

Numerical Investigation of Impedance Variation Effect on Surface-Waves' Propagation for Characterization of Very Soft Soils



Díaz-Durán, F., Cascante, G., & Pandey, M.
University of Waterloo, Waterloo, Ontario, Canada

ABSTRACT

The characterization of soils by using near-surface geophysical techniques has been gaining attention in the geotechnical engineering practice in the last two decades. The multi-channel analysis of surface waves (MASW) is a geophysical technique for profiling shear wave velocity in a soil by applying inversion techniques from the dispersion curve. Numerical simulations were performed to understand the limitations of MASW technique when characterizing very-soft soils. Results show that shear wave velocities and layer thicknesses of very-soft materials can be estimated with uncertainty about 2%. However, the acoustic impedance ratio between adjacent soil layers has effect in the MASW results interpretation. From the results it is possible to conclude that the lower the impedance ratio the higher the diffusion of seismic energy. Based on these results it is possible to define a threshold value for the impedance ratio beyond which the use of MASW technique becomes unfeasible to characterize very-soft soils.

RÉSUMÉ

La caractérisation des sols en utilisant des techniques géophysiques proches de la surface a attiré l'attention dans la pratique de l'ingénierie géotechnique au cours des deux dernières décennies. L'analyse multicanal des ondes de surface (MASW) est une technique géophysique pour profiler la vitesse de l'onde de cisaillement dans un sol en appliquant des techniques d'inversion à partir de la courbe de dispersion. Des simulations numériques ont été réalisées pour comprendre les limites de la technique MASW lors de la caractérisation de sols très mous. Le rapport d'impédance acoustique entre les couches de sol adjacentes a un effet sur l'interprétation des résultats MASW. À partir des résultats, il est possible de conclure que plus le rapport d'impédance est faible, plus la diffusion de l'énergie sismique est élevée. Ainsi, il est possible de définir une valeur seuil pour le rapport d'impédance au-delà duquel l'utilisation de la technique MASW devient impossible pour caractériser les sols très mous.

1 INTRODUCTION

Conventional drilling techniques for subsurface exploration are not feasible to be used in very soft soils because of many aspects, including the standing water and the lack of consistency of soil materials, as well as economic and environmental issues. The characterization of soils by using near-surface geophysical techniques has been gaining attention in the geotechnical engineering practice in the last two decades. A very well-known geophysical technique is the multi-channel analysis of surface waves (MASW), which allows the profiling of shear wave velocity in a soil by applying inversion techniques from the dispersion curve. MASW technique has been applied to characterize a wide range of engineering materials, for instance, from very soft to very stiff soils. The main objective of this research is to investigate the MASW geophysical technique through numerical simulations to understand the limitations of this technique when very-soft soils are present.

Numerical simulations were performed to understand the complex wave propagation phenomena in a layered medium involving a very-soft material. Results shown that seismic wave velocities can be estimated with uncertainty less than 2%. Inversion results shown that shear wave velocity profile and layer thickness of very-soft materials

can be estimated with a precision close to 98%. The effect of changes in acoustic impedance ratio in the response of the models could be summarized as follows: the lower the impedance ratio the higher the diffusion of seismic energy. In addition, for very low values of impedance ratio the multiples generated by primary reflections are evident both in the frequency and time domain. Based on this results it is possible to define a threshold for the impedance ratio beyond which the use of MASW technique become unfeasible to characterize very-soft soils.

2 GEOTECHNICAL CHARACTERIZATION OF VERY SOFT SOILS – TRADITIONAL TECHNIQUES

According with Terzaghi & Peck (1948), a clay soil is considered very-soft if the number of blows (N) in a SPT test is less than 2 and if unconfined compression strength (q_u) is less than 25 kPa; in the same way a clay soil is soft if N ranges between 2 and 4, while q_u ranges between 25 – 50 kPa. On the other hand, a sand soil is considered very loose if relative density (D_r) is less than 20% and the number of blows (N) in a SPT test is less than 4; while it is considered loose if D_r ranges between 20 – 40% and N between 4 and 10.

Nevertheless, a better understanding of both dynamic and geotechnical properties of very soft soils is needed.

