Geotechnical Resistance Factors for Seismic Design

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ABSTRACT

The next edition of the Canadian Highway Bridge Design Code will contain a table of geotechnical resistance factors to be used for seismic design. This paper will investigate how the geotechnical resistance factors should change as a result of the target maximum acceptable failure probability, which in turn, depends on the severity of failure consequences. The investigation will include consideration of design lifetime with a goal being to achieve a design reliability which properly accounts for the rare and extreme-value nature of the seismic loading. The preliminary results suggest resistance factors which are lower than commonly used at the moment in Canada and the failure probability is not greatly dependent on the return period of the design earthquake.

RÉSUMÉ

La prochaine édition du Code canadien de conception des ponts routiers contiendra un tableau des facteurs de résistance géotechnique à utiliser pour la conception sismique. Cet article examinera comment les facteurs de résistance géotechniques devraient changer en fonction de la probabilité de défaillance maximum acceptable cible, qui à son tour, dépend de la gravité des conséquences de la défaillance. L'étude comprendra la prise en compte de la durée de vie de la conception dans le but d'obtenir une fiabilité de conception qui tient compte de la nature rare et extrême de la charge sismique. Les résultats préliminaires suggèrent des facteurs de résistance qui sont inférieurs à ceux couramment utilisés actuellement au Canada et la probabilité de défaillance ne dépend pas beaucoup de la période de retour du séisme de conception.

1 INTRODUCTION

Fenton and Naghibi (2017) presented a preliminary reliability analysis of the seismic design of geotechnical systems using total probability theorem of the form:

$$\begin{aligned} p_f &= \mathrm{P}[F] = \mathrm{P}[F \mid R = r_1]\mathrm{P}[R = r_1] \\ &\quad + \mathrm{P}[F \mid R = r_2]\mathrm{P}[R = r_2] + \ldots \\ &= \sum_{i=1}^{\infty} \mathrm{P}[F \mid R = r_i]\mathrm{P}[R = r_i] \le p_m \end{aligned} \qquad [1]$$

where F is the event that the footing fails, p_m is the maximum acceptable failure probability, R is the random return period of the earthquake, and r_i is a specific realization of R. Larger values of r_i imply stronger earthquakes. In their work, a lognormal distribution was assumed for the conditional failure probability $\mathrm{P}[F \mid R = r_i]$ in Eq. 1, and a Poisson distribution was employed for the unconditional probability $\mathrm{P}[R = r_i]$ in Eq. 1. In the present study, the bearing capacity failure probability of a strip footing under seismic loading is estimated by comparing the actual load on the footing (random) to the actual resistance of the footing (also random). The actual resistance is estimated using local averages of the soil properties beneath the footing.

The goal of this work is to determine resistance factors required to achieve certain target failure probabilities for extreme limit state seismic design of shallow foundation by means of the Load and Resistance Factor Design (LRFD) approach. Load factors used are as prescribed by the Canadian Highway Bridge Design Code (CHBDC) (CSA, 2014).

The resistance factors required to achieve target failure probabilities are estimated as a function of the return period of the earthquake being designed against. For example, if the foundation is being designed to resist an earthquake with return period 975 years, then seismic forces, or accelerations, are imposed on the foundation and the design aims to achieve a target failure probability consistent with the performance criteria of the CHBDC. This target failure probability will change as the return period changes, so that the required resistance factor may also change. The failure probability of the foundation, designed using a specific resistance factor, is estimated using theory and Monte-Carlo simulation. If the failure probability is excessive then the resistance factor needs to be reduced.

The load and resistance factor design of shallow foundations against bearing failure has been studied previously by Fenton et. al (2008). The geotechnical design proceeds by ensuring that the factored geotechnical resistance at least equals the effect of factored loads, i.e:

$$\varphi_g \hat{R}_u \ge \sum \alpha_i \hat{F}_i = \hat{F}_T \qquad [2]$$

in which φ_g is the geotechnical resistance factor at ULS, $\hat{R}_u$ is the characteristic ultimate resistance, and $\alpha_i \hat{F}_i$ is the i^{th} factored load. The load factors, α_i, typically account for uncertainty in loads, and are greater than 1.0

for ultimate limit states while the geotechnical resistance factor, φ_g, is typically less than 1.0 and accounts for uncertainties in geotechnical parameters and prediction models used to estimate the characteristic geotechnical resistance.

