Considering Groundwater Advection on the Design of Borehole Heat Exchanger—A Review of Analytical Solutions

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ABSTRACT
The borehole heat exchanger (BHE) is a crucial component of GeoExchange systems because the efficiency of the system relies primarily on the heat transfer between BHE and its surrounding ground. When groundwater advection exists, this heat transfer can be enhanced, providing a synergistic effect on the thermal performance of BHE. As a result, this synergistic effect can shorten the design length of BHE, thereby reducing the drilling and installation costs. Although the benefit of groundwater advection is evident, most current design software have not considered the effects of groundwater advection due to its complexity and insufficient research in this area. Until now, there are only a few analytical solutions that have addressed this issue. However, these solutions were developed under different assumptions and boundary conditions, and there is little research to summarize and compare them. In the present study, the objective is to conduct a review of the analytical solutions that have accounted for the effects of groundwater advection on BHE. Firstly, a description was performed for various analytical models that involved groundwater advection and for different application conditions of these models (i.e., assumptions and boundaries). Then customized MATLAB coding was deployed to analyze the mathematical solutions for these analytical models using the same input parameters from a GeoExchange project in Edmonton (AB, Canada). After that, the results were summarized to reveal the advantage and disadvantage of these analytical solutions. Based on that, recommendations have been made for further study that includes groundwater advection in the BHE design.

Nomenclature

c	specific heat capacity	[J/kgK]
$E_1(x)$	exponential integral function	
$erf(x)$	error function	
$erfc(x)$	complementary error function	
H	borehole length	[m]
$I(x)$	$=\int_x^\infty \beta^{-1}e^{-\beta^2}\,d\beta$	
I_0	modified zero order Bessel function of the first kind.	
k	thermal conductivity	[W/mK]
q_l	heat flow rate per unit length of the borehole	[W/m]
r, r'	distance to the source	[m]
r_b	radius of the borehole	[m]
t	time	[s]
T	average temperature of the medium	[K]
T_0	undisturbed ground temperature	[K]
ΔT	temperature difference	[K]
U	$=u_d\rho_w c_w/(\rho c)$	[m/s]
u_d	groundwater velocity	[m/s]
x,y,z,z'	coordinates	[m]
Greek symbols		
α	thermal diffusivity	[m^2/s]
β	$=\frac{r}{2\sqrt{\alpha(t-t')}}$	
γ	Euler's constant	
η	$=\frac{1}{2\sqrt{\alpha t}}$	[m^{-1}]
ξ	$=4\alpha(t-t')/r^2$	
ρ	density	[kg/m^3]
Subscripts		
w	water	
b	borehole	
Superscripts		
$<\ >$	$=\frac{1}{H}\int_0^H \cdots dz'$ integral mean	

1 INTRODUCTION

Nowadays, new buildings have been increasingly utilizing GeoExchange system, which use the shallow ground—typically less than 150 meter—as an energy reservoir for space cooling and heating (Zhao et al. 2016). The shallow ground is a desirable energy reservoir because the ground temperature remains relatively constant below a certain depth throughout the year (Koohi-Fayegh and Rosen 2013). For example, In Canada, the shallow ground temperature is commonly within a range of 6 to 12 °C, which is colder in summer and warmer in winter than the common ambient air temperatures (Grasby et al. 2011). Using the shallow ground, the GeoExchange system provides higher energy efficiency and lower electricity consumption, resulting in a substantial reduction of GHG (greenhouse gas) emissions (amount depending on local grid emissions factors and use of renewable energy generation) (Ahmadi et al. 2017).

Within the GeoExchange system, a critical component is the ground heat exchanger (GHE), which is used for heat extraction from (or injection into) the ground (Luo et al. 2013). The most common form of GHEs is a vertical closed loop system which has single U-tube installed into a vertical borehole down to a depth ranging from 50-150 m (Bernier 2006). Through U-tubes, pure water or water with antifreeze fluid (e.g., propylene glycol) is circulated to transfer heat (Yang et al. 2010). In this way, the GHE system is also referred as the borehole heat exchanger (BHE). The efficiency of the BHE—heat transfer—plays a critical role in the performance of GeoExchange system and the required borehole length (Capozza et al. 2013; Li et al. 2017).