2.1 Geotechnical Properties of Very Soft-Soils

In specialized literature there are not too many studies reporting geotechnical characterization of very-soft soils. The reason for that could be the very low interest of geotechnical engineers in the geomechanical behavior of these materials, as they behave in a very weak manner. Nonetheless, some values of geotechnical properties for very soft soils are summarized in Table 1, for which the mass density values ranges from 11.6 to 18.8 kN/m³.

Table 1. Reported Values for Geotechnical Properties of Very Soft-Soils

Author	Material	γ (kN/m ³)	c' (kN/m ²)	ϕ' (°)	C_u (kN/m ²)
Youdeowei and Nwankwoala (2013)	Silty Clay	--	58	3.0	--
Nwankwoala, et al. (2014)	Sand, Clay and Silts	18.8	--	--	48
Awwenagha, et al. (2014)	Organic Clay	17.6	16*	2.5*	22
Salami, et al. (2012)	Peats, Organic and Silty Clays	15.6*	66*	6.0*	8
Almeida, et al. (2010)	Soft to Very Soft Clays	12.8*	--	--	12.5
Baroni, M. and Almeida, M. (2013)	Peats and Clays	11.6*	--	--	3.0
Jung, et al. (2013)	Very soft and Silty Clays	15.8*	--	--	<65
Masad, F. (2009)	Sediment Clays	14.9*	--	--	35
Takaki, et al. (2013)	Sediment Clays	15.0*	--	--	17.6**

*average value

**parameter varying with depth: $C_u = 17.6 + 1.16 * z$, where z =depth

2.2 Dynamic Properties of Very Soft-Soils

Several building and construction codes, all around the world, have adopted the average properties for the top 30 meters as criteria for seismic site classification. According to the National Building Code of Canada, (NBCC, 2015), the shear wave velocity is one of these properties for which classes range from A (hard rock, $V_s > 1500$ m/s) to E (soft soil, $V_s < 180$ m/s), and including a specific class for very-soft soils (F), for which V_s values are not specified.

A summary of reported dynamic properties for very soft soils is presented next in Table 2, in which V_s reported values range from 25 m/s to 173 m/s. All these values for V_s actually correspond with materials type E and F in the NEHRP seismic site classification (BSSC, 2003).

Table 2. Reported Values for Dynamic Properties of Very Soft-Soils

Author	Material	V_s (m/s)	ζ (damping)	G_{max} (MPa)
Borcherdt, et al. (1994)	Very Soft to Silty Clay	94*	--	--
Borcherdt, et al. (1994)	Loose Sand	173	--	--
Campanella, et al. (1994)	Peat and Organic Silty Clay	25	3.5*	--
Campanella, et al. (1994)	NC Clay to Silty Clay	46	1.0	--
Likitlersuang, et al. (2013)	Silty Clays ($z < 10$ m)	--	--	5-15
Hunter, J. and Motezedian, D. (2006)	Soft Soils ($z < 3$ m)	145	--	--
Hunter, J. and Motezedian, D. (2006)	Soft Soils ($z > 3$ m)	110	--	--
Prasad, et al. (2010)	Sand and Clay	90	--	--

*average value

Conventional drilling techniques are not feasible to be used for subsurface exploration of aforementioned materials. Very soft soils are usually found in wetlands where the standing water and the lack of consistency of soil materials represent the main constrains for traditional drilling techniques. Furthermore, the use of traditional techniques in wetlands imply economic and environmental issues which are many times difficult to overcome.

3 GEOTECHNICAL CHARACTERIZATION OF VERY SOFT SOILS – GEOPHYSICAL TECHNIQUES

The characterization of soils by using near-surface geophysical techniques has been gaining attention in the geotechnical engineering practice in the last two decades. A very well-known geophysical technique is the multi-channel analysis of surface waves (MASW), which allows the profiling of shear wave velocity in a soil by applying inversion techniques from the dispersion curve.

3.1 Multi-Channel Analysis of Surface Waves (MASW)

The multi-channel analysis of surface wave (MASW) is a very well-known near-surface geophysical technique used for shear wave velocity profiling. Nazarian et al. (1984) first reported surface waves analysis for a one-channel at the time, that was known as SASW. On the other hand, MASW method requires the use of several transducers to effectively remove the noise usually captured by the SASW technique. In addition, the use of a multiple array helps in

the identification of higher order vibration modes for Rayleigh waves (Park et al., 1999).

