Examples of slug tests in textbooks: given interpretation and correct interpretation

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ABSTRACT
When a slug test is performed in a monitoring well, the test data of the water column $Z(t)$ versus time t may be analyzed using several methods. These use three types of graphs, which provide a clear diagnosis for the test data when used all together. According to experience, most test data contain a systematic error H_0 on the water column data, which is due to five sources of error. According to experience, most plots of log $Z(t)$ versus t yield an upward curvature, a minority give a downward curvature, and very few provide a straight line. The upward and downward curvatures correspond to positive and negative values of H_0, whereas a straight line means no piezometric error. For this paper, 21 sets of slug test data found in textbooks were analyzed, using the three diagnostic graphs. For these textbook data, the plots of log $Z(t)$ vs. t are either curved upward or straight, but no data set has the downward curvature. The optimization method easily found the H_0 value in all cases, and the velocity graph for almost all sets. Therefore, the authors of textbooks have ignored the upward curvatures, and often have provided an incorrect interpretation for their examples.

RÉSUMÉ
Quand on fait un essai de perméabilité à niveau variable dans un piézomètre, les données de la colonne d'eau $Z(t)$ vs le temps t peuvent être analysées par diverses méthodes. Celles-ci utilisent trois types de graphes qui, lorsque utilisés ensemble, donnent un diagnostic clair. Selon l'expérience, la plupart des données contiennent une erreur systématique H_0 sur les valeurs de colonne d'eau, qui est due à cinq sources d'erreur. Selon l'expérience, les graphes de log $Z(t)$ vs t sont en majorité courbés vers le haut, en minorité vers le bas, et très peu donnent une ligne droite. Les courbures vers le haut et le bas correspondent à des valeurs positives et négatives de H_0, tandis qu'une droite signifie qu'il n'y a pas d'erreur piézométrique. Pour cet article, 21 ensembles de données trouvées dans des livres ont été analysés, avec les trois graphes diagnostiques. Pour les données des livres, les graphes de log $Z(t)$ vs t sont soit courbés vers le haut soit rectilignes, mais aucun n'avait une courbure vers le bas. La méthode d'optimisation a facilement trouvé la valeur de H_0 pour tous les cas, et la méthode des vitesses pour la plupart des cas. Les auteurs des livres ont donc ignoré les courbures vers le bas, et ont souvent fourni des interprétations incorrectes pour leurs exemples.

1 INTRODUCTION

For at least one century, specialists in geotechnique and groundwater have routinely performed slug tests in driven flush-joint casings, driven field permeameters, monitoring wells, and between packers in boreholes. Slug tests assess the local value of hydraulic conductivity K in tested aquifers. After the initial rapid change in water level, the water column Z varies versus time t within the solid pipe. A slug test has a major advantage over pumping tests: it is fast and does not require disposal of large volumes of potentially polluted groundwater. Its major drawback is that it tests a small volume of aquifer, at a small radial distance around the monitoring well (MW) filter pack, where the material has been most remoulded by drilling operations (Chapuis 2001; Chapuis and Chenaf 2002, 2003, 2010). All advantages and drawbacks can be assessed by examining in detail the drilling procedures, MW installation procedures, testing methods, and aquifer characteristics (Nielsen and Schalla 2005).

This paper deals with the slug test data of examples used in textbooks. All data here are for overdamped slug tests, in which the water column slowly returns to equilibrium. There is no underdamped test here, where the water column oscillates back to equilibrium.

The paper briefly presents the available theories and current findings relative to overdamped slug tests in aquifers. Then, it analyzes in detail a few examples of textbooks before making a general picture of all examples used in textbooks.

2 BACKGROUND, THEORIES

Several methods are used to analyze overdamped slug tests in aquifers. They are known to yield different results for the K value, and to be user-dependent. The methods belong to either group 1, which neglects the influence of solid matrix strains, or group 3, which tries to consider them, whereas group 2 is for aquitards with delayed and irreversible consolidation strain (Chapuis 1998), thus not used below. Chapuis (2015) proposed to clarify which theory should be used by using three diagnostic graphs, including the derivative or velocity graph (Schneebeli 1966). The diagnostic graphs are non dimensional when using the ratio $Z^* = Z(t) / Z(t = 0)$. After analyzing thousands of tests, two major conclusions emerged. The first major conclusion, all field test data follow a single theory, that of group 1 (Lefranc 1937; Hvorslev 1951; Bouwer and Rice 1976). This is a practical proof that the group-3 theory (Cooper et al. 1967), which involves storativity, S, is incorrect, mostly because it mistreats solid mechanics. The second major conclusion is that the three-diagnostic graphs approach yields a user-independent K value.

