Investigating the Crushing Behavior of Weathered Phyllite Fills Under Compaction

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ABSTRACT

Weathered phyllite fill is crushable and will be further broken during the compaction process. In this paper, standard ASTM compaction tests were performed on weathered phyllite fills with 45%, 55% and 65% rock content, to investigate the crushing behavior of weathered phyllite fill under compaction. The particle size distributions before and after compaction were observed and then the particle breakage of weathered phyllite fills were analyzed by fractal dimension. Furthermore, relative fractal dimension was proposed in this paper and was adopted to evaluate the effect of initial rock content and initial gradation on the particle breakage. It was found that fractal dimension well characterized the crushing behavior of weathered phyllite fills. The weathered phyllite fills became finer with the increased number of times it was compacted. Under the first compaction, the degree of crushing was greatly affected by the initial rock content and increased with greater rock contents. However, under the second compaction, the degree of crushing rate was dominated by initial particle gradation, and the better the initial particle gradation, the lower the rate of crushing. The findings in this paper may provide guidance for the construction of weathered phyllite subgrade fills.
Key words: weathered phyllite fills, crushing behavior, fractal dimension, relative fractal dimension.

RÉSUMÉ

Les charges de phyllite altérées sont écrasables et seront encore brisées pendant le processus de compactage. Dans cet article, les essais de compactage ASTM standard ont été effectués sur des charges de phyllite altérées avec 45%, 55% et 65% de contenu de roche pour étudier le comportement de concassage des charges de phyllite altérées par compactage. Et les distributions granulométriques avant et après compactage ont été observées, puis la rupture des particules de phyllite altérée a été analysée par dimension fractale. En outre, la dimension fractale relative a été proposée dans cet article et a été adoptée pour évaluer l'effet de la teneur initiale en roches et de la gradation initiale sur la rupture des particules. Il a été constaté que la dimension fractale pourrait bien caractériser le comportement d'écrasement des charges altérées. Les charges de phyllite altérées sont devenues plus fines avec une augmentation des temps de compactage. Lors du premier compactage, le degré d'écrasement était grandement affecté par la teneur initiale en roches et augmentait avec l'augmentation de la teneur en roches. Cependant, dans le cadre du deuxième compactage, le taux de broyage était dominé par la gradation initiale des particules, et meilleure est la gradation initiale des particules, plus faible est le taux de broyage. Les résultats de ce document peuvent fournir des indications pour la construction d'un sous-sol de remplissage en phyllite altérée.
Mots clés: charges phyllitiques altérées, comportement au broyage, dimension fractale, dimension fractale relative.

1 INTRODUCTION

Weathered phyllite is classified as a soft rock based on its mechanical properties, whose particles are prone to breakage under large stresses (Mao et al., 2011). Where weathered phyllite is commonly encountered, it is inevitable that it will be used as subgrade fill, considering the geological conditions, economic conditions and environmental protection (Chen et al., 2014). During placement, weathered phyllite fill will often be crushed during compaction, resulting in uneven deformations of the subgrade. Therefore, it is of great significance to investigate the disintegration characteristics of weathered phyllite fills.

In recent years, scholars have been carrying out a great deal of research on the crushing behavior of soil and rock particles (Ueng et al., 1988; Lade & Yamamuro, 1996; Gao et al., 2009; Li et al., 2015). Nakata & Miura (2007) pointed out that particle breakage characteristics had a close relationship with the intra-particle void ratio. Particle breakage is strongly associated with the mechanical behavior of a soil (Bolton, 1986; Karube et al., 1999; Vilhar et al., 2013; Qin et al., 2014). In addition, particle breakage influences a soil's volumetric dilation, peak stress ratio and more importantly will change the plastic deformation mechanisms and the shear failure modes (Kwok & Bolton, 2011; Wang & Yan, 2013; Yu & Towhata, 2016). Moreover, particle breakage resulted in an increase of the mobilized basic friction angle, especially before failure (Fang & Li, 2016). The relationship between particle breakage and the critical state of sand was investigated by Luzzani & Coop (2002), Ghafghazi et al. (2014), and Xiao et al. (2016).