MASW technique takes advantage of the fact that surface waves are dispersive in order to profile the shear wave velocity of the shallow materials. The effective depth to which the surface waves are useful in MASW technique actually depends on the materials' stiffness and on the wavelength of the waves generated by the input force.

On one side, the shear stiffness and the mass density of the materials define the shear wave velocity. On the other hand, the shear wave velocity is also related to the wavelength and frequency of the shear waves. The lower the frequency the longer the wavelength and the faster they propagate.

Could be said that the first third of the wavelength carries most of the energy in a Rayleigh wave. Thus, as a rule of thumb, the effective penetration depth in MASW test is considered to be one third of the longest wave generated in the actual test. However, that is just a wise assumption which could be better analyzed when data from many test are analyzed. Now, because of MASW test are expensive to be performed in the field, numerical simulations could be a suitable option in order to analyze the effect of changes in the wavelength when profiling shear wave velocities.

When dealing with body waves propagating through different materials, transmission and reflection of energy at the interfaces are well understood. However, the transmission of energy carried by surface waves is not very well understood when the interfaces are reached by those waves. Transmission and reflection of energy in body waves is calculated by using the impedance (Z) of the materials, which depends on mass density (ρ) and shear wave velocity (V_s) of the materials.

$$Z = \rho \times V_s \quad [1]$$

In this numerical study different materials are used for the top layer in order to simulate the effect of variation of material's impedance in surface waves propagation. The results from different MASW simulations are then compared in order to see how the impedance variation affects the propagation of surface waves.

4 MODEL DEFINITION

The numerical model is defined to include six layers horizontally distributed. The materials' properties in the layers of the model were defined in such a way that they included different soils ranging from soft soils to soft rock. The Poisson's ratio (ν) values were carefully selected in order to guarantee the elastic properties corresponded to saturated materials. The impedance was calculated for shear waves and the impedance ratio is calculated between adjacent layers.

$$i = \frac{Z_i}{Z_{i+1}} \quad [2]$$

Geotechnical and dynamic properties of the materials are presented in Table 3.

Table 3. Properties of materials in the Numerical Models

Layer (i)	ρ (Kg/m ³)	V_s (m/s)	ν	G (MPa)	K (MPa)	Z (MRayl)	i
1	2000	240	0.45	115.2	1113.6	0.48	0.78
2	2050	300	0.45	184.5	1783.5	0.62	0.81
3	2100	360	0.45	272.2	2630.9	0.76	0.84
4	2150	420	0.45	379.3	3666.2	0.90	0.86
5	2200	480	0.45	506.7	4899.8	1.06	0.56
6	2500	750	0.40	1406.2	6562.5	1.88	--

A sketch of the geometry of the initial model is presented in Figure 1. The wave propagation process was simulated by using an axisymmetric model and the input force was applied at axis of symmetry.

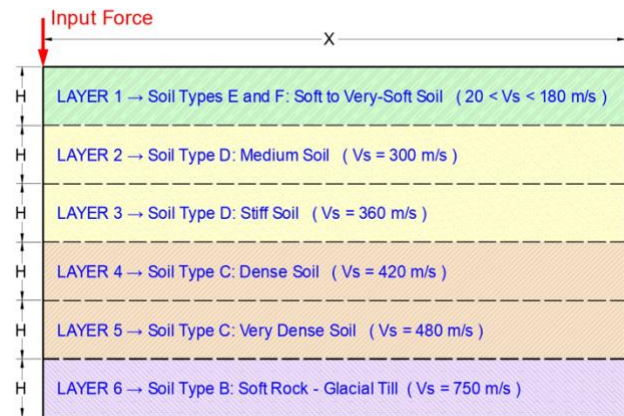


Figure 1. Initial model for the numerical simulations (X=40m, H=4m)

Lamb and Sine pulses were used as input force. In order to generate different wavelengths in the material, different frequencies in the input force were used. At the end Sine pulse was preferred over the Lamb pulse because the aforementioned better concentrate the energy at lower frequencies, which makes it more effective.