The paper is organized as follows: In Section 2, the random fields used to model the soil supporting the foundation are described along with random load model. In Section 3, the reliability-based footing design approach is discussed and a theoretical model is developed to estimate the failure probability of a footing. The simulation model used to access footing failure probability is described in Section 4. The results are presented in Section 5 and Conclusions are summarized in Section 6.

2 RANDOM MODELS

A random field $X(\underset{\sim}{t})$ is a collection of random variables $X_1 = X(\underset{\sim}{x}_1), X_2 = X(\underset{\sim}{x}_2), \ldots$, whose values are associated with each spatial location $\underset{\sim}{x}$. In this paper, two random fields are used to represent the soil properties cohesion and friction angle. The cohesion field, c, is assumed to be lognormally distributed while friction angle field, ϕ, is assumed to be bounded between 10 and 30 degrees using a bounded tanh distribution (see Fenton and Griffiths, 2008).

Values within each random field are correlated with one another as a function of the distance between them. In this paper, an isotropic exponentially decaying Markov correlation function is used, defined by

$$\rho(\tau_{ij}) = \exp\left\{\frac{-2\left|\tau_{ij}\right|}{\theta}\right\} \qquad [3]$$

where τ_{ij} is the distance between any two points, X_i and X_j, in the field, and θ is the correlation length (Fenton and Griffiths, 2008). The same correlation length is used for both cohesion and friction angle fields.

Since the cohesion field is lognormally distributed with mean and standard deviation μ_c and σ_c, then $\ln c$ is normally distributed with parameters

$$\begin{aligned} \sigma_{\ln c}^2 &= \ln(1 + v_c^2) \\ \mu_{\ln c} &= \ln(\mu_c) - \frac{1}{2}\sigma_{\ln c}^2 \end{aligned} \qquad [4]$$

where $v_c = \sigma_c / \mu_c$ is the coefficient of variation of c.

The random load applied to the footing is equal to the sum of the maximum life time live load, F_L, and the relatively static dead load, F_D, i.e.

$$F_T = F_L + F_D \qquad [5]$$

where F_L and F_D are each assumed to be lognormally distributed. The mean and variance of the total load, F_T, assuming live and dead loads are independent, are given by

$$\mu_T = \mu_L + \mu_D, \qquad \sigma_T^2 = \sigma_L^2 + \sigma_D^2 \qquad [6]$$

3 THEORETICAL MODEL

In this section, a theoretical model for estimating the designed footing width as well as the failure probability of the footing is developed. The objective is to estimate the resistance factor required to achieve a certain maximum tolerable failure probability, p_m.

3.1 Design footing

In general, the seismic design of a footing involves both the seismic loading and the static loading. The static loading in turn generally involves both dead and live loads. Live loads are usually deemed to lead to negligible horizontal inertial forces on a foundation, due to the fact that the live loads are typically able to move. However, it is almost certain that there will be at least some vertical live loading during an earthquake event. In this paper, we will take the vertical design load during an earthquake event to be equal to:

$$\hat{F}_T = \alpha_L \hat{F}_L + \alpha_D \hat{F}_D \qquad [7]$$

where $\hat{F}_L$ is the characteristic live load, $\hat{F}_D$ is the characteristic dead load, and α_L and α_D are the live and dead load factors, respectively. For earthquake loading, the load factors given by the CHBDC (CSA, 2014) are $\alpha_L = 0$ and $\alpha_D = 1.25$. We will conservatively use $\alpha_L = 1$ to account for both vertical seismic loading as well as the component of live load that is present during an earthquake event. The characteristic loads, $\hat{F}_L$ and $\hat{F}_D$, are obtained by applying bias factors to the means of the load distribution: $\hat{F}_L = \dfrac{\mu_L}{0.9}$, $\hat{F}_D = \dfrac{\mu_D}{1.05}$ (Fenton et al.,2015), where μ_L and μ_D are the means of the maximum lifetime dead and live loads, respectively.

