The assessment of the overall probability of failure of hard-rock slopes using the First Order Reliability Method and the Hunting Equation Method

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ABSTRACT
The overall probability of failure of a hard-rock slope is assessed using a combination of the First Order Reliability Method (FORM) and the Hunting Equation Method. FORM is used to assess the probability for each potential failure mode, which considers the correlation of the variables (e.g. discontinuity friction angle and cohesion). The overall probability of failure of the rock slope is then assessed using the Hunting Equation Method by combining every potential failure mode's probability of failure.

The assessment outlined above is applied to a hard-rock slope that is not susceptible to rotational failure, which allows for the application of simple, closed form limit equilibrium methods for planar- and wedge-type failures in a spreadsheet. All the potential modes of failure for a slope, which could include multiple planar- and wedge -type failure mechanisms must be assessed individually. Consideration must also be given to multiple structural and geological domains within a slope, as the potential failure modes in each domain will need to be assessed and each domain may have a unique set of parameters that need to be assessed and correlated.

The author provides an example assessment of a typical rock slope located in the Coastal and Cascade Mountains, where hard rock slopes are in abundance. The potential failure modes are assessed, including the correlation of the variables. The overall probability of failure is assessed in the absence of rock support.

RÉSUMÉ
La probabilité globale de rupture d'une pente de roche dure est évaluée en utilisant une combinaison de la méthode de fiabilité du premier ordre (FORM) et de la méthode Hunting Equations. FORM est utilisé pour évaluer la probabilité pour chaque mode de défaillance potentiel, qui prend en compte la corrélation des variables (par exemple, l'angle de frottement de discontinuité et la cohésion). La probabilité globale de défaillance de la pente de la roche est ensuite évaluée à l'aide de la méthode Hunting Equations en combinant la probabilité de défaillance de chaque mode de défaillance potentiel.

L'évaluation décrite ci-dessus est appliquée à une pente de roche dure qui n'est pas susceptible de subir une défaillance rotationnelle, ce qui permet l'application de méthodes simples d'équilibre limite pour les défaillances de type plan et coin dans un tableur. Tous les modes de défaillance potentiels d'une pente, qui peuvent inclure plusieurs mécanismes de défaillance de type plan et coin, doivent être évalués individuellement. Il faut également tenir compte de multiples domaines structurels et géologiques dans une pente, car les modes de défaillance potentiels dans chaque domaine devront être évalués et chaque domaine pourra avoir un ensemble unique de paramètres qui devront être évalués et corrélés.

L'auteur donne un exemple d'évaluation d'une pente rocheuse typique située dans les montagnes Coastal et Cascade, où les pentes rocheuses sont abondantes. Les modes de défaillance potentiels sont évalués, y compris la corrélation des variables. La probabilité globale de défaillance est évaluée en l'absence de support rocheux.

1 INTRODUCTION

This paper sets out with the goal to assess the probability of failure (PoF) of a rock slope using the First Order Reliability Method (FORM) (Fenton and Griffiths 2008) in combination with the Hunting Equation Method (Du 2017). Although the quantification of PoF can be done in many geotechnical software packages, many do not provide meaningful insight into the methods used to derive the PoF of a given problem.

The ease at which FORM and the Hunting Equation Method can be executed in a spreadsheet program is remarkable given the complexity of other POF assessment methods. However, not all failure mechanisms that can develop in a rock slope can easily be assessed analytically in a spreadsheet. As such, the author limits the design example described below to planar- and wedge-type failure mechanisms in hard rock. For example, limit equilibrium methods for rotational failures, like the method of slices, require the user to solve the problem iteratively. The same can be said for current methods for topple-type failure mechanisms where the stability of a slope is controlled by a joint set dipping into the slope with an orthogonal joint set that allows for either rotational or sliding failure. Iteration to determine the results of an assessment can be time consuming unless user-defined tools (i.e. macros in a spreadsheet) or commercially available software is used.

The author considers a theoretical slope based on recent experience to illustrate the use of FORM to assess the probability of failure due to planar- and wedge-type failure. The theoretical slope is shown to be susceptible to both types of failure mechanisms, and the Hunting Equation Method is used to estimate the total probability of failure considering multiple failure mechanisms.