The complicated thermal processes consist of two parts—the first one is within the borehole, and the second is between the borehole and its surrounding ground (Li and Lai 2015). To accurately estimate the second part, a broad range of models have been developed using analytical, numerical, or hybrid approaches. In particular, a hybrid approach (i.e. Eskilson's model) has been widely implemented in commercial BHEs design software (e.g., GLHEPro 4.1), which incorporated both analytical and numerical techniques (GLHEPro 2014) . In Eskilson's model, a two-dimensional (radial-axial) finite difference model is used to model the G-function, which represent the temperature response to the unit-step heat flux of a single borehole (Eskilson 1986). In addition, since the heat flux varies according to the heating and cooling loads of a building in actual applications, the spatial and temporal superposition techniques are employed to calculate the thermal response (i.e., g-function) of a given physical layout of multiple boreholes to time-varying heat flux (Eskilson 1986; He 2012). In order words, once the time-varying building loads and borehole configuration are determined, the heat transfer problem can be easily tackled using G-function solutions. The G-function can be obtained numerically, such as finite-difference method (Eskilson 1986), or analytically (Carslaw and Jaeger 1959; Diao et al. 2004; Ingersioll et al. 1954; Molina-Giraldo et al. 2011), this paper mainly focuses on analytical solutions.

The current available G-function from analytical solutions have been evolved from the Kelvin infinite line source model (ILS) (Ingersioll et al. 1954) and the infinite cylindrical source (ICS) model (Carslaw and Jaeger 1959). However, there are many assumptions during the development of the ILS and ICS models: one assumes that the ground is a purely conductive medium without groundwater flow (Tye-Gingras and Gosselin 2014). This assumption has been widely accepted in both the ASHRAE (American Society of Heating, Refrigerating and Air-Conditioning Engineers) handbook and commercial software packages for BHE design (Angelotti et al. 2014; ASHRAE 2015). The ASHRAE handbook and commercial software packages only employ the purely heat conductive model and neglect the effects of groundwater advection (Bandos et al. 2009; Diao et al. 2004). However, when groundwater exists, the heat exchange between borehole and ground is inherently coupled with gross heat advection by the movement of groundwater, providing a synergistic effect on the thermal performance of BHE. This synergistic effect can shorten the design length of BHE while giving the same performance, so the initial cost spent on drilling and installation can be reduced. Despite the benefits, there are only a few refined G-functions that have addressed the influence of groundwater in BHE design. Furthermore, these refined G-functions have not been reviewed and compared with the current design approach (Erol and François 2018; Molina-Giraldo et al. 2011; Rivera et al. 2015). In this regard, the comparison of these refined G-functions is significant because it will preliminarily highlight the role of groundwater flow on the design of the BHE.

In the present study, the objective is to conduct a review of four analytical models for a single borehole. Among these models, two of them include the effects of groundwater advection. One of them is used as the reference model—the modified finite line source model (FLS), which is the current design approach used in commercial software (i.e., GLD2016) (Ground Loop Design 2016).

2 ANALYTICAL MODELS

In this section, four analytical models have been briefly reviewed—the infinite line source model (ILS), the finite line source model (FLS), the moving infinite line source model (MILS), and the moving finite line source model (MFLS). Among these models, ILS is the earliest version of G-function; FLS is the current design approach used in commercial software (i.e., GLD2016); and the MILS and MFLS are refined G-functions that include the effects of groundwater flow.

2.1. Infinite line source model (ILS)

One dimensional infinite line source model (ILS) (in the radial direction) is the first analytical model to evaluate the thermal interactions between a single borehole and its surrounding ground (Priarone and Fossa 2015). It is called "infinite line source" because the radius of the borehole is tiny when compared to its length. As a result, the thin borehole is considered as an infinite line source in an infinite medium (Ingersoll and Plass 1948). In ILS, Kelvin's line source theory was reiterated to solve the pure conduction problem for thermal process analysis (Ingersioll et al. 1954; Monzó 2011):

$$\begin{cases} \rho c \dfrac{\partial T}{\partial t} = k\left(\dfrac{\partial^2 T}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial T}{\partial r}\right) \\ r \to 0 \quad -2\pi k \lim_{r\to 0} r\dfrac{\partial T}{\partial r} = q_l \end{cases} \tag{1}$$

where T [K] denotes the temperature of the ground, q_l [W/m] is the rate of heat production or withdrawal, k [W/mk] is the thermal conductivity, ρc $[J/m^3K]$ is the volumetric heat capacity of the medium, and $r = \sqrt{x^2 + y^2}\ [m]$ is the distance to the line source.