For an overdamped response, CAN/BNQ 2501-135 is the standard in Canada (CAN/BNQ 1988, 2008, 2014). Although slightly modified along the years this standard has always retained the derivative or velocity graph method. The derivative method is also used in old French standard NF P94-132 (AFNOR 1992) and the present European standard 22282-2 (ISO 2012).

2.1 The three diagnostic graphs

Chapuis (2015) introduced the three diagnostic graphs as follows. The first graph, usually called Hvorslev's graph, is much older than the report by Hvorslev (1951). It appears in Fig. 1 with several possibilities including the straight line as predicted by theory, and the two types of curves as obtained in the field with either upward or downward curvature.

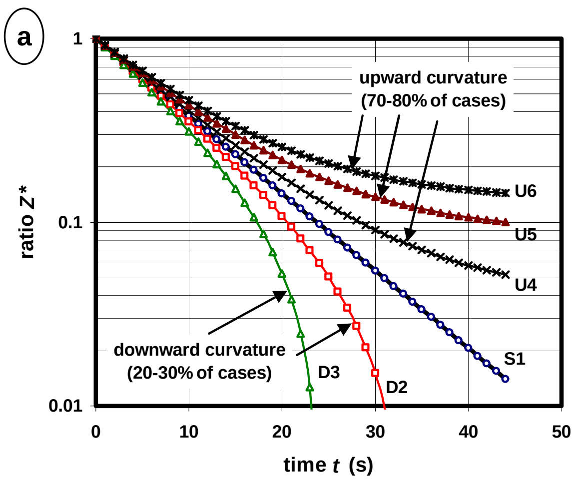

Figure 1. First diagnostic graph: six examples are presented with symbols S1 to U6 (S = straight; D = downward curvature; U = upward curvature).

The second diagnostic graph is the non-dimensional graph of the group-3 basic theory (Cooper et al. 1967). It appears in Fig. 2, similar to that of the original paper.

The third diagnostic graph is the derivative or velocity graph, of $Z(t)$ vs. dZ/dt, useful for the derivative sensitivity. Derivatives are used for pumping tests (Bourdet et al. 1989; Bourdet 2002; Renard et al. 2009). For slug tests, the derivative plot was proposed by Schneebeli (1966) and improved by Chapuis et al. (1981) for several aspects of field tests in flush-joint driven casings. These include assessing seepage conditions within a succession of aquifer and aquitard layers, and also hydraulic fracturing or separation. The 3rd diagnostic graph appears in Fig. 3 for the group-1 theory (Lefranc 1937; Hvorslev 1951; Bouwer and Rice 1976) and for the group-3 theory (Cooper et al. 1967).

The 3rd diagnostic graph uses a normalized derivative defined as $V^* = (dZ^*/dt)/(dZ^*/dt)_{95}$ where $(dZ^*/dt)_{95}$ is the velocity when $Z^* = 0.95$ (Fig. 3). A 0.95-value was retained instead of 1 because initial readings are rarely reliable, due to dynamic initial effects and release of bubbles, which are not considered in theories. In Fig. 3, the group-1 or Hvorslev's solution is a straight line.

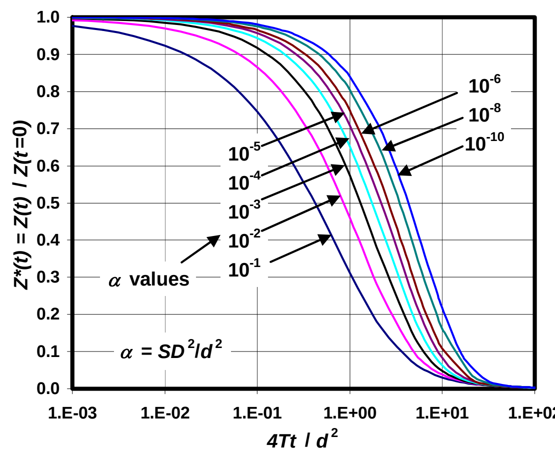

Figure 2. Second diagnostic graph for the group-3 theory. The group-1 theory plot is slightly to the right of the last master curve for $\alpha = 10^{-10}$. The abscissa involves the aquifer transmissivity T.

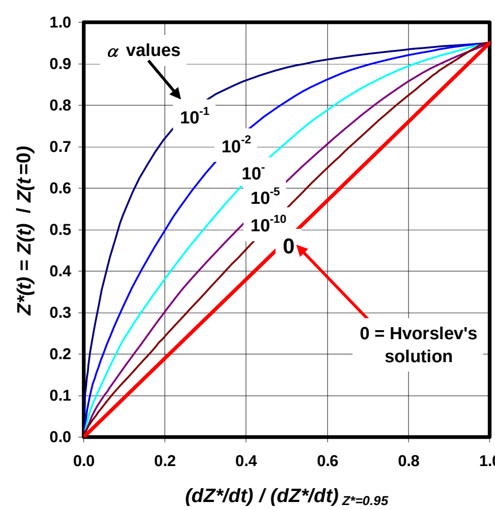

Figure 3. Third diagnostic graph for the group-1 and group-3 theories.

