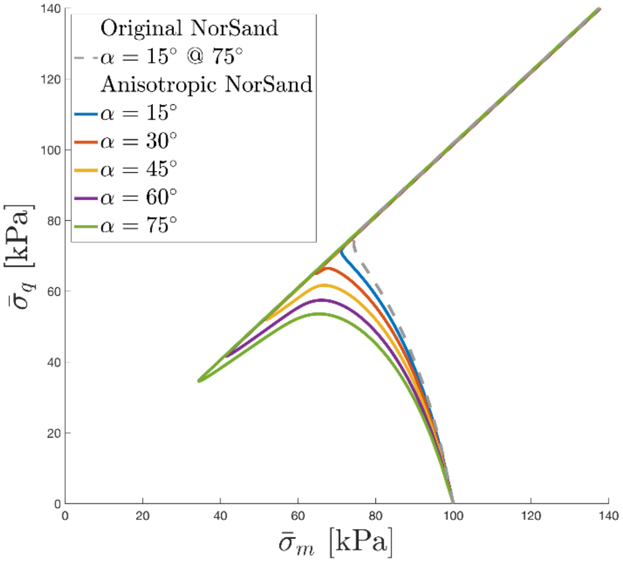

Figure 6. Original vs anisotropic NorSand: Stress path for five principal stress directions

5 COMPARISON WITH LABORATORY RESULTS

Results of numerical modelling of fixed principal stress direction shear tests were discussed in the previous section. These results will be compared to laboratory measurements made by Yoshimine *et al.* (1998) in the present section. Void ratios used for each loading direction considered are indicated in Figure 1. The intent of this comparison exercise is not to demonstrate the quality of the fit obtained between the numerical model and the laboratory results, but rather to highlight the good performance of the anisotropic critical state theory implemented in NorSand.

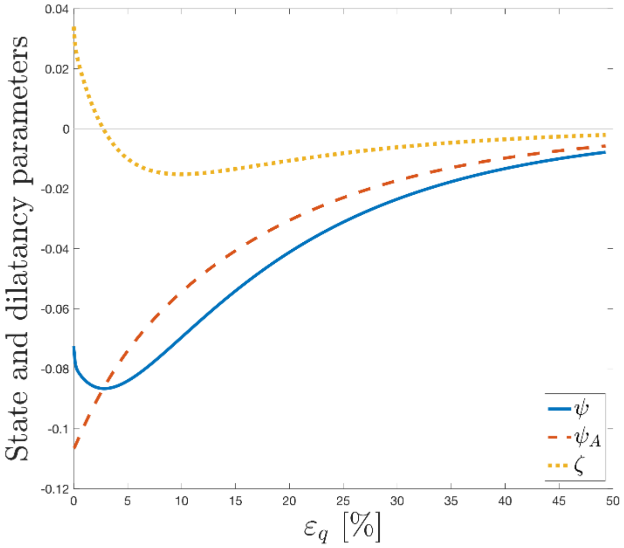

Figure 7. Evolution of state and dilatancy parameters for $\alpha = 60°$

On Figure 8 and 9, the comparison between the predicted and measured stress-strain responses and stress paths for a direction of loading of $60°$ are presented. On both figures, the laboratory results show a very contractive response. The measured stress path shows a rapid accumulation of pore-water pressure, similar to flow liquefaction, followed by a dilatant phase. Similar behaviour is obtained by the anisotropic NorSand model, although not as pronounced. Overall, trends are similar and satisfactory. The original NorSand model wrongly predicts a very dilative behaviour. The apparent brittleness observed on the stress-strain curve and the stress path of the laboratory results is completely absent from the isotropic model predictions.

Lastly, a comparison of the anisotropic NorSand model and the measured laboratory results for each loading direction considered is presented in Figure 10 and 11. Overall, the modified model acceptably predicts the trends observed in the laboratory data. The model is able to predict important features of anisotropic soil behaviour. Most notably, dilative behaviour is predicted for low principal stress direction angles (such as $15°$) and conversely, highly contractive behaviour is predicted for higher principal stress direction angles (such as 60° and 75°). These behaviour features match what is shown by the laboratory stress-strain and stress path curves.

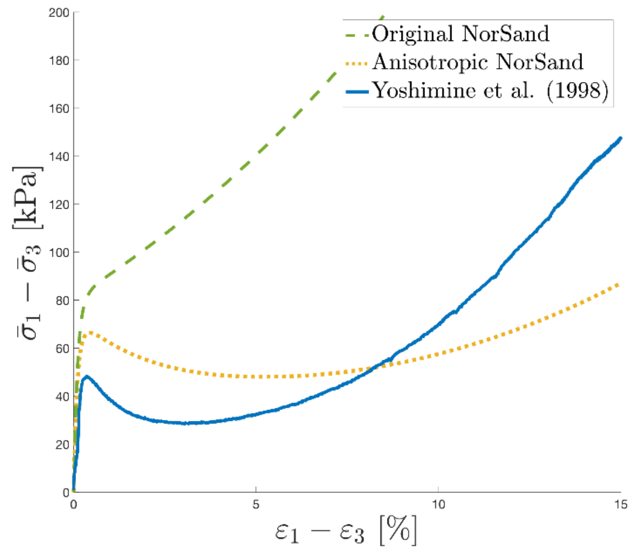

Figure 8. Comparison of predicted and measured stress-strain responses for $\alpha = 60°$