2 RISK

Risk is defined as the product of consequence of a certain hazard causing harm and the likelihood of that hazard occurring. It is coming more important with each passing year for geotechnical engineers to be aware of able to communicate efficiently with their clients, the public, and regulating bodies about risk (Guthrie 2017). In the past, it has been widely acceptable to qualify risk, but as the management of risk becomes more and more expensive, geotechnical engineers are being called upon to quantify risk for more than the most complex projects. Owners, in many cases, are wanting, if not outright demanding, less conservative designs, as traditional conservatism can be costly in the face of riding construction costs.

Recent developments have indicated that engineers will be held accountable for when their estimates of risk are wrong and there is an incremental loss of life, economic value, or environmental habitat (AAP 2016, Guthrie 2017). In fact, many engineering codes provide acceptance criteria for acceptable levels of risk for a given consequence (NRCC 2015). Nevertheless, as the cost of litigation and the reduction of risk have driven up the cost of construction, historic use of conservative design methods are becoming more and more unacceptable to owners.

Geotechnical engineers need to evolve their current tools to include the quantification of risk. This does not necessarily mean spending more money on analyses, but quite often can be achieved by carrying out thorough geotechnical site data collection programs to increase one's knowledge of the site, thereby increasing the reliability in the design parameters through the use of probabilistic methods.

2.1 Quantification

Quantifying risk can be challenging when knowledge of a site is limited by a lack of available geotechnical information. In many cases the acceptable level of consequences is provided by way of codes and guidelines, but the assessment of hazards and their likelihood of occurrence are site specific.

Geotechnical engineers are well advanced, in general, for the identification of hazards. It is the occurrence of geotechnical hazards that motivated civil engineers to develop the field of geotechnical engineering. Classic examples include the Pisa Tower, the digging of the first tunnel under the River Thames and many others.

The likelihood of occurrence requires much knowledge about a site that often excludes many small projects from being able to cover the costs of site specific risk assessment. But as construction has occurred on most of the suitable land, the demand for more infrastructure is pushing projects into areas that are ripe with danger. Many regulatory bodies, and owners, require detailed risk assessments before approving projects. And as most people are willing to admit, geotechnical hazards are some of the most challenging to assess with respect to likelihood of occurrence.

2.2 Likelihood Of Occurrence

The estimate of likelihood of occurrence requires some form of statistical measurement of the input parameters for a given method of assessment or design. Statistical methods require that a minimum number of data points are collected to provide a meaningful statistical representation of a heterogeneous and/or anisotropic material. Soil and rock may display homogeneous and isotropic properties at a large scale, such as over hundreds of square meters, but across the footprint of a building foundation, the subgrade is more than likely to exhibit enough variation to cause most engineers consternation regarding the selection of one set of input parameters.

Although most methods of analysis used in industry require the geotechnical problems to be simplified, this only leads to the use of conservative, deterministic methods of analysis. Selection of material parameters for a soil or rock, that exhibit a large degree of variability, using statistical methods, allows the user to better assess variability of a site's geomaterials and determine the best means of analysis and develop more appropriate measures to mitigate the likelihood that a failure could occur. For example, FORM allows a user to assess the PoF of a particular failure mechanism regardless of the factor of safety or value of the limit state function.

2.3 Reliability

Reliability can be defined as the measure of the distance between the mean value point, which is the result of a calculation using the mean design parameters, and the design point, which either falls on the limit state function (i.e. the failure surface) or the point calculated using parameter values reduced by some amount based on their statistical distributions. In other words, reliability can be assumed to be a measure of the confidence level one can have in their design (Harish Jose 2015).

As will be shown below, measure of reliability can be used to 1) provide a measure of spatial probability of occurrence, 2) provide guidance in the development of future site investigations, and 3) develop partial factors for use in design.

3 FIRST ORDER RELIABILITY METHOD

3.1 Definition

The FORM is a method that determines the shortest path to the failure surface or limit state surface, as shown in Figure 2, from the mean point of stability (Low 2008). That is, the factor of safety is defined using the mean of the variables and then the standard deviations are used to determine how far said point is from the limit state surface.

That distance is then assessed to determine the likelihood of the data defining the mean actually being at or past the failure surface. The closet point on the limit state surface to the mean point is known as the design point, as shown in Figure 2 (Low 2008), which denotes the position of unity with respect to the working limit state (i.e. factor of safety equal to one).

