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A numerical study of mechanical impact from ice loss from fractures in thawing rock

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ABSTRACT

In this paper, we introduce a numerical model to analyse the thermo-mechanical interactions present during the thawing of an ice-filled discontinuity. The proposed model is applicable to near-surface ice-filled discontinuities exposed to ambient environmental conditions where the convective heat flux at the ice-air boundary causes melting and the meltwater can freely evacuate from the melting front (e.g. into an excavated void). This model includes both a mechanical analysis and an ablation approach for approximating the melting process. The stresses and strains developed as thawing occurs are explored in relation to the applied conductive and convective heat fluxes. Using a specific example, we show that the model accurately approximates the thermo-mechanical behaviour of a thawing discontinuity. This model is presented as a step towards the comprehensive simulation of thawing fractured rock masses.

RÉSUMÉ

Un modèle numérique développé pour l'analyse des interactions thermo-mécaniques découlant de la fonte de glace présente dans les discontinuités est présenté dans ce papier. Le modèle proposé est applicable dans les cas où les discontinuités remplies de glace se trouvent près de la surface et sont exposées aux conditions environementales où le flux convectif de chaleur sur l'interface air-glace cause la fonte et où l'eau de fonte peut s'évacuer librement vers des vides. Le modèle comprend deux méthodes d'analyse pour l'évaluation du processus de fonte de la glace: l'analyse mécanique et la méthode d'ablation. Les contraintes et les déformées développées durant le processus de fonte de la glace sont mises en relation avec les flux convectifs et conductifs de la chaleur. En utilisant un exemple concret, nous démontrons que le modèle estime de manière juste les comportements thermo-mécaniques d'une discontinuité dans laquelle la glace fond. Ce modèle est présenté avec l'objectif d'augmenter la compréhension et la capacité de simulation du processus de fonte des glaces dans les massifs rocheux.

1 INTRODUCTION

In a recent study on the economic implications of thawing permafrost, it was estimated that infrastructure in the Northwest Territories will require approximately $51 million annually to repair damages caused by thawing permafrost linked to climate change (Brown 2017). Although most infrastructure such as buildings, sewers, and bridges are developed in or on soils overlying bedrock, there is also a significant amount of engineered structures in or on rock masses, including open pit or underground mines and transportation links. Because the thawing of frozen rock can strongly alter its mechanical characteristics, improving our understanding of the behaviour of thawing rock masses is important for minimising the economic implications of climate change on engineered structures in or on bedrock permafrost.

As ice thaws in a rock mass, meltwater and advective heat transport can form channels of flowing liquid within ice-filled discontinuities (Gruber and Haeberli 2007, Hasler et al. 2011) and thus create voids or zones of minimal shear strength along a discontinuity. The mechanical weakness of these voids or zones means that their formation may have significant impact on the engineering behaviour of a thawing rock mass. The thawing of ice-filled discontinuities causing rock mass instability is a complex thermo-hydro-mechanical (THM) process, and thus requires particular investigation.

When designing for structures in a frozen or thawing, discontinuous material, adhering to modern geotechnical engineering design standards, such as the Canadian Foundation Engineering Manual or Eurocode 7, is difficult since the behaviour of ice-filled discontinuities during temperature fluctuations seems to be epistemically uncertain rather than aleatory as these codes require. Current engineering design methods include design by calculation, prescriptive measures, design by experimental tests, the observational method, and empirical design (CEN 2004, CGS 2006), and a review of current rock engineering design in permafrost rock (Gambino and Harrison 2017) has called attention to the need of advanced numerical modelling for investigating the performance of engineered structures in frozen and thawing rock to support these design methods.