This approach has been derived with the following simplifications (Meester 2013):

1. The entire borehole is considered as an infinite line source with a constant heat flux, q_l;
2. The surrounding ground is considered as an infinite, isotropic and homogeneous medium; and
3. The heat flow only involves heat conduction in one-dimension, which is normal to the line source.

If the medium has a uniform initial temperature T_0, the temperature difference of the surrounding ground ($\Delta T = T - T_0$) can be obtained at any later time t [s] and any radius r [m] around the line source by solving equation (1) (Ingersioll et al. 1954):

$$\Delta T = \frac{q_l}{2\pi k}\int_{r\eta}^{\infty}\frac{e^{-\beta^2}}{\beta}d\beta \equiv \frac{q_l}{2\pi k}I(r\eta) \tag{2}$$

where $= \frac{r}{2\sqrt{\alpha(t-t\prime)}}$, $\eta = \frac{1}{2\sqrt{\alpha t}}\ [m^{-1}]$, and $I(x) = \int_x^{\infty}\beta^{-1}e^{-\beta^2}d\beta$. Introducing the exponential integral function $E_1(x) = \int_x^{\infty}\frac{e^{-t}}{t}dt$, which is a special function in mathematics, Carslaw and Jaeger (Carslaw and Jaeger 1959) described Equation (2) as:

$$\Delta T = \frac{q_l}{4\pi k}\int_{r^2/4\alpha t}^{\infty}\frac{e^{-u}}{u}du = \frac{q_l}{4\pi k}E_1\left(\frac{r^2}{4\alpha t}\right) \tag{3}$$

where $\alpha = k/\rho c\ [\mathrm{m}^2/s]$ is the thermal diffusivity. At the time $t \geq 5r^2/\alpha$, the exponential integral $E_1(x)$ in equation (3) can be approximated as:

$$E_1\left(\frac{r^2}{4\alpha t}\right) = ln(\frac{4\alpha t}{r^2}) - \gamma \tag{4}$$

where γ donates the Euler's constant (approximating to 0.5772…).

Gehlin (Gehlin 2002) reported that the natural logarithm approximation gives errors less than 10%. The maximum error drops to 2.5% for the time $t \geq 20r^2/\alpha$.

The ILS model is characterized by its simplicity. However, these assumed simplifications may also restrict its applications. First, the ILS model ignored the presence of groundwater flow which has considerable influence in the long-time period. Besides, for yearly temperature response simulation, the solution of Equation (4) will never reach a steady-state value because the effect of ground surface as a boundary is neglected (Bandos et al. 2009; Zeng et al. 2002). In addition, the heat conduction at the bottom of the borehole cannot be quantified. This has been proven by Marcotte et al. (Marcotte et al. 2010), who concluded that the borehole length could be 15% shorter when the heat conduction is included at the bottom and top of boreholes. Despite the above shortcomings, the ILS model is still widely applied in the in-situ thermal response test (TRT) to determine the ground thermal conductivity (Zhang et al. 2014).

2.2 Finite line source model (FLS)

A step forward to the ILS, the finite line source model (FLS) has two major changes: (1) assuming that the interface between ambient air and ground surface is maintained at constant temperature (as same as initial ground temperature) (Priarone and Fossa 2015); (2) considering the borehole as a continuous point heat source to solve the governing equation of purely heat conduction (Lienhard 2013) as below.

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) \tag{5}$$

First proposed by Eskilson (Eskilson 1986), a general solution of the FLS model was later constructed by Zeng et al. (Zeng et al. 2002). Setting a virtual line heat sink on symmetry to the ground surface, the constant temperature boundary condition at the surface is compiled by the symmetrical distribution of line heat source and sink. Then the temperature rise of the surrounding ground can be obtained by integrating contributions of the continuous point heat sources at the point coordinates $(0, 0, \mathrm{z}')$ of the line heat source and sink (Zeng et al. 2002):