3.2 Formulation

FORM is a better representation of the probability of occurrence, however it does assume a linear limit state function (Fenton and Griffiths 2008). In the case of a non-linear limit state function, multiple local minima may occur which could result in an under estimation of the probability of occurrence (Fenton and Griffiths 2008). Although the shape of the limit state function for a given problem may not be known, the author considers FORM to be a suitable method for at least preliminary design, if not detailed design. FORM is defined by Equation 6.

$$\beta = \min_{M=0} \sqrt{\left(\frac{x - E[\boldsymbol{X}]}{\sigma_X}\right)^T \boldsymbol{C}^{-1} \left(\frac{x - E[\boldsymbol{X}]}{\sigma_X}\right)} \quad [6]$$

Where β is the reliability index (discussed further in Section 2.7), M is the limit state surface, x is the vector of number of standard deviations from the mean for each independent random variable, and E[X] is the vector of means for the variables, C is the inverse of the correlation matrix. The superscript T indicates the transform of the matrix of reduced variables (discussed below). The author uses C to indicate the correlation matrix, for clarity, however, other authors have used C for the covariance matrix, and R for the correlation matrix (e.g. Low 2008).

1.1 Probability of Failure

The probability of failure (PoF) is determined by assuming the reliability index, β, is normally distributed and the probability of exceeding β is equivalent to failure when the mean values are considered. This requires the use of the inverse standard normal distribution function.

This is only valid when the stability of the failure mechanism in question has a factor of safety equal to or greater than one. The value of the PoF calculated would be close to or equal to zero and would need to be subtracted from one to have real meaning.

4 P(F) FOR FAILURE MECHANISMS

4.1 Rock Slopes

FORM can be used to assess the spatial probability of rock slope failure mechanisms. Planar- and wedge-type failure mechanisms can be assessed analytically using close form solutions (Hoek et al. 1973, Pariseau 2017, Wyllie 2017). This allows one to easily deploy the solution for each type of slope failure mechanism is a spreadsheet which can then be coupled with FORM to assess the reliability of the solution and the probability of that solution exceeding the geotechnical resistance for given failure mechanism.

Direct topple is explicitly identified as solutions for flexural topple are much more difficult to implement in a spreadsheet. Additionally, rotational failures in weak rock are also excluded due to the indeterminate nature of the solutions making spreadsheets ill equipped to provide rigorous methods of analysis. Use of a spreadsheet provides a user a means of a simple and reliable method without the need of iterative solutions of a deterministic software solution or using Monte Carlo. Both of which can be time consuming. For most engineering projects, the use of advanced statistical methods and or numerical methods are not justified. The prolific availability of basic spreadsheet programs for desktop computers allows every geotechnical to develop simple, time saving tools for use on projects with budget conscience clients.

Rock slopes, either natural or excavated in hard rock, are ideal for assessment with spreadsheet deployed close form solutions coupled with FORM. Furthermore, hard rock slopes tend to provide the best exposures for geological mapping and when necessary, recovery of good quality core is readily achievable in such rock formations.

4.2 Application of FORM

The analytical solution for each failure mechanism must be computed in the spreadsheet in such a way that the standard deviation, σ, can be easily calculated at each step. However, consideration must be given to the potential for over calculating σ with each step. For this assessment, σ was calculated for each variable in the assessment, for instance the weight of a block, using the independent variables that were used to calculate each of the intermediate variables used to derive W. For instance, when calculating the weight of a wedge, the value of σ was derived from the independent variables defining each plane of the wedge, the height of the wedge, and the unit weight of the rock. However, many other intermediate variables were needed to calculate the magnitude of W, but their σ values were used to calculate the σ value for W, it would have been very conservative.

The author used the First Order Approximation method (Fenton and Griffiths 2008) to calculate the mean, μ, and σ for each variable. The correlation between variables was calculated using the parametric method where variable were linearly correlated (Fenton and Griffiths 2008) and the Spearman method where variables were non-linearly correlated (Corder and Foreman 2014).

5 P(F) FOR ROCK SLOPES

5.1 Risk Assessment Extended to Full Rock Slope

In many jurisdictions in the world, and in particular a large portion of BC, linear infrastructure such as highways, railways, and energy transmission corridors, are constructed immediately on or adjacent to rock slopes. This requires long stretches of natural and excavated rock slopes to be assessed for risk. Many rock slopes propagate more than one failure mechanism. Tools are needed to quantify not only the PoF of an individual failure mechanism, but multiple types for a given geological and/or structural domain.