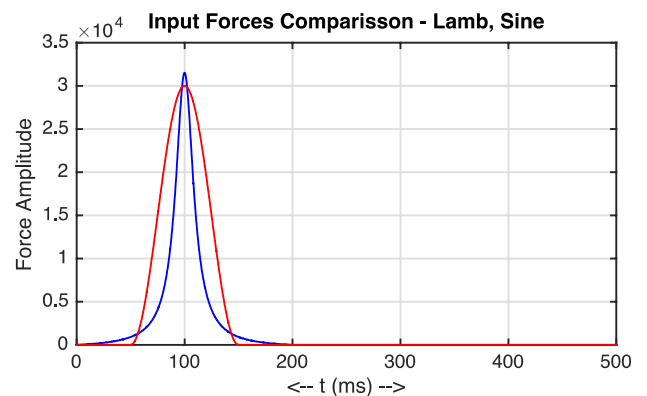


Figure 2. Pulses used as input force in the model.

Once the initial model was run, the top layer was replaced by soft and very-soft materials which have impedance ratios ranging from 0.04 to 0.56 (see Table 4).

Table 4. Properties of Soft to Very-Soft soils in layer 1 of the Numerical Models

Soils in Layer 1	ρ (Kg/m ³)	V_s (m/s)	ν	G (MPa)	K (MPa)	Z (MRayl)	$i = \frac{Z_1}{Z_2}$
I	1100	20	0.499	0.44	1833.3	0.02	0.04
II	1200	40	0.499	1.92	1742.4	0.05	0.08
III	1300	60	0.498	4.68	1416.2	0.08	0.13
IV	1400	80	0.496	8.96	1117.5	0.11	0.18
V	1500	100	0.491	14.99	827.8	0.15	0.24
VI	1600	120	0.484	23.04	690.4	0.19	0.31
VII	1700	140	0.476	33.33	674.7	0.24	0.39
VIII	1800	160	0.465	46.10	643.2	0.29	0.47
IX	1900	180	0.452	61.58	627.7	0.34	0.56

As a result, in addition to the initial model nine more models were run, which allowed the analysis of the effect of variation of impedance of shear waves on the effect of surface waves.

5 NUMERICAL SIMULATIONS

The wave equation is a linear second-order partial differential equation (PDE) with two independent variables on a domain Ω in the form:

$$\frac{\partial^2}{\partial t^2} u(x_i, t_n) - c_p^2 \frac{\partial^2}{\partial x^2} u(x_i, t_n) = 0 \quad [3]$$

Where u is the displacement in x coordinate, and c_p is the wave velocity.

To perform the numerical simulations the finite difference method was used. In this method, each derivative in the wave equation is replaced by an algebraic expression relating variables at specific locations in the grid.

$$\frac{\partial^2}{\partial t^2} u(x_i, t_n) \approx \frac{u_i^{n+1} - 2u_i^n + u_i^{n-1}}{\Delta t^2} \quad [4]$$

$$\frac{\partial^2}{\partial x^2} u(x_i, t_n) \approx \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2} \quad [5]$$

5.1 Initial and Boundary Conditions

Initial conditions defined for the model were:

- *First Initial Condition:* Function evaluating displacements $u(x)$ at the initial time $t_{n=0}$ for different nodes. The displacement for every node at the initial time is assumed to be zero.

- *Second Initial Condition:* Function evaluating velocity $u'(x)$ at initial time $t_{n=0}$ for different nodes. The velocity for every node at the initial time is assumed to be zero.

Boundary conditions defined for the model were:

- *First Boundary Condition (Essential):* Function evaluating displacements $u(t)$ at the fixed boundaries. The displacement for every node at the bottom $x_{i=0}$ for different times was assumed to be zero.
- *Second Boundary Condition (Natural):* The external force times EA at certain nodes n is a given value. In this model the only where a force was applied was at the top of the model on the axis of symmetry.