$$\Delta T = \frac{q_l}{4\pi k}\int_0^H\left[\frac{erfc\left(\frac{\sqrt{r^2+(z-z')^2}}{2\sqrt{\alpha t}}\right)}{\sqrt{r^2+(z-z')^2}} - \frac{erfc\left(\frac{\sqrt{r^2+(z+z')^2}}{2\sqrt{\alpha t}}\right)}{\sqrt{r^2+(z+z')^2}}\right]z' \tag{6}$$

where $erfc(x)$ is the complementary error function in mathematics, which is defined as:

$$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty}e^{-t^2}dt \tag{7}$$

However, Equation (6) only gives the temperature response at a single point. One additional integration is required to estimate the average temperature response along the borehole wall. Accordingly, this double integration results in a remarkable increase of computational efforts. Therefore, Lamarche and Beauchamp (Lamarche and Beauchamp 2007) and Bandos et al. (Bandos et al. 2009) simplified the double integrals into a single integral. A more straightforward expression for integrated average temperature introduced by Bandos et al. (Bandos et al. 2009) is:

$$\begin{aligned} &< \Delta T > \\ &= \frac{q_l}{4\pi k}\int_{r/\sqrt{4\alpha t}}^{\infty}\left[4\,erf\left(\frac{Hu}{r}\right) - 2\,erf\left(\frac{2Hu}{r}\right) - (3 \right. \\ &+ exp(-\frac{4H^2u^2}{r^2})\cdots \\ &\left. - 4\,exp(-\frac{H^2u^2}{r^2}))\frac{r}{\sqrt{\pi}Hu}\right]\frac{e^{-u^2}}{u}du \end{aligned} \tag{8}$$

where $erf(x)$ is the error function in mathematics, which is defined as:

$$erf(x) = 1 - erfc(x) \tag{9}$$

Note that this modified formulation, Equation (8), have been implemented in GLD2016 as the current design approach.
The FLS model has considered the finite length of the borehole, through which the effects of axial heat conduction is taken into account. In particular, the axial heat conduction accelerates the heat exchange at the bottom of the borehole and transfers the imbalance heat of extraction and injection on a year-round basis to the ambient air through the ground surface (Molina-Giraldo et al. 2011; Zeng et al. 2002). However, the FLS model is still a solution to the purely heat conductive problem, which neglects the influence of groundwater flow.

2.3 Moving infinite line source model (MILS)

The moving infinite line source model (MILS) was firstly explored by Diao et al. (Diao et al. 2004) to address the influence of groundwater flow. In the MILS, the heat transfer is composed of (1) heat conduction through the solid and water in the pores and (2) the heat advection through the flowing groundwater. The two-dimensional equation for this relationship can be expressed as follow (Diao et al. 2004):

$$\rho c \frac{\partial T}{\partial t} + u_d \rho_w c_w \frac{\partial T}{\partial x} = k(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) \qquad (10)$$

where u_d [m/s] denotes the uniform groundwater velocity which can be determined using Darcy's law.
The following assumptions have been made to derive the MILS model (Diao et al. 2004):

1. The groundwater is parallel to the ground surface and defined in the *x*-direction;
2. The entire borehole is considered as an infinite line source with a constant heat flux, q_l;
3. The surrounding ground is considered as a semi-infinite, isotropic and homogeneous medium, and its thermal properties is independent of the temperature changes; and
4. The surrounding ground has a uniform initial temperature, T_0.

Diao et al.(Diao et al. 2004) and Sutton et al.(Sutton et al. 2003) have solved the partial differential equation (4) for the saturated porous medium using Green's function method (Carslaw and Jaeger 1959):

$$\Delta T = \frac{q_l}{4\pi k} exp(\frac{Ux}{2\alpha}) \int_0^{r^2/4\alpha t} \frac{1}{\xi} exp(-\frac{1}{\xi} - \frac{U^2 r^2 \xi}{16\alpha^2})\, d\xi \qquad (11)$$

where $\mathrm{U} = u_d \rho_w \mathrm{c}_w/(\rho\mathrm{c})$ [m/s] and $\xi = 4\alpha(\mathrm{t}-\mathrm{t}')/\mathrm{r}^2$.
The MILS model has incorporated the effect of moving groundwater. However, this model has many limitations. For example, results produced by the MILS model usually overestimate the long-term temperature response (years and decades), because the axial effects are not taken into account (Molina-Giraldo et al. 2011). This is a common disadvantage of all the analytical models considering the borehole as an infinite line source.