It was found that particle breakage of a coarse-grained soil increased with increased compactive effort (Hua et al., 2009; Wang et al., 2009; Chu, 2014). The initial gradation of a soil sample also had a great effect on the particle breakage (Zhang & Baudet, 2013; Chen et al, 2015). Generally, there are two methods for evaluating the degree of particle crushing: the breakage potential as proposed by Hardin (1985) and the fractal dimension as proposed by Tyler (1992). For many granular materials, it was found that the limiting grading during compaction was the fractal distribution of particle sizes with a fractal dimension of around 2.5–2.6 (McDowell et al., 1996; McDowell & Bolton., 2000).

Previous studies on particle breakage have mainly focused on granular materials. Nevertheless, weathered phyllite has a flaky structure and it may differ from the properties of granular materials. This paper presents experimental data on the breakage of weathered phyllite fills under compaction. The application of fractal dimension in evaluating the crushing behavior of weathered phyllite fills was also examined. Furthermore, relative fractal dimension was adopted to evaluate the effect that different initial rock contents and initial gradations of weathered phyllite fills had on their particle breakage.

2 FRACTAL DIMENSION

Soils or geological materials in nature are self-similar or scale-invariant. Therefore, fractal geometry (Mandelbrot, 1983) has become a widely accepted descriptive tool of physical systems and has been widely used in micro and macro experiments. On this basis, Tyler & Wheatcraft (1992) assumed that different soil particles had the same density, and then established a fractal model of particle mass-particle distribution, as shown in Eq. 1.

$$\frac{M_1(\mathrm{d_i})}{M_\mathrm{t}} = 1 - \frac{M_2(\mathrm{d_i})}{M_\mathrm{t}} = \left(\frac{\mathrm{d_i}}{\mathrm{d_{max}}}\right)^{3-D} \qquad [1]$$

where d_i is the sieve size, dmax is the maximum particle size of the weathered phyllite fill, $M_1(d_i)$ is the mass of the weathered phyllite fill with a particle size of less than di, $M_2(d_i)$ is the mass of a weathered phyllite fill with particle sizes of larger than d_i, M_t is the total mass of the weathered phyllite fills, and $M_t = M_1(d_i)+M_2(d_i)$, D is the fractal dimension of the weathered phyllite fills.

Taking the logarithm on both sides of Eq. (1), we get:

$$\lg\left[M_1(\mathrm{d_i})/M\right] = (3-D)\lg\left(\mathrm{d_i}/\mathrm{d_{max}}\right) \qquad [2]$$

As is shown in Eq. 2, if $\lg\left[M_1(\mathrm{d_i})/M\right]$ is linearly correlated with $\lg\left(\mathrm{d_i}/\mathrm{d_{max}}\right)$, then the weathered phyllite fills under study possess fractal characteristics and the fractal dimension can be obtained if the particle size and mass of the weathered phyllite fills is known.

3 MATERIALS AND METHODS

3.1 Test Material

The weathered phyllite fills under study were chosen randomly from the eastern Ankang section of Shitian highway in China (see Figure 1). The mineralogy of the weathered phyllite fill consisted predominantly of clay minerals, quartz and sericite quartz.

Figure 1. Weathered phyllite fillers under study

According to ASTM D1557–12, test soils (materials) should have 30% or less by mass of their particles retained on the 3⁄4in. (19.0mm) sieve. It was necessary to equivalently replace the over sized particles of the fill so that the maximum particle size was 37.5mm after replacement. Particles that are retained on a No. 4 (4.75mm) U.S. standard sieve are defined as rock (ASTM D 2488-17), hence, weathered phyllite particles with a diameter larger than 4.75mm were defined as rock in this paper. Three groups of weathered phyllite fills with different initial gradations of 45%, 55% and 65% rock content were prepared for this study. The grain size distributions for the three weathered phyllite fills is shown in Figure 2.