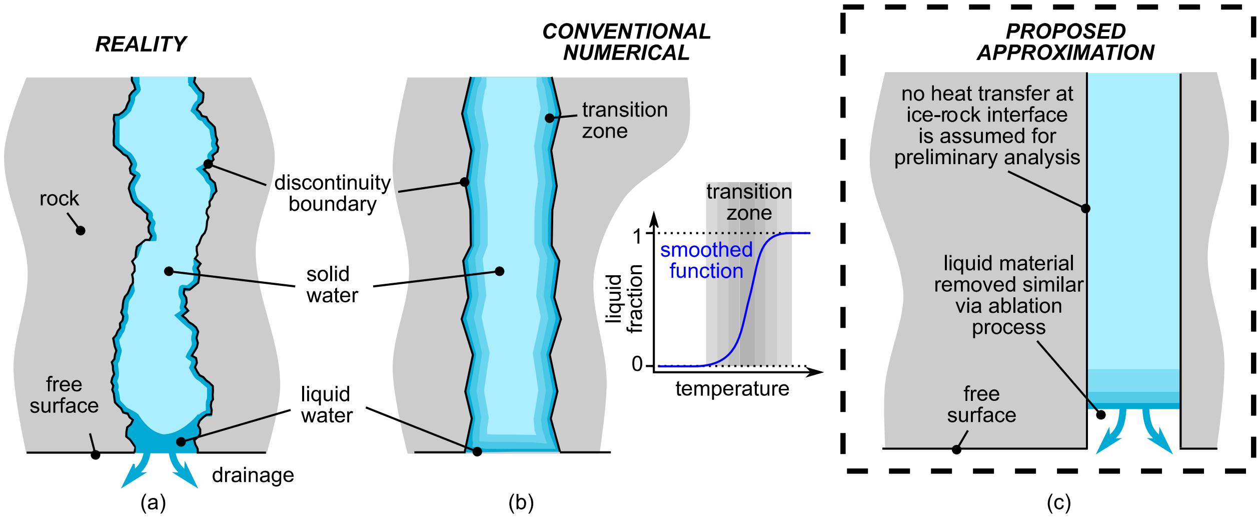

Figure 1: Conceptual comparison between (a) reality, (b) the conventional numerical approach, and (c) the effective ablation approach

Here, we present a numerical model that uses ablation for investigating the melting of ice in a fracture. Ablation (i.e. material removal) modelling is rooted in the material science and biomedical science literature, and has a variety of applications including laser or frequency heating for ablation of metals or tissue (Chen and Beraun 2001, Berjano 2006). The application of the ablation model to the melting of ice in a fracture will be demonstrated using a simply supported beam example, in which the change of stresses can be examined in response to the melting process. The numerical modelling of the thermal and mechanical processes involved in melting an ice-filled discontinuity are precursors to laboratory experiments that will test and parameterise the ablation modelling approach.

2 ABLATION: A CONCEPTUAL MODEL

Ablation due to heating can be a complicated physical mechanism to numerically replicate, but is commonly applied when investigating material decomposition due to heating (Radice et al. 2012, Ewing et al. 2013). For the purposes of this paper, ablation is regarded simply as the loss of material due to heating, and it is equivalent to a well-drained ice-filled discontinuity where any meltwater is instantaneously removed once both the material temperature reaches 0°C and a solid-liquid phase change occurs. This ablation methodology, which is further discussed in the following section, imitates the removal of material by implementing a deformation of the discretised geometry that does not affect the material density. Figure 1 illustrates the primary differences between (a) reality, (b) a conventional phase change numerical simulation, and (c) the proposed ablation approach. As shown in Fig. 1, the proposed approximation assumes the liquid is removed from the system and does not provide a material for stress to act in; it is thus simpler than the conventional numerical approach which includes an incompressible liquid in the force calculations. We assume a one-dimensional problem, with heating and ablation occurring on only one side of the model; future analyses will include conductive coupling with the surrounding rock.

Overall, our conjecture is that the process of melting a well-drained ice-filled discontinuity can be simulated by modelling the removal of liquid water using a procedure similar to that of thermally ablating metals. This conjecture will be tested by performing advanced numerical simulations that progressively incorporate more complicated geometries and thermo-mechanical loading conditions and thus more accurately replicate reality. Furthermore, this numerical modelling will be supplemented with laboratory testing to parameterise the ablation methodology.