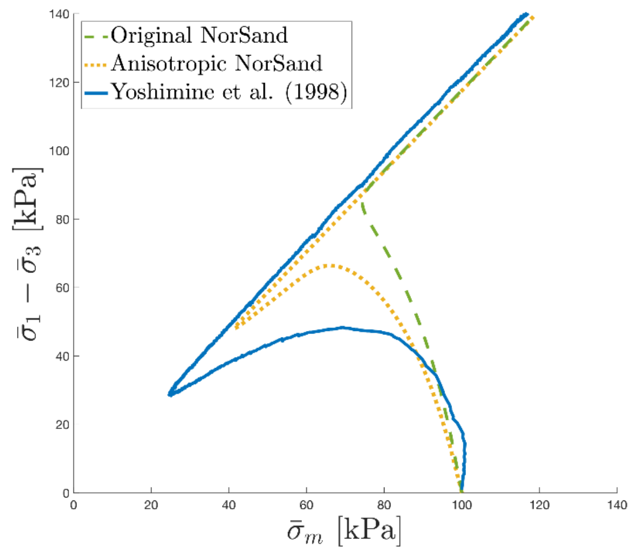

Figure 9. Comparison of predicted and measured stress paths for $\alpha = 60°$

However, it is notable that the anisotropic NorSand modelling results do not cover as wide a range of behaviour as the laboratory data, both in terms of stress strain behaviour and stress path followed. As is evident from Figure 10 and 11, dilative laboratory results (i.e. $\alpha = 15°$) are more dilative than their corresponding anisotropic NorSand modelling. The same applies for contractive laboratory results (i.e. $\alpha = 75°$). While the behaviour difference between low and high principal stress direction is extremely important within the laboratory results, the behaviour difference is not as dramatic for the modified NorSand results.

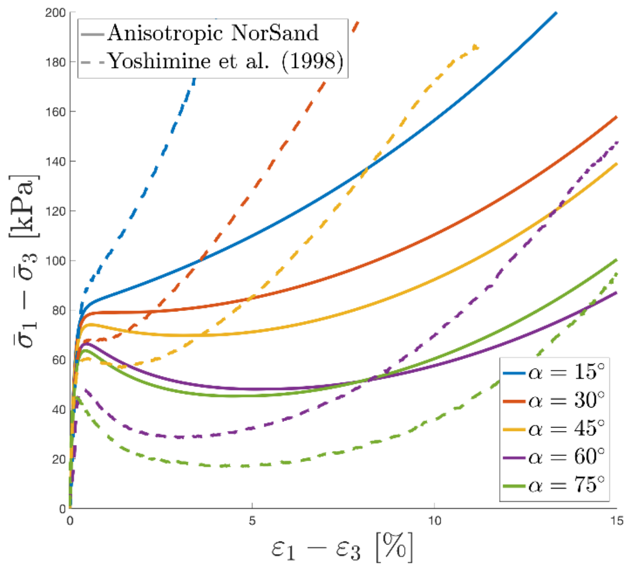

Figure 10. Stress-strain comparison for multiple principal stress directions

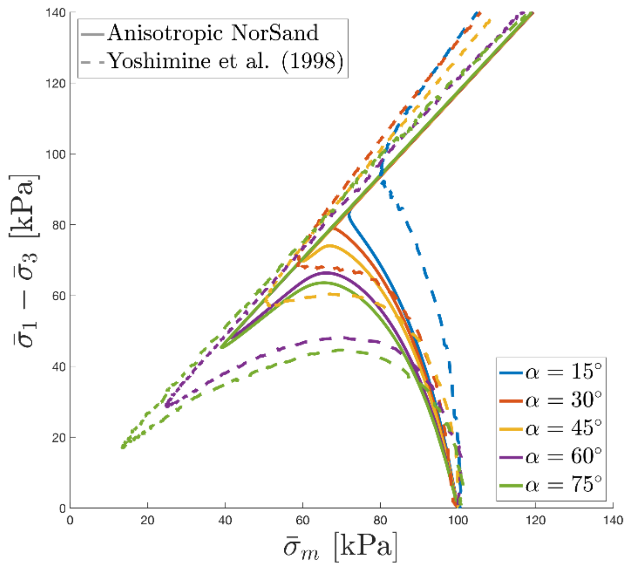

Figure 11. Stress path comparison for multiple principal stress directions

6 CONCLUSION

The numerical modelling of fixed principal stress direction shear tests was presented in this article. To adequately predict anisotropic sand behaviour, the NorSand soil model was modified by integrating the anisotropic critical state theory (Li & Dafalias, 2012). This new anisotropic NorSand model showed promising results in satisfactorily predicting the anisotropic behaviour of Toyoura sand. Modelling results correctly showed contractive behaviour for tests where the loading direction was close to 90° from the consolidation direction. Such contractive behaviour was impossible to obtain using the isotropic NorSand model.

There is however still room for improvement in accurately predicting the wide range of measured response from laboratory tests.

The work presented in this article sought to showcase potential applications of the anisotropic critical state theory (Li & Dafalias, 2012). The modifications made to the original isotropic NorSand model to account for anisotropic behaviour were modest and easily implemented. The newly developed anisotropic soil model could readily be used to simulate the behaviour of sands for loading situations where anisotropy greatly affect the soil's response (i.e. simple shear tests, earthquake loading, etc.).

7 ACKNOLEDGEMENTS

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