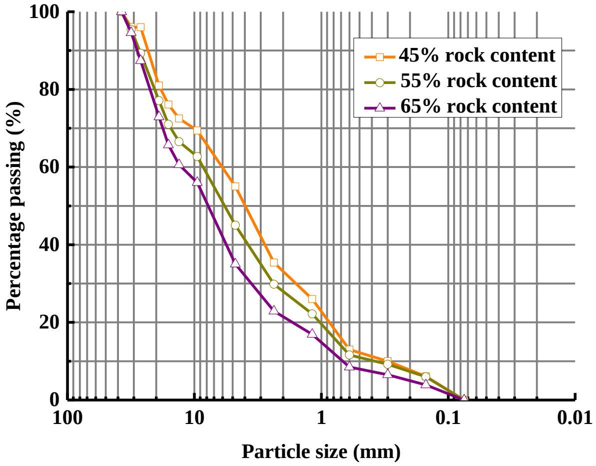


Figure 2. Grain size distributions for weathered phyllite fillers

As illustrated in Figure 2, the gradation curve of weathered phyllite with 65% rock content is at the bottom, while gradation curve of weathered phyllite with 45% rock content is at the top. It shows that the larger the rock content, the coarser the particles are in the weathered phyllite fills. In addition, the coefficients of uniformity (U) of the three groups were all larger than 10, while the coefficients of curvature (Cc) ranged from 1 to 3 (see Table 1), indicating that the three groups of fills were well-graded. In addition, It should be noted that the coefficients of uniformity (U) of weathered phyllite fills with 55% rock content was larger than other ones, meaning that weathered phyllite fills with 55% rock content had a wider particle distribution than other two groups.

Table 1. Coefficients of uniformity and curvature of weathered phyllite fills

Rock content	d_{60}(mm)	d_{30}(mm)	d_{10}(mm)	U	Cc
45%	6.40	1.68	0.30	21.33	1.47
55%	8.78	2.39	0.40	21.94	1.63
65%	12.72	3.76	0.70	18.08	1.58

3.2 Preparation of Soil Samples

Three groups (45%, 55%, and 65% rock content) of weathered phyllite fills were prepared to be tested. First the weathered phyllite fills were dried at 105℃. Then the fills were mixed with distilled water to achieve their optimum moisture contents, which were 8.0%, 7.3% and 6.7% respectively. The prepared fills were then placed in closed containers for 24 hours to make their moisture contents as uniform as possible.

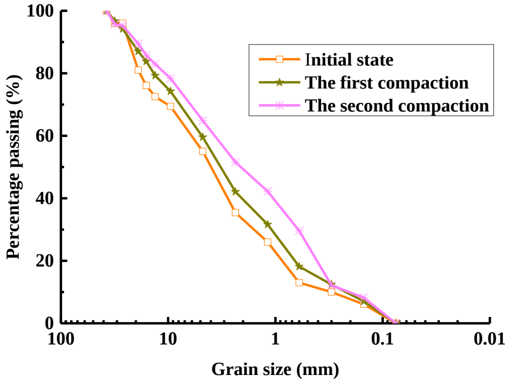
3.3 Test Method

Standard ASTM compaction tests were carried out according to ASTM D 1557-12, using a mold with an internal diameter of 6 in. (152.4mm), and a height of 4.584 in. (116.4mm). The prepared weathered phyllite fills were placed in five layers into the mold, with each layer compacted by 25 or 56 blows with a 10.00-lbf (44.48N) rammer dropped from a distance of 18.00 in. (457.2mm). Therefore, the weathered phyllite fills were each subjected to a total compactive effort of about 56 000 ft - lbf / ft3 (2700kN-m/m3). After the ASTM compaction test, the weathered phyllite fills were placed in oven for at least 8 hours, and then they were screened to determine their particle-size distribution. This whole process was called the first compaction. After that, the samples were again recompacted a second time according to ASTM D 1557-12, using the same compaction process and equipment as with the first compaction test and was called the second compaction.