3 NUMERICAL IMPLEMENTATION AND ANALYSIS

In reality, discontinuities in rock have complicated geometries and are thus usually simplified to allow numerical simulation. This is the case here, where the ice-filled discontinuity is simplified to a parallel-sided opening. The aim of the ablation technique is to approximate the behaviour of a melting front as ice transforms from solid to liquid and instantly drains from a discontinuity. The heat exchange and its effect on the movement of the melting front are of specific interest, as are the changed stresses resulting from loss of ice. As melting occurs, several phenomena may significantly affect the behaviour of the model, including: the temperature-dependent mechanical properties of ice (e.g. shear strength); the decreasing volume of infilling material due to melting; and the reduced ice-rock contact area on which stresses develop. The processes are incorporated in a model of a simple beam incorporating a discontinuity at mid-span, the analysis of which allows the potential mechanical influence caused by melting to be highlighted. Here, we concentrate on implementing ablation modelling to analyse the effects of melting in discontinuities; more advanced thermo-hydro-mechanical (THM) phenomena and more complicated geometries will be investigated in future simulations.

The thermo-mechanical (TM) and potential THM extension to this work requires the ability to analyse heat transfer via convection and conduction, fluid flow, and non-linear stress-strain response. To model ablation, a technique for removing material from the model is required. We have chosen to use a finite element (FE) approach, as these phenomena can be modelled simultaneously within this. To incorporate ablation, we use an arbitrary Lagrangian-Eulerian (ALE) scheme for mesh geometry deformation. An ALE scheme introduces the benefits of both a Lagrangian description (i.e. precise tracking of free surfaces and interfaces between different materials) and an Eulerian description (i.e. handling of large distortions in the continuum) (Donea et al. 2004), both of which are crucial for accurate modelling of a melting front. All modelling has been performed using COMSOL Multiphysics FE (COMSOL 2017).

3.1 Numerical description of an ice-filled discontinuity

In this paper, we present a heat transfer analysis performed using a 2D FE model comprising an unstructured grid of approximately 1100 triangular elements representing the filling of a 2.5 cm wide fracture. The ice domain is more finely discretized at the boundary exposed to the convective heat flux to improve the numerical approximation near the location of applied heating and geometry deformation. A model height of 10 cm is used here; however, an insulated boundary condition on the top edge represents a fracture extending semi-infinitely. The heat transfer is governed by the weak form of the heat balance equations derived from the Fourier equation and the laws of thermodynamics (COMSOL 2017). In this model, conductive heat transfer occurs in the solid material and properties such as heat capacity, density, and thermal conductivity are the primary parameters governing the heat transfer process. These prescribed heat transfer conditions as well as the assumptions that heat is transferred in one direction and the ice freely drains at the melting surface, allow the appropriate thermal boundary conditions to be specified.

3.1.1 Thermal boundary conditions

The ice surface exposed to the atmosphere is simulated using a convective heat flux boundary condition. The conventional form of the convective boundary condition used in the presented model is

$$q_0 = h\left(T_{ext} - T\right), \qquad [1]$$

where q_0 is the incident or inward convective heat flux with units $\mathrm{W} \cdot \mathrm{m}^{-2}$, h is the heat transfer coefficient ($\mathrm{W} \cdot \mathrm{m}^{-2} \cdot \mathrm{K}^{-1}$), and T_{ext} and T are the external temperature and material temperature, respectively (K).

Here, the thermal boundary conditions applied for simulating the ablation due to melting include a uniform heat flux which approximates a natural convection setting where air is in contact with the ice surface. The material ablation is controlled by a particular heat transfer coefficient h_a, which is zero when the material temperature T is less than the temperature of ablation T_a (i.e. melting temperature of ice or 0°C). Table 1 lists the parameters required for undertaking the ablation simulation.

The remaining boundaries are insulated, so heat is not transferred across them. Future simulations will include the heat transfer through these boundaries to simulate melting at the rock-ice interface.

Table 1: Summary of simulation parameters

Parameter	Value
Thermal conductivity of ice, k	2.2 W/(m °C)
Heat capacity of ice, C_p	2050 J/(kg °C)
Density of ice, ρ	918 kg/m^3
Heat transfer coefficient, h	100 W/(m^2 °C)
Ablating temperature, T_a	0°C
External temperature, T_{ext}	20°C
Specific heat of fusion ΔH_f	333 kJ/kg
Initial model height, l	100 mm
Initial model width, w	25 mm