2.4 Moving finite line source model (MFLS)

For long-term thermal response simulation, both the axial heat conduction and groundwater flow can considerably change the temperature regime around the borehole. Hence, a general solution of MFLS model has been developed by Molina-Giraldo et al. (2011) using the same theory as the MILS model. To do so, the MFLS model has inherited the same assumptions made in MILS, expect that the borehole is considered as a line source with finite length H in the MFLS model. This MFLS model is valid for all groundwater flow velocities and borehole lengths (Liuzzo-Scorpo et al. 2015). In addition, Molina-Giraldo et al. (2011) also claimed that for low groundwater flow scenarios (i.e., Peclet numbers < 1.2), the FLS model yields acceptable results because the heat transfer process is dominated by heat conduction; while for high groundwater flow scenarios (i.e., Peclet numbers > 10), the heat transfer process is convective dominated and the difference between MILS and MFLS models can be neglected. For a Peclet numbers range between 1.2 and 10, only MFLS can accurately evaluate the temperature response.
The solution from MFLS model turns out to be (Molina-Giraldo et al. 2011):

$$\Delta T = \frac{q_l}{2\pi k} exp\left(\frac{Ux}{2\alpha}\right)\left[\int_0^H f(x,y,z,t)\, dz' - \int_{-H}^0 f(x,y,z,t)\, dz'\right] \qquad (12)$$

$$f(x,y,z,t) = \frac{1}{4r}\left[exp\left(-\frac{Ur'}{2\alpha}\right) erfc\left(\frac{r'-Ut}{2\sqrt{\alpha t}}\right) + exp\left(\frac{Ur'}{2\alpha}\right) erfc\left(\frac{r'+Ut}{2\sqrt{\alpha t}}\right)\right] \qquad (13)$$

where $\mathrm{r}' = \sqrt{x^2+y^2+(z-z')^2}$ $[m]$ is the distance to the point source at the coordinates $(0,0,\mathrm{z}')$.
Similar to Equation (6), Equation (11) needs integrating itself over the borehole length and the borehole circumference for the average borehole wall temperature. Therefore, inspired by Lamarche and Beauchamp (Section 2.2) (Lamarche and Beauchamp 2007), Rivera et al. (Rivera et al. 2016) reformulated the triple integral formulation and introduced an alternative one to describe the integrated average temperature:

$$<\Delta T> = \frac{q_l}{8\pi k} I_0(\frac{Ux}{2\alpha}) \int_{r^2/4\alpha t}^{\infty} \frac{1}{\varphi} exp\left[-\varphi - (\frac{R_b Pe}{4})^2 \frac{1}{\varphi}\right]\left\{4\, erf\left(\frac{H\sqrt{\varphi}}{r}\right) \cdots - 2\, erf\left(\frac{2H\sqrt{\varphi}}{r}\right) \cdots + \frac{r}{\sqrt{\pi\varphi}H}\left[4\, exp\left(-\frac{H^2\varphi}{r^2}\right) - exp(-\frac{4H^2\varphi}{r^2}) - 3\right]\right\} d\varphi \qquad (14)$$

where *$I_0(x)$* is the modified zero-order Bessel function of the first kind.
The MFLS model requires a massive demand for computational resources and a well understanding of the complex groundwater flow (Zhang et al. 2016). For this reason, no current commercial design tools have used the MFLS model (Wang et al. 2009).

In brief, this section introduced four analytical models. In the following sections, the long-term (ten years) heat transportation processes were simulated using a real GeoExchange system project in Edmonton (AB, Canada) to obtain the thermal responses on the borehole wall. In the simulation, a single vertical borehole was modeled with a length of 70m through the homogenous and semi-infinite ground. Based on this, a comparison was made between these models and the GLD2016 result to evaluate the effects of groundwater flow.