5.2 Stability Criterion

Grigoryan (2012), mentioned that the value of the stability parameter (s) has a crucial effect on the stability of the numerical scheme. When (s) > 1 the scheme leads to unexpected large values, and hence is unstable. The stability condition is:

$$s = c^2 \frac{(\Delta t)^2}{(\Delta x)^2} \leq 1 \quad [6]$$

If we define the speed of the numerical scheme to be $(\Delta x/\Delta t)$, then the stability condition implies that the speed of the numerical scheme must be at least as large as the speed of the exact equation (wave velocity). A different way of understanding stability is comparing the domains of dependence of the exact equation and the numerical scheme.

5.3 Spatial Discretization

Kuhlemeyer and Lysmer (1973) showed that the wavelength (λ) determines the accuracy for wave propagation problems. The value of (λ) relates to the mesh element length in the direction of propagation (Δx) by a factor of one-tenth to one-eighth.

5.4 Time Discretization

A numerical scheme for finite differences is not unconditionally stable. A time step must be small enough so that the speed of the calculation front is greater than the speed of the faster existing wave. Thus, a critical time step is defined as:

$$\Delta t_{crit} = \min \left(\frac{A}{c_p \Delta x_{max}} \right) \quad [7]$$

Where Δx_{max} is the maximum zone dimension, which is usually a diagonal distance and A is the area of the triangle. The $\min(\cdot)$ function is taken over all zones. For a right angle triangle with two equal sides ($\Delta s = \Delta x$) the area would be equal to $(1/2 \Delta x^2)$ and the maximum dimension would be equal to $(\Delta x \sqrt{2})$. Hence the following stability condition is obtained for a factor of safety FS = 2:

$$\frac{\Delta t}{\Delta x} \leq \frac{1}{4c_p\sqrt{2}} \quad [8]$$

This equation requires smaller time increments than the ones introduced in previous sections. However numerical dispersion should still be considered. Also this equation is set for a homogeneous medium with no damping, hence it should be used cautiously.

6 RESULTS

From the literature review is clear that there are not too many studies reporting geotechnical characterization of very-soft soils, which are the materials in wetlands. For dynamic properties the scenario is even worst, because there is just one paper reporting characterization of very-soft materials in the laboratory; few papers are reporting correlation for dynamic properties and results of field tests like CPT.

To evaluate the accuracy of the models, waves velocities were measured in the travel-time curve and compared against the theoretical ones. In this case errors were defined as the difference between the theoretical velocities and the measured values. As a result, it can be said the errors are in most cases less than 1%, which is a very good result showing the calibration of the model is very reliable.

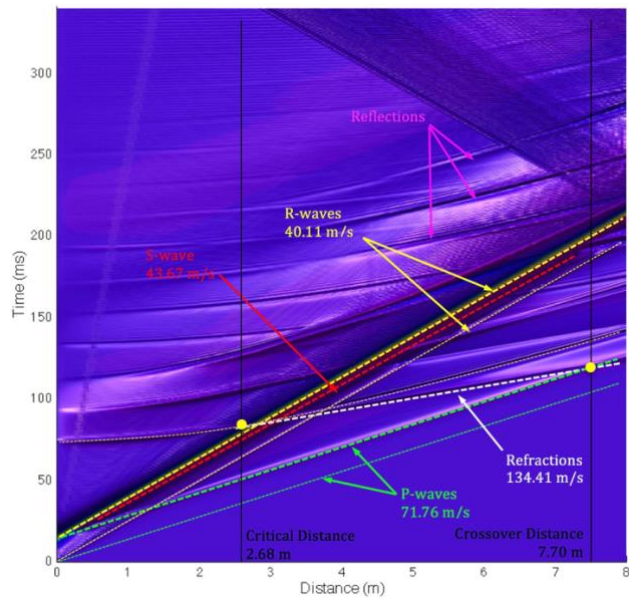


Figure 3. Example of identification of seismic waves by using a travel-time curve for a model with impedance ratio $i > 0.50$, in which the two top layers were medium sand overlaid by a very-soft clay.

Energy scattering in the numerical models simulating MASW tests could be analyzed in the Frequency vs. Wave Number spectrum. An example of that spectrum is presented in Figure 4.