3.1.2 Material ablation using mesh deformation

The removal of material, or ablation, can be simulated numerically by using mesh geometry deformation (i.e. ALE). Applying particular domain and boundary conditions to the numerical model enables the user to deform the geometry according to prescribed measures such as a critical ablation temperature. For the case of thermal ablation, we assume the melting is due to the convective heat flux applied uniformly across the exposed boundary. The energy associated with the ablative heat flux discussed in the previous section can be used to define the ablation rate (or prescribed normal mesh velocity), v_0 via the relation

$$v_0 = \frac{q_a}{\rho \Delta H_f}, \qquad [2]$$

where ρ is the material density and ΔH_f is the specific heat of fusion describing the heat requirement of a unit mass to transform from solid to liquid.

The normal mesh velocity is prescribed to the geometry nodes of this boundary using the relation

$$\frac{\partial \mathbf{Y}}{\partial t} \cdot \mathbf{n} = v_0 \qquad [3]$$

where $\mathbf{n}$ is the outward unit normal vector of the ablating surface and $\partial \mathbf{Y} / \partial t$ is the partial velocity term in the y-direction. Since this simulation investigates the melting of a single boundary, the mesh is deformed parallel to the normal of the heat flux (i.e. in the y-direction).

4 ABLATION MODEL RESULTS

The results summarized in Fig. 2 illustrate the transient process of heating and subsequent melting of the ice-air interface. There is a rapid increase in surface temperature until the material reaches 0 °C, after which point the temperature remains constant (solid line). Once this temperature has been attained, additional heat transferred to the ice is in the form of latent heat resulting in a constant temperature at the exposed surface. Since the melting front continually recedes while heat is applied, the ice surface exposed to the heat flux remains at 0 °C.

The dashed line represents the rapid decay of the sensible heat flux as the ice surface reaches the melting

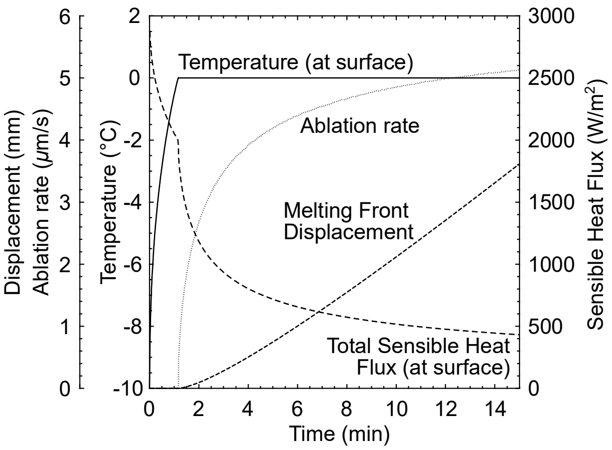

Figure 2: Transient values of mesh displacement, temperature, and sensible heat flux at the exposed surface

temperature (at approximately 68 s). Furthermore, as the ice temperature increases to 0°C, the ablation rate (dotted line) and the melting front displacement (dash-dotted line) also begin to increase. The large initial increase in the ablation rate reduces as it becomes controlled by the applied heat flux (see Eq. 2), while the melting front displacement, which represents the receding melting front, increases accordingly. This displacement approximates the removal of ice with respect to time and heat applied.

This simulation approximates the melting process over 15 minutes, and it demonstrates the efficacy of the numerical approach. The total volume of material melted is computed as the product of the total melting front displacement, the sample width, and thickness, (i.e. 25 mm × 3.6 mm × 1 m) which is approximately 90 cm^3 for this analysis. This volume calculation would be made more complicated in the case of an irregular discontinuity geometry, but it could similarly be approximated using an average aperture, melting front displacement, and thickness (or persistence) of the discontinuity. The melted volume and stress development may be critical parameters to monitor and investigate for rock engineering design.

The shear strength of ice in discontinuities is known to be a function of the temperature and normal stress at the interface (Davies et al. 2000), and so the stress increases as the contact area of the ice-rock interface decreases due to melting may be important in rock engineering design. Developing large tensile stresses at the ice-rock boundary may lead to ice detachment by exceeding the adhesive strength. Furthermore, pressure melting may occur when excessive compressive stresses are applied (Davies et al. 2000). In the following section, we investigate the effect of compressive and tensile stresses at the rock-ice interface by using a simply supported cylindrical beam.