The bearing capacity of a strip footing subjected to static loading is formulated by Meyerhoff (1963) as

$$q_u = cN_c s_c d_c i_c + qN_q s_q d_q i_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma i_\gamma \qquad [8]$$

where q_u denotes the ultimate bearing capacity of the foundation under a vertical centered load, c is the soil's cohesion, $q = \gamma D$ is the total pressure on the unit length of the bearing surface, B is the footing width, γ is the soil's unit weight, D is the depth of foundation, s_c, s_q, s_γ are the shape factors, d_c, d_q, d_γ are the depth factors, i_c, i_q, i_γ are the load inclination factors, and N_c, N_q, N_γ are the bearing capacity factors which only depend on the soil's friction angle, ϕ, defined as follows:

$$\begin{aligned} N_q &= e^{\pi \tan\varphi} \tan^2\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) \\ N_c &= (N_q - 1) / \tan\phi \\ N_\gamma &= 2\left(N_q + 1\right)\tan\phi \end{aligned} \qquad [9]$$

Budhu and Al-karni (1993) introduce seismic factors in Eq. 8 to account for seismic loading as follows:

$$q_{uE} = cN_c s_c d_c e_c + qN_q s_q d_q e_q + 0.5\gamma BN_\gamma s_\gamma d_\gamma e_\gamma \qquad [10]$$

or alternatively

$$q_{uE} = cN_{cE} s_c d_c + qN_{qE} s_q d_q + 0.5\gamma BN_{\gamma E} s_\gamma d_\gamma \qquad [11]$$

where the seismic factors are defined as:

$$\begin{aligned} e_c &= \exp\left(-4.3k_h^{1+\frac{c}{\gamma H}}\right) \\ e_q &= (1-k_v)\exp\left(\frac{-5.3k_h^{1.2}}{1-k_v}\right) \\ e_\gamma &= \left(1-\frac{2}{3}k_v\right)\exp\left(\frac{-9k_h^{1.1}}{1-k_v}\right) \end{aligned} \qquad [12]$$

and k_h and k_v are horizontal and vertical acceleration coefficients, $N_{cE} = N_c e_c, N_{qE} = N_q e_q, N_{\gamma E} = N_\gamma e_\gamma$ are seismic bearing capacity factors, and

$$H = \frac{0.5B}{\cos\left(\frac{\pi}{4}+\frac{\phi}{2}\right)}\exp\left(\frac{\pi}{2}\tan\phi\right) + D \qquad [13]$$

is the depth of the soil's failure zone from the ground surface under the seismic loading. As far as the authors can tell, the seismic factors developed by Budhu and Al-karni (1993) include the effects of load inclination arising from the seismic inertial forces, and so the horizontal seismic loads do not need to be explicitly considered. The bearing capacity predicted by Eq. 10 is to be compared to the vertical component of the applied load on the footing, F_T.

The choice of design k_h and k_v depends directly on the peak ground acceleration (PGA) calculated as (Melo and Sharma, 2004):

$$\begin{aligned} \hat{k}_h &= 0.5\hat{a}_p \\ \hat{k}_v &= 0.25\hat{k}_h \end{aligned} \qquad [14]$$

where $\hat{a}_p$ is the PGA. The following regression was fit to the PGA values estimated for the Vancouver area by NRCan website for the four earthquake return periods 100, 475, 975, and 2475 years (http://www.earthquakescanada.nrcan.gc.ca/hazard-alea/interpolat/index_2015-en.php),

$$\hat{a}_p = 0.15505 - 0.075897\ln(r_m) + 0.014632\ln^2(r_m) \qquad [15]$$

where r_m is the return period of an earthquake having magnitude at least m.