Depending on the rock slope, the summation of the probabilities of failure for a slope with multiple failure mechanisms, and possibly multiples of the same type, could equate to a PoF greater than 100%, which is not realistic. For example, for a slope with three failure mechanisms, each with an equivalent PoF of 35%, the total PoF for the slope would be 105%. The extension of FORM to a full rock slope is complimented by a method of summing the multiple PoF's for a slope is a manner that results in a defensible assessment.

5.2 Hunting Equation Method

The Hunting Equation Method was developed to extend assessment of PoF to rock slopes using any number of stability assessment methods available to a designer (Du 2017). This could include the limit equilibrium, finite element, finite difference, discrete fracture network methods, or others. At the heart of the method, the Hunting Equation Method provides an easily deployed solution for quantify the PoF for a rock slope given the PoF for one or more failure mechanisms.

The Hunting Equation Method is defined by Equations 2, 3, and 4 (Du 2017).

$$PoF_{overall} = 1 - PoNF \quad [2]$$

$$= (1 - PoNF_{Mechanism1}) \times (1 - PoNF_{Mechanism2}) \times \ldots \times (1 - PoNF_{Mechanismn}) \quad [3]$$

$$= (1 - PoF_{PL}) \times (1 - PoF_{WD}) \times \ldots \times (1 - PoF_{Mechanismn}) \quad [4]$$

$$PoF_{PL} = 1 - (1 - PoF_{PL1}) \times (1 - PoF_{PL2}) \times \ldots \times (1 - PoF_{PLn}) \quad [5]$$

$$PoF_{WD} = 1 - (1 - PoF_{WD1}) \times (1 - PoF_{WD2}) \times \ldots \times (1 - PoF_{WDn}) \quad [6]$$

Where:
PoNF: Probability of no failures
$PoNF_{Mechanismn}$: Probability of no failure of Failure Mechanism n
$PoF_{Mechanismn}$: Probability of failure of Failure Mechanismn
n: the total number of failure Mechanisms
PoF_{PL}: Probability of failure for all planar-type failure mechanisms
PoF_{WD}: Probability of failure for all wedge-type failure mechanisms

A detailed description of the Hunting Equation Method is provided by Du (2017).

6 DESIGN EXAMPLE

6.1 Problem Setting

British Columbia is well known for its mountain ranges, particularly the hard rock of the Coastal and Cascade Mountains. Here you can find all manner of rock slopes – natural and man made. An iconic photo of one of the more well known rock slopes in British Columbia can be found on the cover of one of the first in depth books on rock slope engineering (Hoek and Bray 1981) – the Porteau Cove rock slope along the scenic Sea to Sky Highway 99.

A typical rock slope, broken into three segments, is considered in this example. The slope is considered to have one geological domain, in other words, only one rock type is present. Each segment is then considered to reside in a separate structural domain. For the purposed of this example, this will provide the various failure mechanisms for which the individual PoF values will be calculated and then used to calculate the overall PoF for the rock slope.

6.2 Geological Domain Parameters

The rock slope segments are summarized in Tables 1, 2, and 3, which provides the μ and σ values for each of the independent (those variables that are measured, not calculated) and deterministic variables (those values that are not random – no standard deviation).

Table 1. Rock Slope Segment Geometry

Parameters	Segment 1	Segment 2	Segment 3
Slope Height (m)	37, 2.1[1]	53, 3	62, 3.5
Slope Face			
Dip (radians)	1.3, 0.06		
Dip Direction (radians)	1.7, 0.06		
Slope Upland[2] (radians)	0.17, 0.04		
Distance to Vertical Tension Crack from Crest of Face (m)	20, 5	25, 6	30, 8

[1]37, 2.1 = mean, standard deviation for the given parameter
[2]descibes the dip of the surface behind the crest of the slope and is assumed to dip in the same direction as the face of the slope

Table 2. Discontinuity Shear Strength

Parameters	Segment 1	Segment 2	Segment 3
JRC	15	16	17
JCS	35	30	35
φ_r	25	28	30

The values on Table 2 were taken as deterministic values (i.e. a standard deviation of zero). The author did not have access to a large enough set of JRC, JCS, and residual friction angle values for this study. Additionally, by assuming deterministic values for the parameters in Table 2, the author could then have some control over the results of the assessment to provide meaningful examples.

The values in Tables 1, 2, and 3 are used to calculate the variables for the stability of each failure mechanism, as well as the standard deviation for each variable used with FORM to determine the PoF for a given mechanism (Fenton and Griffiths 2008). The method provided by Wyllie (2017) was used to assess planar-type failure. The method by Hoek et al. (1973) was primarily used to assess wedge-type failures, with the exception of the water pressure acting on the planes, which was assessed using the method by Pariseau (2017).