5 APPLICATION TO A SIMPLY SUPPORTED BEAM

This example of a simply supported beam comprises two cylindrical lengths of rock separated by a thin ice-filled discontinuity at mid-span, similar to the conditions shown in Fig. 1c. By placing the discontinuity at the mid-span of the beam we are able to investigate the effect of melting on the development of compressive and tensile stress without shear stress. Using the transient ablation model results (Fig. 2), the location of the melting front can be related to the radius of the ice segment between the rock cores. As the volume of ice decreases due to melting, the area of the ice disk also decreases resulting in increased compressive and tensile stresses acting at the ice-rock boundary. This is shown conceptually in Fig. 3.

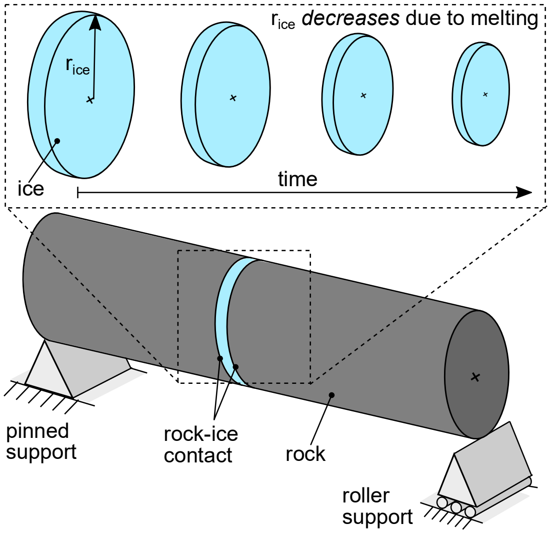

Figure 3: Simply supported beam example with decreasing volume of ice.

Applying the ablation model to a simply supported beam provides a simple method to analyse the transient development of compressive and tensile stress by using the fundamental beam equation

$$\sigma_{\max} = \frac{M \cdot r_{ice}}{I}, \qquad [4]$$

where, M is the bending moment, r_{ice} is the radial distance from the longitudinal axis (i.e. radius of ice core), and I is the area moment of inertia around the longitudinal axis. For a cylindrical core, this is

$$I = \frac{\pi \cdot r_{ice}^4}{4}.$$

For this example, the radius of the rock and ice segments are selected such that they are representative of a common NQ-size (approx. 50 mm) geotechnical core. An ice-filled aperture of 25 mm is selected to be an acceptable thickness that may occur in reality. Here, a length of 150 mm for each rock segment is selected; however, for a decreasing ratio of rock length to ice-filled aperture, the difference between the maximum stress at the mid-span and the maximum stress at the rock-ice boundary increases. For this example, the ratio of rock and ice lengths are sufficiently large such that the maximum stress magnitudes are approximately the same with a slightly greater stress at the mid-span. It is important to recognize that although greater stresses occur at the mid-span of the ice segment, the interfacial strength is typically less than the tensile strength of ice

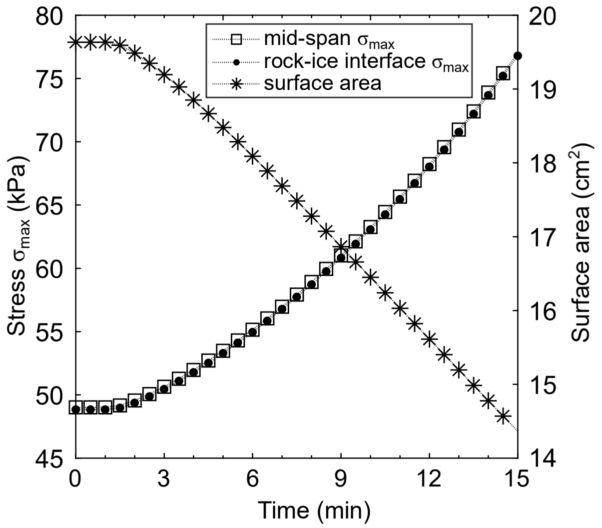

Figure 4: Analytical results for maximum stress magnitudes at the ice-rock interface and midspan as the ice-rock contact surface area decreases due to melting.