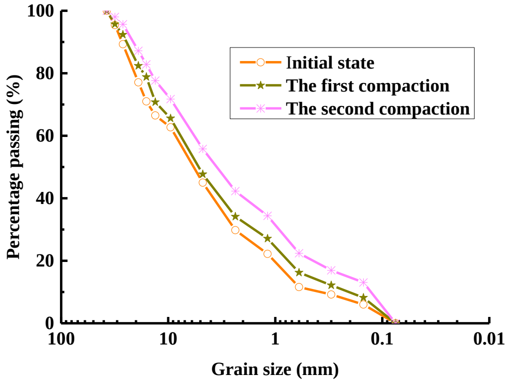
4 RESULTS AND DISCUSSION

4.1 Grain Size Changes Before and After Compaction

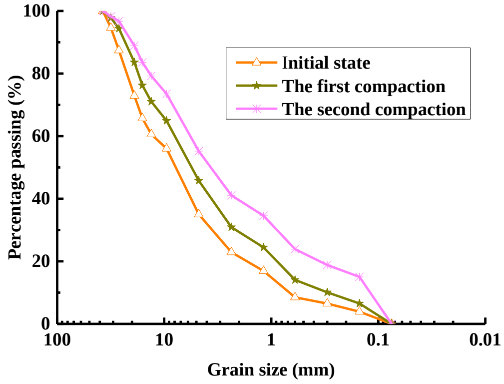
Grain size distributions for the weathered phyllite fills before and after compaction are shown in Figure 3. As shown in Figure 3, the gradation curves of weathered phyllite fills after compaction were above the gradation curves of the weathered phyllite before compaction, indicating that the particles of the weathered phyllite fills were being broken by compaction. In addition, the gradation curves of the weathered phyllite fills after the second compaction tests were above those of the first compaction tests, showing that the weathered phyllite fills were being further broken down during the second compaction test as compared with the first compaction test. All in all, the weathered phyllite fills became finer after each compaction test.



(a) 45% rock content



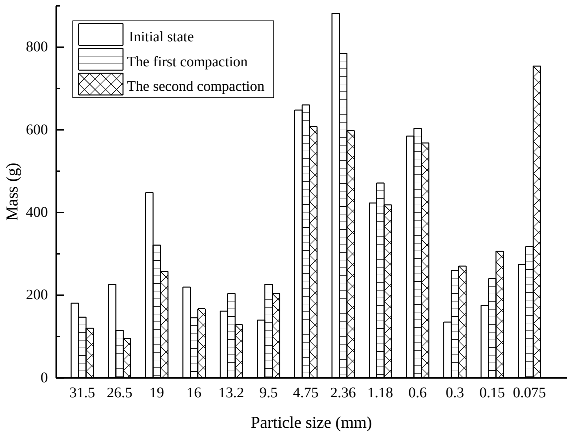
(b) 55% rock content



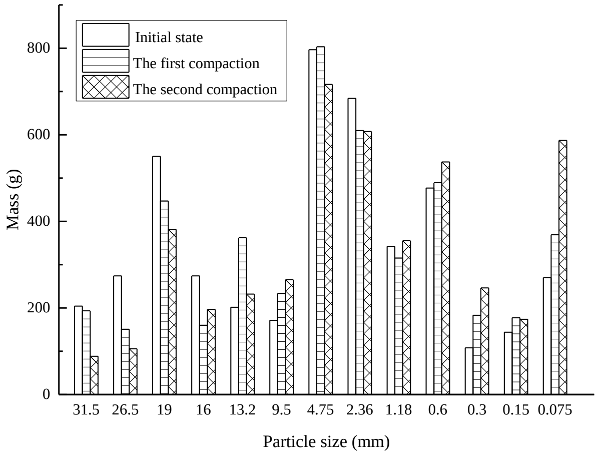
(c) 65% rock content

Figure 3. Grain size distributions for the weathered phyllite fillers before and after compaction