3. PARAMETER SETTINGS USED IN THE CALCULATION

As shown in Table 1, a typical borehole used in a real GeoExchange system project in Edmonton is chosen: borehole length ($\mathrm{H} = 70.0\ \mathrm{m}$), and borehole radius ($r_b = 0.0919\ m$). The thermal properties are obtained from an in-situ thermal conductivity test: the undisturbed ground temperature tested ($T_0 = 6℃$); the thermal conductivity ($k = 1.59\ W/mK$); the thermal diffusivity of the borehole ($\alpha = 6.944 \times 10^{-7}\ \mathrm{m^2/s}$); the volumetric heat capacity of the bulk porous medium ($\rho c = k/\alpha = 2.290 \times 10^6\ \mathrm{J/m^3K}$). In addition, the ground is assumed as an isotropic and homogeneous medium. The hydraulic and thermal properties are assumed to be independent with the temperatures. The groundwater is assumed in the *x*-direction and parallel to the ground surface with a uniform velocity, $u_d = 1 \times 10^{-7}\ m/s$. Then a customized MATLAB coding was deployed, by which the mathematical solutions for the borehole wall temperature were assessed.

Table 1 Parameters used in the calculations (REVOLVE 2018)

Parameter	**Value**
Length of the borehole (H)	70 m
Borehole radius (r_b)	0.0919 m
Thermal conductivity ground (k)	$1.59\ \mathrm{W/mK}$
Thermal diffusivity of the ground (α)	$6.944 \times 10^{-7}\ \mathrm{m^2/s}$
Volumetric heat capacity-ground (ρc)	$2.290 \times 10^6\ \mathrm{J/m^3K}$
Volumetric heat capacity-water ($\rho_w c_w$) at 6 °C	$4.200 \times 10^6\ \mathrm{J/m^3K}$

4. RESULTS AND DISCUSSION—A COMPARATIVE STUDY OF VARIOUS G-FUNCTIONS FOR BHES

In this section, the mathematical solutions of these four analytical models are presented as the integrated average temperature along a typical borehole wall over the time (from one hour to ten years). In actual applications, the heat flux varies according to the heating and cooling loads of a building. To compare the results of different G-functions, a constant heat transfer rate, $q_l = 1W/m$, is applied in this study. As shown in Figure 1 and Table 2, all temperature responses increase with time, and they are in close agreement with each other in the first three months. In particularly, in the first 50 hours (a typical duration of TRT test), the results of ILS and other models (FLS/MILS/MFLS) differ by less than 0.4%. This small difference indicates that, in short-term, the other models cannot provide much more information regarding the influence of axial heat transfer and groundwater. .After the third month, the temperature responses from FLS and MFLS start to be more different (~1%), and the dispersion gets larger as the simulation time increases. This dispersion can be explained by the synergistic effect of groundwater flow, by which the accumulated heat is balanced (Diao et al. 2004). After ten years, the ILS model yields the temperature of 0.548 ºC without convergence; the FLS model reaches a lower temperature of 0.513 ºC without convergence; and the MILS and MFLS model obtain the temperature responses to steady-state values of 0.453 ºC and 0.437 ºC respectively. Note that after ten years the FLS/MFLS results have a 14.8% difference. This substantial difference between FLS and MFLS indicates that groundwater flow has a large effect on the thermal process in high groundwater scenario (i.e., Darcy velocity of 10^{-7} m/s). This effect becomes more evident if the Darcy velocity increases (Molina-Giraldo et al. 2011). In general, neglecting groundwater flow can cause an overestimation of ground temperature variation. This ultimately results in an underestimation of geothermal potential, leading to an oversized borehole design which increasing the first cost of the system (Rivera et al. 2015). ASHREA has an elaborate design procedure to size BHEs, in which the heat transfer rate is divided into a series of constant heat rate "pulses" (ASHRAE 2015). In our case, the presence of groundwater can approximately reduce the borehole by about 5m/borehole calculated using the ASHREA procedure (compared with the results from GLD2016 using FLS).

Table 2 Integrated average borehole wall temperature (ºC) with various specified periods

	Integrated average borehole wall temperature (ºC)						
Duration	1-hour	1-day	50-hour	1-month	3-month	1-year	10-year
ILS	0.0144	0.1404	0.1762	0.3089	0.3639	0.4332	0.5480
FLS (Ref)	**0.0143**	**0.1400**	**0.1755**	**0.3059**	**0.3584**	**0.4222**	**0.5128**
MILS	0.0144	0.1403	0.1761	0.3074	0.3593	0.4160	0.4530
MFLS	0.0143	0.1399	0.1754	0.3044	0.3540	0.4062	0.4370