(Raraty and Tabor 1958), and thus the interface is the likely location of instability occurring.

The results shown in Fig. 4 demonstrate the development of the maximum stress magnitude at the top or bottom of the simply supported beam. Since the maximum stress is symmetric about the long axis, the tensile and compressive stresses are equal. In agreement with the results shown in Fig. 2, the melting begins at approximately 68 seconds causing the ice-rock contact surface area to decrease and as a result, the maximum stresses to increase.

6 SUMMARY AND CONCLUSIONS

The proposed ablation approach is seen to simulate the melting of ice-filled discontinuities, and results obtained illustrate the ability to approximate the time required to melt a given geometry of ice under particular environmental conditions. Appropriate boundary conditions are specified in the COMSOL Multiphysics FE software such that the applied convective heat flux produces a temperature increase in the ice body and the subsequent geometry deformation imitating the recession associated with the melting front. The progressive uniform melting process is illustrated in Fig. 5.

As shown, the unstructured discretized material and exposed bottom boundary temperature increases until melting begins. Heat is continually provided to the ice through the exposed boundary and causes the boundary to displace upward, parallel to the direction of the applied heat flux. The melting front recedes at a rate in accordance with the amount of heat supplied and is governed by the material density and heat of fusion (see Eq. 2 and 3). This ablative modelling approach approximates the time required—under specific thermal conditions—to thaw an ice-filled discontinuity in one direction, using a simplified example.

The eventual goal of this work is to improve the understanding of the behaviour of a thawing rock mass by investigating different thermo-mechanical interactions in a single discontinuity, beginning with a one-dimensional heat transfer and ablation model. Advancing this model will require coupling the thermo-mechanical behaviour between rock and ice and allow heat to transfer laterally through solid material surrounding the ice as well as at the exposed free surfaces of the rock and ice.

Additionally, the current and future numerical modelling will be accompanied by parameterised laboratory testing to identify and isolate influential or controlling factors. Understanding the effect that these factors have on the behaviour of the system will be important for developing more complicated geometries and incorporating additional phenomena, such as the effect of fluid pressure from meltwater to build a hydro-thermo-mechanical model. Implementing these advancements will lead to an improved ability to predict when rock mass instabilities may occur due to the thawing of ice-filled discontinuities.

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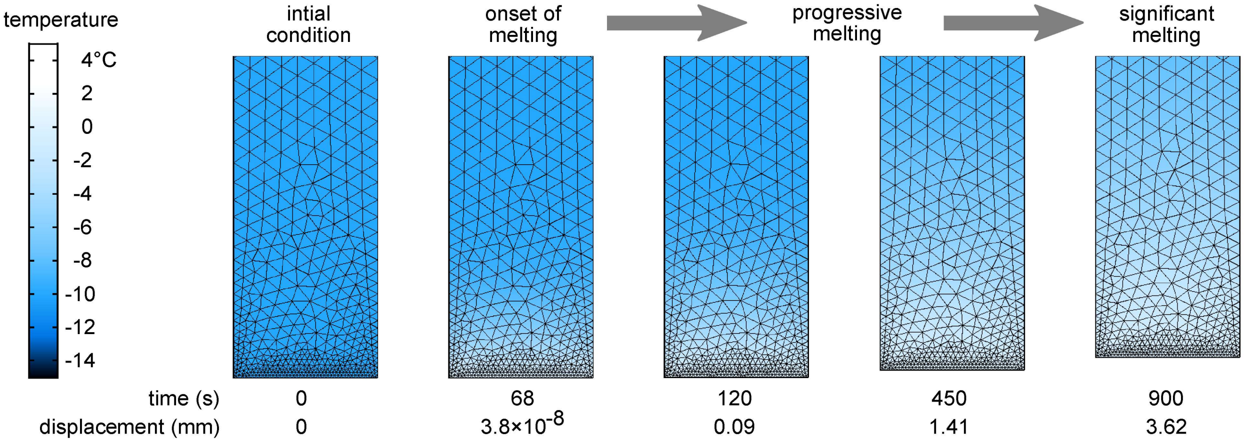

Figure 5: Summary of resulting geometry deformation in accordance with the exposed surface temperature

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