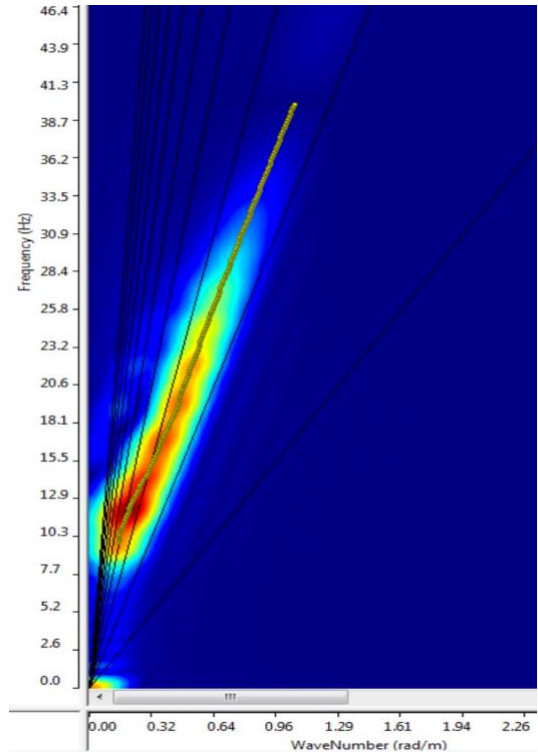


Figure 4. Frequency vs. Wave Number spectrum for model VIII (impedance ratio $i = 0.47$).

From the results in the Frequency vs. Wave Number spectrum, the dispersion curve of Rayleigh wave velocity could be extracted. From the dispersion curve an inversion process could be followed in order to get the shear wave velocity profile. An example of that dispersion curve is presented in Figure 5.

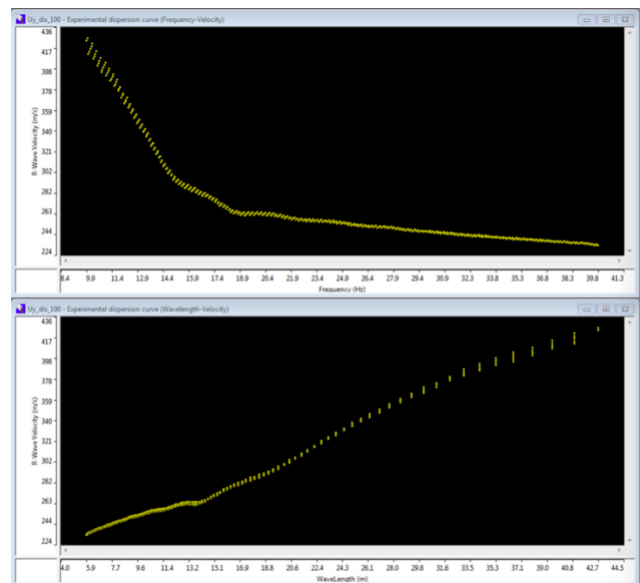


Figure 5. Dispersion curve of Rayleigh wave velocity for model VIII (impedance ratio $i = 0.47$).

Root means square (RMS) values between dispersion curves gotten from the numerical simulations and the theoretical dispersion curve were calculated when the spacing was modified. From results presented in Table 5, it could be concluded that when the spacing between channels is 1.0 meter the RMS values are the lowest ones for most of the models.

Table 5. RMS of dispersion curve for MASW models simulated with different spacing between channels (transducers). Input force was a Sine pulse ($f=20\text{Hz}$).

Model	$i = \frac{Z_1}{Z_2}$	dx=0.6 (m)	dx=1.0 (m)	dx=2.0 (m)	dx=4.0 (m)	dx=8.0 (m)
Initial	0.78	3.53	3.97	3.98	4.01	4.85
IX	0.56	2.58	6.65	3.04	2.95	3.80
VIII	0.47	1.00	0.81	1.31	1.44	1.60
VII	0.39	0.89	0.78	0.79	1.12	1.15
VI	0.31	0.95	0.53	1.49	1.62	1.36
V	0.24	0.37	0.33	0.42	0.42	0.69
IV	0.18	0.56	0.65	0.80	0.97	1.51
III	0.13	1.97	1.90	2.05	2.59	2.38
II	0.08	2.44	2.46	2.92	3.40	3.36
I	0.04	2.97	--	3.30	3.25	5.15

In a similar way, RMS values between dispersion curves gotten from the numerical simulations and the theoretical dispersion curve were calculated when the central frequency in the input force (Sine pulse) was modified. From results presented in Table 6, it could be concluded that for soft soils with impedance ratio between the two top layers $i > 0.15$, frequencies higher 20 Hz in the input source result in lower values for RMS. On the other hand, for very-soft soils with impedance ratio between the two top layers $i < 0.15$, frequencies less than 20 Hz result in lower values of RMS.