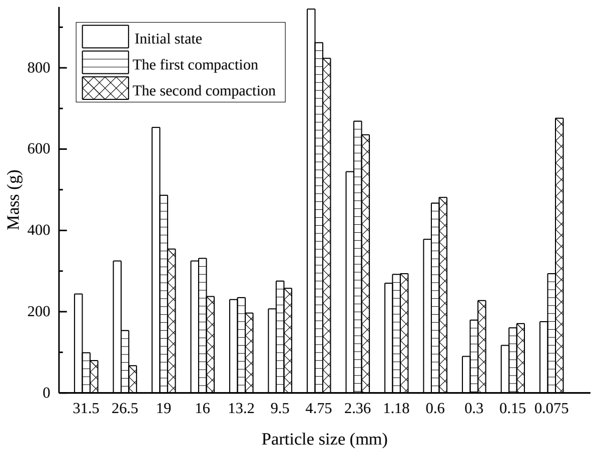
In order to further investigate the crushing characteristics of the weathered phyllite fills, the particle mass changes of the weathered phyllite fills with different rock content were counted (see Figure 4). For weathered phyllite fills with 45% and 55% rock contents, there was an observable decrease in the mass of particles retained on the 16~31.5mm sieve and the 2.36mm sieve, with a remarkable increase in the mass of particles retained on the 9.5~13.2mm sieve and 0.15~0.075mm sieve. For weathered phyllite fills with a 65% rock content, the mass of the particles retained on the 19~37.5mm and 4.75mm sieves were significantly reduced, while mass of particles groups retained on the 9.5mm sieve and 2.36~ 0.075mm sieves were dramatically increased. It is interesting to note that mass of larger size particles decreased, while mass of smaller size particles increased, indicating that the larger size particles broke into smaller ones under compaction. This is due to that smaller particles exhibiting higher strengths than larger ones (Nakata et al., 2001; McDowell, 2002).



(a) 45% rock content



(b) 55% rock content



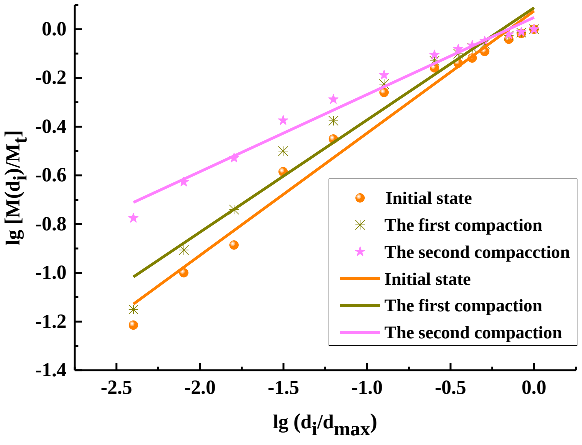
(c) 65% rock content

Figure 4. Mass of weathered phyllite fills retained on various sieves before and after compaction

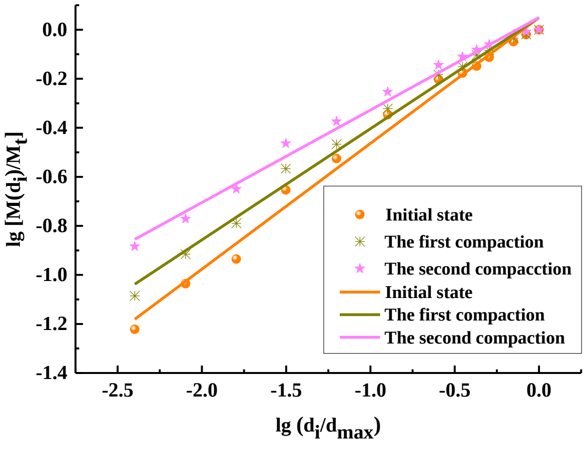
Furthermore, for weathered phyllite fills with a 45% rock content, the mass of the particles retained on the 0.075mm sieve after the first compaction and second compaction increased by 38.83% and 174.75% respectively compared to the initial state samples. For weathered phyllite fills with a 55% rock content, the mass of particles retained on the 0.075mm sieve after the first compaction and second compaction increased by 36.70% and 117.44% respectively compared to the initial state samples. For the weathered phyllite fills with a 65% rock content, the mass of particles retained on the 0.075mm sieve after the first compaction and second compaction was 67.29% and 285.24% respectively compared to the initial state samples. It was found that the amount of small grains that were created increased with greater rock contents, similarly to the findings of Sammis & Stacey (1995) and Zhang & Baudet (2013). Therefore, not only compactive effort, but also the initial rock content of the weathered phyllite fills had an effect on the amount of particle crushing.