In this work, D, the embedment depth of the footing in Eq. 13 is assumed to be zero for simplicity, and all shape and depth factors are set to 1. Thus, the simplified equation

$$\hat{q}_{uE} = \hat{c}\hat{N}_{cE} + 0.5\gamma B\hat{N}_{\gamma E} = \hat{c}\hat{N}_c\hat{e}_c + 0.5\gamma B\hat{N}_\gamma\hat{e}_\gamma \qquad [16]$$

will be used here for the seismic design of a strip footing under earthquake loading. The characteristic ultimate geotechnical resistance of the strip footing becomes,

$$\hat{R}_{uE} = B\hat{q}_{uE} \qquad [17]$$

The hat parameters in Eq. 16 are obtained by sampling the soil in the vicinity of the footing leading to m observed values of the soil's properties as shown in Figure 1. For instance, $\hat{c}$ is estimated as the geometric average of m observations $\hat{c}_1, \hat{c}_2, ..., \hat{c}_m$ of soil cohesion taken at the site:

$$\hat{c} = \left[\prod_{i=1}^{m}\hat{c}_i\right]^{1/m} = \exp\left\{\frac{1}{m}\sum_{i=1}^{m}\ln\hat{c}_i\right\} \qquad [18]$$

Similarly, $\hat{\phi}$, is computed as the arithmetic average of m observed friction angle values, $\hat{\phi}_1, \hat{\phi}_2, ..., \hat{\phi}_m$, as

$$\hat{\phi} = \frac{1}{m}\sum_{i=1}^{m}\hat{\phi}_i \qquad [19]$$

The earthquake parameters $\hat{e}_c$ and $\hat{e}_\gamma$ in Eq. 16 are obtained by Eq. 12 using the design earthquake coefficients $\hat{k}_h$ and $\hat{k}_v$.

The goal of the design is to determine the footing width, $\hat{B}$, which satisfies the LRFD Eq. 2, using $\hat{R}_u$ replaced by $\hat{R}_{uE}$. Since the seismic factors in Eq. 12 involve the footing width (see Eq. 13), the determination of $\hat{B}$ involves an iteration. The one-point iteration method was found to converge very quickly. The basic idea of one-point iteration is to start with an initial guess, compute seismic coefficients in Eq. 12, then solve for an updated $\hat{B}$ using the LRFD equation, and repeat until $\hat{B}$ remains stable. A trial design footing width of

$$B_o = \hat{F}_T / \varphi_g \mu_c \mu_{Nc} \qquad [20]$$

with a moderate resistance factor of $\varphi_g = 0.7$ was used as the initial guess where μ_{Nc} is approximated by using mean soil properties (μ_c and μ_ϕ) in Eq. 9:

$$\mu_{Nc} = \left\{e^{\pi\tan\mu_\phi}\tan^2\left(\frac{\pi}{4}+\frac{\mu_\phi}{2}\right)-1\right\}/\tan\mu_\phi \qquad [21]$$

In the one-point iteration, the design footing width, $\hat{B}$, is obtained by substituting Eq.'s 7 and 17 into the LRFD Eq. 2, solved at the equality, which in turn leads to solving the following quadratic equation

$$0.5\gamma\hat{N}_{\gamma E}\varphi_g\hat{B}^2 + \hat{c}\hat{N}_{cE}\varphi_g\hat{B} - \hat{F} = 0 \qquad [22]$$

for $\hat{B}$, giving the following solution:

$$\hat{B} = \frac{-\hat{c}\hat{N}_{cE} + \sqrt{c^2\hat{N}_{cE}^2 + 2\hat{F}\gamma\hat{N}_{\gamma E}/\varphi_g}}{\gamma\hat{N}_{\gamma E}} \qquad [23]$$

3.2 Estimation of actual footing resistance

Fenton et. al (2008) found the effective averaging domain to be best approximated by local average of the soil properties over a region of size $V = W \times W$, centered directly under the footing. W is taken as 80% of the average mean depth of the wedge zone, as given by the classical Prandtl (1921) failure mechanism:

$$W = \frac{0.8}{2}\mu_B \tan\left(\frac{\pi}{4} + \frac{\mu_\phi}{2}\right) \qquad [24]$$

where μ_ϕ is the mean friction angle (in radians), within the zone of influence of the footing, and μ_B is an estimate of the mean footing width obtained by using mean soil properties (μ_c and μ_ϕ) in Eq. 23.