Table 6. RMS of dispersion curve for MASW models simulated with different frequencies in the input force (Sine pulse). Spacing between channels was $dx=1.0\text{m}$.

Model	$i = \frac{Z_1}{Z_2}$	f = 5 (Hz)	f = 10 (Hz)	f = 20 (Hz)	f = 40 (Hz)
Initial	0.78	5.02	5.93	3.97	7.25
IX	0.56	3.37	9.11	6.65	1.89
VIII	0.47	2.17	8.26	0.81	0.85
VII	0.39	1.26	2.80	0.78	0.58
VI	0.31	6.04	2.28	0.53	1.26
V	0.24	5.29	2.18	0.33	1.84
IV	0.18	4.61	1.95	0.65	1.41
III	0.13	4.14	0.58	1.90	3.68
II	0.08	1.75	0.67	2.46	3.58
I	0.04	1.15	1.48	--	4.09

By analyzing the dispersion curves, it is possible to extract the maximum identifiable values for the shear wave velocity. These maximum values are a good indicator of the actual penetration of the Rayleigh waves in the MASW test. From results presented in Table 7, it could be concluded that the frequency in the input source actually has an impact in the results. (see Table 7)

Table 7. Maximum values of shear wave velocity (m/s) identified in the dispersion curves for MASW models simulated with different frequencies in the input force (Sine pulse). Spacing between channels was $dx=1.0\text{m}$.

Model	Layer 1 V_s (m/s)	$i = \frac{Z_1}{Z_2}$	f = 5 (Hz)	f = 10 (Hz)	f = 20 (Hz)	f = 40 (Hz)
Initial	240	0.78	450	438	430	430
IX	180	0.56	445	425	420	400
VIII	160	0.47	440	420	380	380
VII	140	0.39	390	410	320	340
VI	120	0.31	390	380	250	250
V	100	0.24	340	290	200	230
IV	80	0.18	280	230	180	180
III	60	0.13	200	130	140	180
II	40	0.08	120	90	90	90
I	20	0.04	40	45	40	30

* Green: models allowing the resolution of only one layer

* Yellow: models allowing the resolution of two layers

* Orange: models allowing the resolution of three layers

* Red: models allowing the resolution of four layers

7 CONCLUSIONS

Based on the numerical simulation results, the following conclusions can be drawn:

- i. To evaluate the accuracy of the models, waves velocities were measured in the travel-time curve and compared against the theoretical ones. Errors are in most cases less than 1%, which is a very good result showing the calibration of the model is very reliable.
- ii. From the parametric study was evident that any increment in Poisson's ratio is generating an increment in wave velocity. Which is something we already knew. However, what is new here is that the higher the Poisson's ratio, the higher the diffusion of energy around the P-wave
- iii. The spacing between channels has effect on the results of dispersion curves. For the model considered in this numerical study a channels spacing of 1.0 meter leads to the lowest values of root mean square (RMS).
- iv. From the results in the numerical simulations it is clear that frequencies of 20 Hz and 40 Hz in the input source are actually generated very close results to each other. This fact could mean that above 20 Hz the frequency has no effect on the results and that for impedance

ratios less than 0.24 the MASW test will be ineffective in resolving a soil profile.

- v. In a similar way, for input source frequency of 10 Hz impedance ratios less than 0.18 the MASW test will be ineffective in resolving any soil profile. Finally, for input source frequency of 5 Hz impedance ratios less than 0.13 the MASW test will be ineffective in resolving any soil profile.

In general, it could be concluded that impedance ratios about 0.5 are the minimum value required in order to properly resolve a three-layers model when a MASW test is carried out for shear wave velocity profiling.