4.2 Fractal Dimensions of Weathered Phyllite Fills

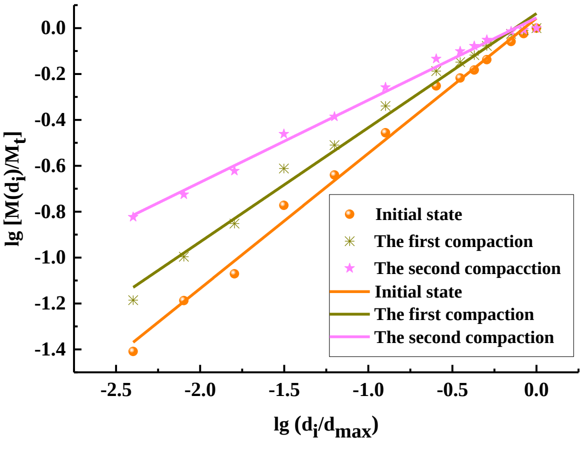
Data calculated according to Eqs. 1 and 2 were plotted in Figure 5, and these data were linearly fitted. It was found that the data had a good linear correlation and the coefficients of correlation were greater than 0.96, suggesting that the weathered phyllite fills before and after compaction had fractal characteristics. The fractal dimensions of the weathered phyllite fills are shown in Table 2.



(a) 45% rock content



(b) 55% rock content



(c) 65% rock content

Figure 5. Fitting curves of $\lg(d_i / d_{\max})$ vs $\lg[M(d_i) / M_t]$ for weathered phyllite fills before and after compaction

Table 2. Fractal dimensions of weathered phyllite fills before and after compaction

State	Rock content		
	45%	55%	65%
Initial state	2.50	2.49	2.41
The first compaction	2.54	2.55	2.50
The second compaction	2.68	2.62	2.64

As shown in Table 2, under the initial state, the fractal dimension of the weathered phyllite fills with a 45% rock content was the largest among the three groups. In contrast, fractal dimension of the weathered phyllite fills with a 65% rock content was the smallest. This is in agreement with the theory of fractal dimension, namely, the coarser the fills, the smaller the fractal dimension. The weathered phyllite fills with a 65% rock content were coarser than other groups, hence, its fractal dimension was the smallest. After compaction, the fractal dimensions of all the weathered phyllite fills increased. It means that weathered phyllite fills were being crushed and got finer after compaction. Moreover, the fractal dimensions of the weathered phyllite fills after the second compaction were larger than that after the first compaction, indicating that weathered phyllite fills were being further broken down compared with the first compaction. This finding is consistent with the results we got in the analysis of the particle mass changes of the weathered phyllite fills (Part 4.1). It shows that fractal dimension can well characterize the fragmentation characteristics of weathered phyllite fills.

4.3 Relative Fractal Dimensions of Weathered Phyllite Fills

It should be noted that fractal dimension can only reflect the crushing characteristics of a certain group of weathered phyllite fills and it cannot be adopted to directly compare and evaluate the degree of crushing of different groups of weathered phyllite fills. Therefore, on the basis of previous studies, this paper puts forward the relative fractal dimension (as shown in Eq. 3) to compare the fragmentation degree of different groups of weathered phyllite fills.

$$D_{\mathrm{r}} = \frac{D - D_0}{D_0} \qquad [3]$$

where, D_r is the relative fractal dimension of weathered phyllite fills; D_0 is the fractal dimension of weathered phyllite fills before compaction; D is the fractal dimension of weathered phyllite fills after compaction.

According to Eq. 3, the relative fractal dimensions of weathered phyllite fills are shown in Table 3.