Table 3. Discontinuity Dip, Dip Direction, and Spacing

Parameters	Segment 1	Segment 2	Segment 3
Wedge 1			
Plane 1 Dip	0.87, 0.31	0.78, 0.31	0.87, 0.31
Plane 1 Dip Direction	1.4, 0.31	1.5, 0.31	1.4, 0.31
Plane 2 Dip	1.0, 0.31	1.1, 0.31	1.0, 0.31
Plane 2 Dip Direction	2.7, 0.31	2.8. 0.31	2.7, 0.31
Wedge 2			
Plane 1 Dip	0.87, 0.31	0.78, 0.31	0.87, 0.31
Plane 1 Dip Direction	1.4, 0.31	1.5, 0.31	1.4, 0.31
Plane 2 Dip	1.0, 0.31	1.1, 0.31	1.0, 0.31
Plane 2 Dip Direction	2.7, 0.31	2.8. 0.31	2.7, 0.31
Wedge 3			
Plane 1 Dip	0.87, 0.31	0.78, 0.31	0.87, 0.31
Plane 1 Dip Direction	1.4, 0.31	1.5, 0.31	1.4, 0.31
Plane 2 Dip	1.0, 0.31	1.1, 0.31	1.0, 0.31
Plane 2 Dip Direction	2.7, 0.31	2.8. 0.31	2.7, 0.31
Planar 1			
Plane 3 Dip	0.78, 0.24	0.78, 0.24	0.78, 0.24
Planar 2			
Plane 3 Dip	0.70, 0.21	0.70, 0.21	0.70, 0.21
Planar 3			
Plane 3 Dip	0.61, 0.19	0.61, 0.19	0.61, 0.19

6.3 Using FORM

Once all the variables used in the analytical solutions for the failure mechanisms under consideration are calculated and the correlation matrix is formed, the vector of reduced variables is then compiled (Fenton and Griffiths 2008). To assess the PoF using the mean values, the author used a reduced value of 0.1 for each variable. This allowed for the calculation of β without reducing the variables such that the limit state function was equal to zero, which results in the calculation of the expectant or characteristic values for the solution (Fenton and Griffiths 2008). This method provides the PoF for the mean values that would otherwise be used for design. Other methods of design, such as limit states design (Becker 1996), would require the assessment of the characteristic values, which was not considered in this study.

Reducing the variables so that the limit state function is zero would be equivalent to assessing the characteristic values for use in reliability based methods for limit states design, which is discussed below.

The PoF calculated based on a vector of reduced variables described above is the PoF of the given mechanism driving forces exceeding the resisting forces.

6.4 Results

The PoF for each of the failure mechanisms listed in Table 3 are summarized in Table 4. These are the PoF values based on the mean values for the variables used to calculate the resistance and load portions of the stability calculations.

Table 4. PoF Values for each Failure Mechanism Considered

Parameters	Segment 1	Segment 2	Segment 3
Wedge 1	39%	39%	38%
Wedge 2	41%	42%	39%
Wedge 3	44%	45%	32%
Planar 1	42%	34%	36%
Planar 2	31%	36%	37%
Planar 3	33%	36%	36%

The PoF values in Table 4 were used with the Hunting Equation Method (Du 2017) to calculate the overall PoF of the rock slope. Using Equation 2, the $POF_{overall}$ for the rock slope was calculated to be 97%. Although this value is less than 100%, the slope would still be likely considered as very unstable.

A sensitivity analysis was completed considering the above data. In order to for a slope, with one planar- and one wedge-type failure, to have a PoF of five percent, each of the failure mechanisms would require a PoF of two or three. Increasing the number of failure mechanisms in the slope would require each to have individual PoF values lower than the two or three.

7 DISCUSSION

7.1 Collection of Field Data

Data collection in the field must be done in such a manner to provide consistency and reliability so that statistical methods can be used to determine the needed values to carryout probabilistic analyses. The quantity of data must also be of a magnitude that allows for meaningful statistical interpretation. The cost of collecting more data than normally contemplated would be typically unacceptable to many clients, engineers must take the time to educate clients as to the value of the additional data to reduce uncertainty and conservatism in design.