The actual ultimate resistance is thus estimated to be,

$$\bar{R}_{uE} = \hat{B}\bar{q}_{uE} = \hat{B}\left(\bar{c}\bar{N}_{cE} + 0.5\gamma\hat{B}\bar{N}_{\gamma E}\right) \qquad [25]$$

where the bar parameters in Eq. 25 are obtained by averaging the soil properties $\bar{c}$ and $\bar{\phi}$ over the region V underneath the footing, as depicted in Figure 1. For instance, $\bar{c}$ is estimated as the geometric average of soil cohesion over V according to:

$$\bar{c} = \exp\left\{\frac{1}{V}\int_V \ln c(\underset{\sim}{x})\, d\underset{\sim}{x}\right\} \qquad [26]$$

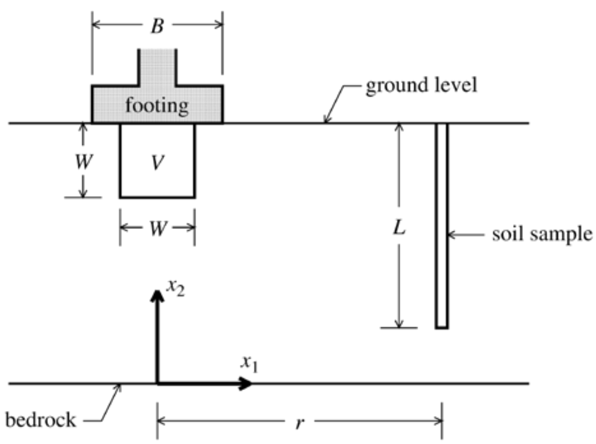

Figure 1: Averaging regions used to predict probability of bearing capacity failure

Similarly, $\bar{\phi}$, is computed as the arithmetic average over the same region

$$\bar{\phi} = \frac{1}{V}\int_V \phi(\underset{\sim}{x})\, d\underset{\sim}{x} \qquad [27]$$

The bar parameters are now defined as:

$$\begin{aligned} \bar{N}_q &= e^{\pi \tan\bar{\phi}} \tan^2\left(\frac{\pi}{4} + \frac{\bar{\phi}}{2}\right) \\ \bar{N}_c &= (\bar{N}_q - 1) / \tan\bar{\phi} \\ \bar{N}_\gamma &= 2\left(\bar{N}_q + 1\right)\tan\bar{\phi} \end{aligned} \qquad [28]$$

3.3 Estimation of Failure Probability

Since the earthquake magnitude and its corresponding return period are unknown, we must make use of the total probability theorem to compute the failure probability of the designed foundation for a given resistance factor, φ_g , and design $\hat{k}_h$:

$$p_f = \sum_{i=1}^{n_r} \mathrm{P}\left[F_T > \bar{R}_{uE} \mid R = r_i\right]\mathrm{P}\left[R = r_i\right] \qquad [29]$$

The conditional probability of failure of a footing for a given return period $R = r_i$ is:

$$\mathrm{P}\left[F_T > \bar{R}_{uE} \mid R = r_i\right] = \mathrm{P}\left[F_T > \bar{R}_{uE} \mid k_{h_i}\right] \qquad [30]$$

Eq. 30 has no analytical solution, so far as the authors are aware, and so is estimated by simulation as described in the next Section.