Table 3. Relative fractal dimensions of weathered phyllite fillers

State	Relative fractal dimension		
	45%	55%	65%
The first compaction	0.016	0.024	0.038
The second compaction	0.074	0.055	0.096

4.4 Effect of Initial Rock Content and Initial Gradation on the Degree of Crushing

As illustrated in Table 3, after the first compaction, the relative fractal dimension of the weathered phyllite fills increased with the increase in rock content. Namely, the larger the rock content of the weathered phyllite fills, the greater the degree of crushing. It may be due that high rock content also means a higher number of large particles are also present. According to Nakata et al. (2011) and McDowell (2012), smaller particles exhibit higher strengths than larger ones. Consequently, the breakage of weathered phyllite fills seems to increase with the larger particle content under the first compaction. The data shown in Table 3 also illustrates that after the second compaction, the fractal dimensions of the weathered phyllite fills all increased in relation to that after the first compaction, indicating that the weathered phyllite fills were being further broken down. This is due to the fact that by increasing in the number of compaction times also increases the number of opportunities to break down the larger particles in the phyllite fill.

Nevertheless, the relative fractal dimension of weathered phyllite fills with a 55% rock content after the second compaction was smaller than that with a 45% rock content. In other words, degree of crushing for weathered phyllite fills with a 55% rock content was lower than that for weathered phyllite fills with a 45% rock content. This phenomenon would seem to be not in agreement with the theory put forward by Nakata et al. (2011) and McDowell (2012). This is because that initial particle gradation is of key importance in defining the degree of crushing of soil samples.

According to Muir Wood (2006), initial particle gradation is of key importance in defining the degree of crushing of soil samples, and the wider the particle size range in a single sample, the greater the packing efficiency, resulting in a higher coordination number for larger particles and thus lowering the probability of breakage. In addition, Altuhafi & Coop (2011) pointed out that well graded soil samples exhibited less breakage than those with uniform gradation, even when the gradation is good enough, the gradation of particles will not change even subjected to high stress or strain (Altuhafi et al., 2011). It can be seen from Table 1 that the coefficients of uniformity (U) of weathered phyllite fills with a 55% rock content were larger than those of the other rock contents, meaning that weathered phyllite fills with a 55% rock content had wider particle distribution than the two other groups. Accordingly, the weathered phyllite fills with a 55% rock content experienced the less breakage in comparison to other two groups after the second compaction.

For weathered phyllite fills with a 55% rock content, the smaller particles may be participating in the contact force chain, thus reducing the amount of stress on the larger particles; for weathered phyllite fills with 65% rock content, the smaller particles may not be participating in the contact force chain, leaving the larger particles bearing larger forces and more vulnerable to breakage (Vallejo & Lobo-Guerrero, 2012; Zhang & Baudet, 2013). For weathered phyllite fills with a 45% rock content, there are too many small particles to form a contact force chain, leading to breakage of larger particles more easily. In brief, with increasing the number of compaction times, the effect of rock content of weathered phyllite fills on the degree of crushing was weakened gradually, while the effect of initial gradation of weathered phyllite fills on the degree of crushing increased gradually. In the long run, the impact of the initial gradation of the weathered phyllite fills on the degree of crushing was dominant.

5 CONCLUSIONS

Weathered phyllite fills with rock contents of 45%, 55% and 65% were tested under compaction, and their particle breakage behaviors were analyzed. The following results were obtained.

(1) After compaction, greater particles of weathered phyllite fills broke into smaller particles, resulting in an increase of smaller particles.

(2) Fractal dimension could well reflect the crushing behavior of weathered phyllite fills and the degree of crushing of the weathered fills increased with the number of compaction times. Moreover, relative fractal dimension could directly compare the breakage rate of weathered phyllite fills with different rock content.

(3) For the first compaction test, the initial rock content of weathered phyllite fills played an important role in the particle breakage. The higher the initial rock content, the greater the amount of crushing, with the lowest degree of crushing in the weathered phyllite fills with a 45% rock content.

(4) In the long term, the initial gradation of weathered phyllite fillers dominated the breakage amount. Well graded weathered phyllite fills exhibited less breakage than those with a uniform gradation. After the second compaction, the degree of crushing of weathered phyllite fills with a 55% rock content was the lowest.

6 ACKNOWLEDGEMENT

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