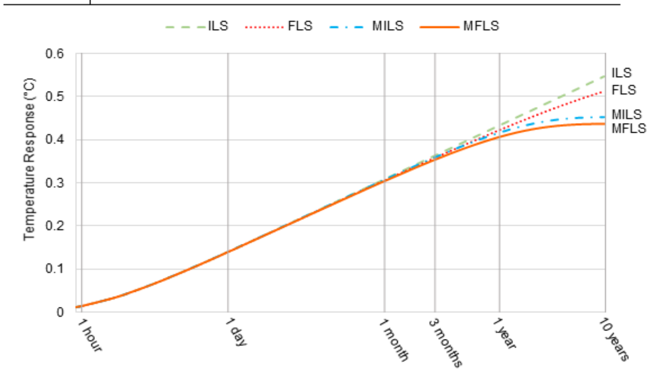

Figure 1 Comparison of various analytical models for temperature response (A log-10 scale of time (year) is used in the *x*-axis)

Despite the improvements, the MFLS model has three major limitations that hindered its application in the current BHE design:

First, the analytical solution of thermal responses is only for one single borehole (or BHE). However, multiple boreholes (or BHEs) are common in large-scale commercial buildings that have high energy demands. In this case, temperature differences of multiple BHEs should be more significant than the one of single BHE due to the thermal interaction between boreholes (Katsura et al. 2009). In practice, commercial design software (i.e., GLD2016) has a built-in function to calculate ground temperature changes according to the BHE field layout. However, this built-in function does not include the existence of groundwater flow.

The second limitation is due to the homogeneous assumption, where the ground is considered as a homogeneous medium. In reality, thermal properties of soil vary with depth, and the homogeneous assumption causes underestimation or overestimation of the temperature variations respect to each soil layer (Abdelaziz et al. 2014). The ground heterogeneity consideration is used to evaluate the long-term performance of BHE. In particular, if a part of the BHE is operated in low thermal conductivity clays, the risk of freezing and thawing has to be evaluated based on correct temperature change estimation. Accordingly, the potential soil swelling and compaction can be avoided under the freezing and thawing cycles. To this end, Erol and Francois (Erol and François 2018) introduced an analytical solution of heat conduction, advection and dispersion problem in a multilayer porous medium with groundwater flow for a finite line source. This model can further optimize the BHE design.

The last limitation is the parallel assumption of groundwater flow in MILS/MFLS models. Among all of the analytical models that applied the methodology of moving heat source, including the models that were not mentioned in this paper (Rivera et al. 2015; Tye-Gingras and Gosselin 2014; Zhang et al. 2016), a common assumption has been adopted—the groundwater is parallel to the ground surface. However, in engineering practice, the moving direction of groundwater may be horizontal, vertical or inclined. In this case, the reliability of moving line source solutions primarily depends on how much the parallel assumption represents the real situation.

To sum up, the ILS model is the simplest, but it is only applicable within a narrow time scale range from a few hours to months. The modified FLS model (with axial effects) has been implemented in current commercial design tools; however, FLS model does not account for the groundwater flow. A step further, the MILS model considered the presence of moving groundwater, but the application is still limited by assuming the borehole as an infinite line source. The MFLS is more accurate for long-term prediction because the axial effects and groundwater flow are included. Although the MFLS is more reliable than the other three, it is also restricted by the homogeneous assumption and parallel assumption (Erol and François 2018; Li and Lai 2015). To address these limitations, numerical methods may be chosen to evaluate the heat transfer process of BHE (Abdelaziz et al. 2014; Lee). Numerical methods can offer a more accurate and detailed evaluation when the three-dimensional model was fully discretized (Bauer et al. 2011). In the future study, the influence of groundwater flow can be quantitatively tested through comparative studies between industry-dominant software, available analytical models, and numerical models. The findings of the study could be used to guide the BHEs design to a better performance.

5. CONCLUSIONS

The following conclusions were drawn from the study:

- A review of analytical models has summarized the infinite line source model (ILS), the finite line source model (FLS), the moving infinite line source model (MILS), and the moving finite line source model (MFLS).
- Groundwater flow has a beneficial effect on the long-term simulation. After ten years, the groundwater flow with Darcy velocity of $1 \times 10^{-7}\ m/s$ can reduce the ground temperature variation by about 14.8% for one single borehole in a local case. And a potential borehole reduction can be achieved.
- To optimize the current BHE design using the MFLS model, there are three major limitations need to be overcome: These limitations would be addressed by numerical modeling in future studies.

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