The unconditional probability $\mathrm{P}\left[R = r_i\right]$ used in Eq. 29 is obtained as follows:

$$\begin{aligned} \mathrm{P}\left[R = r_i\right] &= \mathrm{P}\left[M_{\max}(l) = m_i\right] \\ &\approx F_{M_{\max}}\left(m_i + \frac{\Delta m}{2}\right) - F_{M_{\max}}\left(m_i - \frac{\Delta m}{2}\right) \\ &= \exp\left\{-l / r_{m_{i+0.5}}\right\} \text{-} \exp\left\{-l / r_{m_{i-0.5}}\right\} \end{aligned} \qquad [31]$$

where, according to Fenton and Naghibi (2017),

$$F_{M_{\max}}(m_i) = \exp\left\{-l / r_{m_i}\right\} \qquad [32]$$

$M_{\max}(l)$ is the maximum earthquake magnitude experienced over lifetime l and $F_{M_{\max}}$ is the cumulative distribution function of $M_{\max}(l)$.

In order to compute the sum in Eq. 29, we must discretize the range in return periods. For simplicity, n_r is selected to be 41 subdividing the range $\ln(r_m)$ from 4.0 to 8.0, corresponding to return periods ranging from 55 to 3000 years, into 40 intervals such that,

$$\begin{aligned} r_{m_i} &= \exp\left\{4.0 + (i-1)\Delta\ln(r_m)\right\} \\ &= \exp\left\{4.0 + (i-1)\frac{8.0 - 4.0}{n_r - 1}\right\} \\ &= \exp\left\{4.0 + 0.1(i-1)\right\} \end{aligned} \qquad [33]$$

Once the probability of failure is computed via Eq. 29, it can be compared to the maximum acceptable failure probability, $p_m = \Phi^{-1}(-\beta)$, where β is the target reliability index corresponding to p_m , and Φ is the standard normal cumulative distribution function. If p_f exceeds p_m , then the resistance factor needs to be reduced.

4 SIMULATION MODEL

The simulation involves $n_{sim} = 100{,}000$ realizations. The standard deviation of the failure probability estimate is

$\sqrt{p_f(1-p_f)/n_{sim}} \simeq 0.003\sqrt{p_f}$ for small failure probability p_f. This means that if $p_f = 1\times10^{-4}$, then the standard deviation of its estimate is about 3×10^{-5} and therefore, $n_{sim} = 100,000$ can reasonably resolve probabilities down to about 10^{-4}. The steps involved in the simulation are as follows:

i. Simulate c and ϕ using local average subdivision (LAS, Fenton and Griffiths 2008).
ii. Sample the soil at the distance r from the footing center line to obtain $\hat{c}$ and $\hat{\phi}$.
iii. Obtain the design footing width, $\hat{B}$, for a given resistance factor, φ_g, design $\hat{k}_h$ and actual k_h.
iv. Average the c and ϕ fields over the domain V to obtain $\bar{c}$ and $\bar{\phi}$ and thus the actual footing resistance $\bar{R}_{uE}$.
v. Simulate $F_T = F_L + F_D$. The footing fails if $F_T > \bar{R}_{uE}$, and if so, increment the number of failures counter, n_{fail}.
vi. Repeat, from step i, n_{sim} times.
vii. Estimate failure probability, given φ_g, $\hat{k}_h$ and k_h as $\mathrm{P}\left[F_T > \bar{R}_{uE} \mid k_{h_i}\right] \approx n_{fail}/n_{sim}$.

5 RESULTS AND DISCUSSION

The objective of this section is to determine resistance factors required to achieve a maximum tolerable lifetime failure probability $p_m = 0.01$, corresponding to a target reliability index of $\beta = 2.33$. The parameters used in this case study are listed in Table 1. The design values, $\hat{k}_h$, are obtained for Vancouver using Eq.'s 14 and 15 for return periods $r_m = 475, 975,$ and 2475 years.