8 REFERENCES

- Ali, H., Nasser – Moghaddam, and Cascante, G. (2011). Use of numerical simulation for the identification of underground voids using the MASW test. In *Proceedings of the 14th Pan-American conference on soil mechanics and geotechnical engineering and 64th Canadian Geotechnical Conference*. (2 - 6 October, 2011)
- Ali, H. (2015) Current laboratory and field techniques to measure shear wave velocity in soils. University of Waterloo, Civil and Environmental Engineering Department, PhD. Dissertation.
- Almeida, M., Marques, M. and Baroni, M. (2010). Geotechnical parameters of very-soft clays from CPTu. Proceeding of 2nd International Symposium on Cone Penetration Testing, May 9-11, 2010, Huntington Beach, California. Paper 2-46.
- Amoroso, S. (2014). Prediction of the shear wave velocity VS from CPT and DMT at research sites. *Frontiers of Structural and Civil Engineering*, vol 8(1): 83–92
- Armstrong, K. (2010). Archaeological geophysical prospection in peatland environments. Dissertation for the degree 'Doctor of Philosophy', awarded by Bournemouth University, U.K.
- Avwenagha, E., Akpokodje E. and Tse A. (2014). Geotechnical Properties of Subsurface Soils in Warri, Western Niger Delta, Nigeria. *Journal of Earth Sciences and Geotechnical Engineering*, vol. 4, no. 1, 2014, 89-102. Scienpress Ltd, 2014
- Building Seismic Safety Council – BSSC (2003), NEHRP Recommended Provisions for seismic Regulations for New buildings and other Structures, Part1: Provisions, FEMA 368, Federal Emergency Management Agency, Washington, D.C.
- Campanella, R. G., Stewart, W. P., Roy, D., and Davies, M. P. (1994). "Low Strain Dynamic Characteristics of Soils with the Downhold Seismic Piezocone Penetrometer," *Dynamic Geotechnical Testing II*, ASTM STP 1213, Ronald J. Ebelhar, Vincent P. Drnevich, and Bruce L. Kutter, Eds., American Society for Testing and Materials, Philadelphia, 1994.
- Díaz-Durán, F., Cascante, G. and Pandey, H. (2018) Effect of Input-Source Frequency Content on Results from Seismic Techniques for the Vs Profile Definition in Spatially Variable Soils, *Proceedings from Symposium on the Application of Geophysics to Engineering and Environmental Problems – SAGEEP 2018*.
- Grigoryan, V. (2012) Fourier Series and Numerical Methods, Lecture 18 of Classnotes for *MATH 124B*, University of California – Santa Barbara, Recovered from: <http://web.math.ucsb.edu/~grigoryan/124B/>
- Hoar, R. and K. Stokoe (1984). Field and laboratory measurements of material damping of soil in shear. In *Proc. 8th World Conference on Earthquake Engineering*, Volume Vol III, San Francisco, pp. 47 - 54.
- Itasca (2000). *Fast Lagrangian Analysis of Continua - Users Guide*. Minneapolis, Minnesota: Itasca consulting group Inc.
- Kuhlemeyer, R. and Lysmer, J. (1973) Finite Element Method Accuracy for Wave Propagation Problems, *Journal of the Soil Dynamics Division*, 99: 421-427.
- Nazarian, S., Stokoe, K. H. 1984. In-situ shear wave velocities from spectral analysis of surface waves, *Proc. of the Eighth World Conf. on Earthquake Engineering*, San Francisco, California, III, July 21 - 28, pp. 31 – 38.
- Paoletti, L., Mouton, E. and Liposcak, I. (2013) Comparison of underwater MASW, seismic CTP and downhole methods offshore Croatia. *Geotechnical and Geophysical Site Characterization 4 – Coutinho & Mayne (Editors)*. Taylor & Francis Group, London.
- Park, C. B., Miller, R. D., and Xia J. 1999. Multichannel analysis of surface waves. *Geophysics*, 64(3), 800-808.
- Santamarina, JC, Rinaldi, V., Fratta, D., Klein, K., Wang, Y., Chun, G., and Cascante, G. (2005) "A Survey of Elastic and Electromagnetic Properties of Near-Surface Soils". *Near-Surface Geophysics Part 1: Concepts and Fundamentals*.
- Terzaghi, K. and Peck, R. (1948) *Soil Mechanics in Engineering Practice*. John Wiley & Sons, Hoboken.