New methods are being developed to efficiently collect data, such as photogrammetry. However, these methods must be repeatable and be ground truthed using methods that are accepted in engineering practice. As technology improves data collection methods, costs are likely to decrease over time as more options become available. One only needs to look towards the introduction of the cone penetration test to illustrate how technology can have a meaningful impact on geotechnical engineering.

7.2 Interpretation of Data

Data interpretation can be difficult if suitable tools are not available. Simply assuming that the collected data is normally distributed could be factually wrong and not provide realistic results from a statistical analysis. Given the need to move more towards using probabilistic methods, geotechnical engineers should be educating themselves on the use of statistical tools that may not already be well understood. Although spreadsheet programs are easy to use, many are not intended for detailed statistical analyses. For example, popular spreadsheet programs do not provide a tool for determining the statistical distribution of a data set. Although third-party plug-ins are available, many do not provide the tools that are necessary for completing the required work.

More advanced statistical tools are available such as R (Venables et al. 2018) is a more appropriate tool for analytical methods requiring a significant amount of statistical analyses. The regular use of R would require an engineer to spend time becoming familiar with the software so as to be confident in its use and the reliability of the results.

7.3 Correlation

Correlation between variables is quite easy to calculate when the variables are linearly correlated. This is not the case when the variables are not linearly correlated and other methods are required.

Assumptions regarding the correlation, or lack thereof, between variables in not recommended. For instance, Fenton et al. (2008) assume that friction and cohesion of soil are independent (i.e. not correlated), which they concede is "slightly conservative". This would not be the case when friction and cohesion are assumed to be non-linear, as is the case with the Barton and Bandis method (Hoek 2007) considered herein.

Further work is required to fully understand the correlation between rock mass parameters – for geometry, strength, and discontinuities. The author accepts that the methods employed for this study may not be wholly realistic, but in the absence of examples in literature, the author made assumptions thought to be reasonable at the time the work was completed.

7.4 Variables Considered with FORM

The author is of the opinion, after assessing a number of specific slope problems using FORM, that only the independent variables that are used to determine the size of a rock slope failure mass and the controlling discontinuity shear strength are needed for the analysis. For example, when considering a wedge of rock, only the dip and direction of the planes forming the wedge are needed for an assessment using FORM. All other variables used in a wedge-type slope problem are calculated from those variables (Hoek et al. 1973) and their use only adds the potential for large variances in the solution. In other words, each time a calculation is made with multiple variables with a mean and variance, the result with have a larger variance than the inputs.

By limiting the FORM assessment to the main parameters of the problem, the level of variance considered is minimized. However, simply limiting the assessment to the strength parameters does not indicate the variability in the size of a block of rock susceptible to failure, given that a combination of two or more joint sets can result in a range of block sizes being formed.

Consideration of rock support in a FORM assessment would require a mean and variance for the steel elements, such as the bar tensile or shear strength. It may not be reasonable to expect manufacturers to provide the level of statistical data required for such an analysis. Consideration could be given to treating the steel elements as deterministic values, and assessing the geomaterials that connect the rock support to the rock mass, namely the grout-rock interface friction angle, and the mobilized rock mass volume and tensile strength.

7.5 Incorporating Results into Limit States Design

As referenced above, FORM can be used to assess the characteristic values required for reliability methods that are used for limit states design. Although the author was not aware at the time of writing of any examples of limit states design applied to rock slopes, or any slope for that matter, FORM does act as a tool for such analyses.

By using an automated solution seeking tool in a spreadsheet, a solution can be found where the limit state function is set to zero by minimizing the values used to calculate the stability of a slope. This then forms the vector of reduced variables (Fenton and Griffiths 2008). The minimized values can be considered as the characteristic values, or in other words, those values less than the mean that can be quantified by a percentile of a given distribution.

The use of characteristic values that are less than the mean, but do not result in the failure of a slope increase the reliability one can have in the solution, hence forming the basis of a reliability method.

Limit states design is based on the use of either a global resistance factor applied to the geotechnical resistance values when considering foundation engineering, or partial factors when enough information is available to assess the statistical reliability of individual variables. The use of partial factors has been common practice in structural engineering for quite some time.

Limit states design has been applied to geotechnical design of foundations on soil. The author is of the opinion that this will gain further traction is the near future as owners – government agencies and private companies – move to reduce costs by reducing conservatism. It only makes sense to develop reliability based methods for slope engineering (soil and rock) and rock engineering in general.