Table 1 Input parameters used in simulation

Parameters	Values Considered
μ_c, σ_c	100, 50 (kN/m^2)
μ_ϕ, σ_ϕ	20, 10 (degrees)
μ_L, σ_L	200, 60 (kN)
μ_D, σ_D	600, 90 (kN)
α_L, α_D	1.5, 1.25
$\hat{k}_h$	0.121, 0.163, 0.228
$\hat{F}_T$	936.5 (kN)
$r = \theta$	5 (m)
γ	15 (kN/m^3)
n_r	41
n_{sim}	100,000

Figure 2 depicts the conditional failure probabilities of Eq. 30 as a function of actual lifetime maximum k_h for $\varphi_g = 0.5$ and three design k_h values. The lines show some jitter due to natural sampling variability but are used directly in Eq. 29 to obtain the total probability of footing failure.

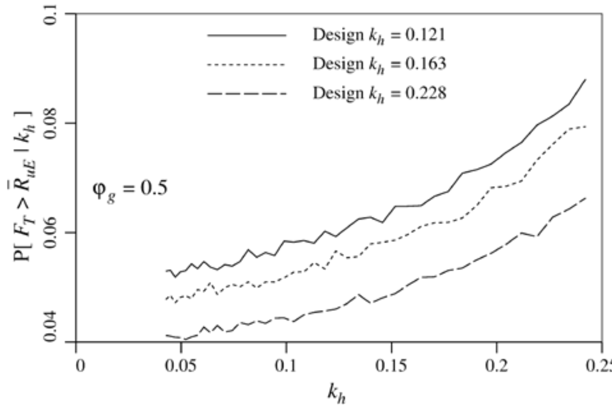

Figure 2: Plot of failure probability vs. actual k_h for $\varphi_g = 0.5$ and three design $\hat{k}_h$ values

Figure 3 shows the total failure probability of the footing, as a function of resistance factor for different values of design $\hat{k}_h$. This figure can be used for design by drawing a horizontal line across the plot at the maximum tolerable failure probability, p_m, and then reading off the required resistance factor for a given design $\hat{k}_h$. For example, if $p_m = 0.01$, it can be seen that the resistance factor is almost between 0.3 and 0.34 for $k_h = 0.121$. For other k_h's considered, the required resistance factor is between 0.34 and 0.4.

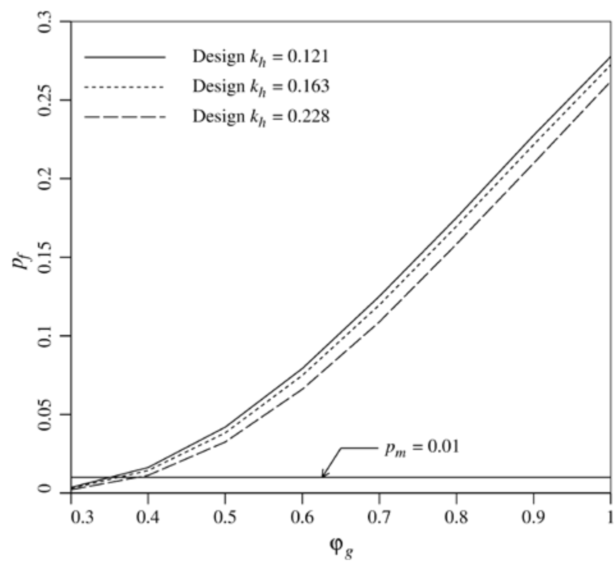

Figure 3: Plot of failure probability vs. resistance factor for three design k_h values

6 CONCLUSIONS AND FUTURE WORK

This paper presents a preliminary investigation into the relationship between geotechnical resistance factor, lifetime and uncertain extreme future events. In principle, the resistance factor should not be involved in the consideration of these extreme loads. However, current practice in Canada seems to involve adjusting the resistance factor to account for the rare nature of seismic loads. The results of the paper are somewhat surprising in that the suggested resistance factors are lower than expected. Further research is required in particular to compare static resistance factors obtained using the same approach to those obtained in this study.

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