7.6 Areas for Further Research

As described above, the use of reliability based methods is mostly confined to foundation engineering as applied to soil sites. More can be done to develop methods to incorporate reliability based methods for slopes and rock engineering.

The low hanging fruit, as it were, would be any methods of analyses that currently provide close formed solutions – analytical or otherwise. The examples in literature of the methods that have been developed to assess reliability for soil can likely be easily revised to be used for rock. Many of the rock foundation methods (Wyllie 2003) are analogous to the methods used for conventional shallow foundations (CGS 2006).

Further consideration needs to be made to problems where the loads, which are provided by a structural engineer for foundation design problems, are derived by the weight of a soil or rock mass that also generates the geotechnical resistance. For example, for the theoretical slope described above, the weight of the planar or wedge blocks generated the normal stress on the sliding planes that developed the shear resistance, but also acted as the load, or driving force. To use limit states design for slope stability problems, an accepted rationale for determining the load and resistance factors for the problem needs to be developed.

8 CONCLUSIONS

This paper provides an example for utilizing the First Order Reliability Method for rock slope engineering where two failure mechanisms, planar and wedge, are present. The Hunting Equation Method is then used to determine the total probability of failure for a rock slope due o the presence of multiple failure mechanisms.

9 REFERENCES

AAP. 2016. BHP-Vale Samarco dam: 21 employees, including Australian, charged over deaths. Available from https://www.smh.com.au/business/companies/bhpvale-samarco-dam-21-employees-including-australian-charged-over-deaths-20161021-gs7b7o.html [accessed 15 April 2018].

Becker, D.E. 1996. Eighteenth Canadian Geotechnical Colloquium: Limit states design for foundations. Part I. An overview of the foundation design process. Canadian Geotechnical Journal, **33**(6): 956–983. NRC Research Press.

CGS. 2006. Canadian Foundation Engineering Manual. *In* 4th edition. Canadian Geotechnical Society.

Corder, G.W., and Foreman, D.I. 2014. Nonparametric statistics: A step-by-step approach. John Wiley & Sons.

Du, J.D. 2017. Estimate of Overall Probability of Failure of Rock Slopes Considering Multiple Failure Mechanisms. *In* GeoOttawa. Canadian Geotechnical Society, Ottawa, ON. p. 8.

Fenton, G.A., Griffiths, D. V, and Zhang, X. 2008. Load and resistance factor design of shallow foundations against bearing failure. Canadian Geotechnical Journal, **45**(11): 1556–1571. NRC Research Press. doi:10.1139/T08-061.

Fenton, G.A., and Griffiths, D.V. 2008. Risk Assessment in Geotechnical Engineering. John Wiley & Sons, Inc., Hoboken, New Jersey.

Guthrie, R.H. 2017. Courting Disaster: The Increasing Challenge of Risk Assessments. Innovation,. Vancouver.

Harish Jose. 2015. Relationship between AQL/RQL and Reliability/Confidence: | Harish's Notebook - My notes... Quality, Data Science, Strategy & Lean. Available from https://harishsnotebook.wordpress.com/2015/06/28/relationship-between-aqlrql-and-reliabilityconfidence/ [accessed 7 July 2018].

Hoek, E. 2007. Practical Rock Engineering. Rocscience, Toronto.

Hoek, E., and Bray, J. 1981. Rock Slope Engineering. Inst. of Mining and Metallurgy.

Hoek, E., Bray, J.W., and Boyd, J.M. 1973. The stability of a rock slope containing a wedge resting on two intersecting discontinuities. Quarterly Journal of Engineering Geology and Hydrogeology, **6**(1): 1–55. Geological Society of London.

Low, B.K. 2008. Efficient Probabilistic Algorithm Illustrated for a Rock Slope. Rock Mechanics and Rock Engineering, **41**(5): 715–734. doi:10.1007/s00603-007-0146-7.

National Research Council of Canada (NRCC). 2015. National Building Code of Canada. *In* 2015th edition. Natioanl Research Council of Canada.

Pariseau, W.G. 2017. Design analysis in rock mechanics. CRC Press.

Venables, W.N., Smith, D.M., and R Core Team. 2018. An Introduction to R. Vienna. Available from https://cran.r-project.org/doc/manuals/r-release/R-intro.pdf.

Wyllie, D.C. 2003. Foundations on Rock: Engineering Practice. CRC Press.

Wyllie, D.C. 2017. Rock Slope Engineering: Civil Applications. CRC Press.