The main role of the 3rd graph is to verify whether the derivative is straight (group-1 theory) or smoothly curved (group-3 theory). If the data do not converge towards the origin of axes but towards $x = 0$ and $y = H_0$, then H_0 is the error made when assessing the piezometric level for the test. There are at least five reasons to make an error H_0 (Chapuis 2009a). When the data are inaccurate, the best-fit method of Chiasson (2005) gives better results than the derivative or velocity graph method. The latter has some advantage when dealing with special cases such as initial consolidation (irreversible strains) at the beginning of a test in an aquitard, and for identifying special cases such as hydraulic fracturing or separation, and hydraulic short-circuiting along a poorly sealed monitoring well.

Chapuis (2015) proved, with thousands of field test data, that the group-3 theory (Cooper et al. 1967), which involves storativity, S, is incorrect, mostly because it mistreats solid mechanics. Recently, Chapuis (2017) added a theoretical proof to this previous practical proof.

He proceeded with the stress-strain analysis around the injection zone during a slug test in an aquifer. The complete stress-strain analysis has shown that the soil (or rock) radial contraction (or dilation) is fully compensated by the tangential expansion or contraction. As a result, there is no volumetric strain for either a cylindrical cavity or a spherical cavity in an elastic medium, infinite or of radius *R*. This outcome is valid for slug test conditions that yield purely elastic strains, which is usually the case in aquifers. This means that the conservation equation to be used for slug tests is the Laplace equation (used by the group-1 theory) and not the too simplified diffusion equation of the group-3 theory. This has shown that the group-2 theory is the only one to be physically correct for slug tests in aquifers.

In a soil or rock undergoing some irreversible strain, a small zone adjacent to the cavity has permanent strain for a certain time at the beginning of the slug test. This yields a non-linear derivative graph at the beginning of the test, caused by a “geometric” drop in water level. However, after a few hours for soft rock aquitard, or 1 or 2 days for clay, the material becomes normally or over consolidated, and thus, the elasticity assumption prevails, which yields a straight-line derivative graph. After a long time, the derivative graph may also change its shape. This is due to the long lag-time of the general response in an aquitard, and the fact that a “static” water level in the riser pipe of any MW in an aquitard is never a piezometric level (Chapuis 2009a; Chapuis et al. 2012).

In this paper, we use the three diagnostic graphs to find the correct and not user-dependent interpretation for each slug test example in textbooks.

2.2 First example: Todd and Mays (2005, p. 266)

The first example comes from the textbook of Todd and Mays (2005, page 266). The usual "Hvorslev" plot presents an upward curvature (Fig. 5). The textbook qualifies this shape as a double straight-line effect. The supporters of the group-3 theory incorrectly interpret this curve as an influence of specific storage, S_s.

The derivative plot, however, does not show a nice regular curve as those in Fig. 3, but two parts: an initial part with high velocities, and then a straight-line plot. The initially high velocities are usually due to dynamic effects of slugging (these are not taken into account by the theories), initial degassing of the water column after swift injection of a water volume, or inexact data (too fast for the manual readings, or transducer readings). The straight-line long-duration portion of the derivative plot is the part used to derive the *K* value for the test.

Finally, the best fit plot for the first example (Fig. 6) plots the raw data for Z* (they are curved upward), and the corrected data after detection of the systematic error, H_0. The optimized data with determination of the H_0-value, form a straight line, except for early times, due to dynamic slugging effects, and probably initial degassing.

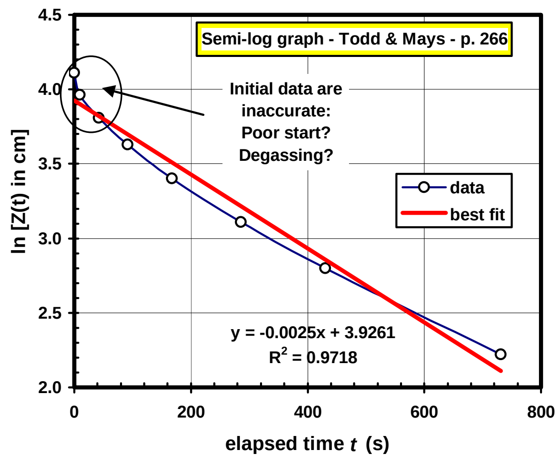

Figure 4. First plot, ln (Z) vs. t, for the 1st example. Note the curvature, and the inaccuracy of initial data.

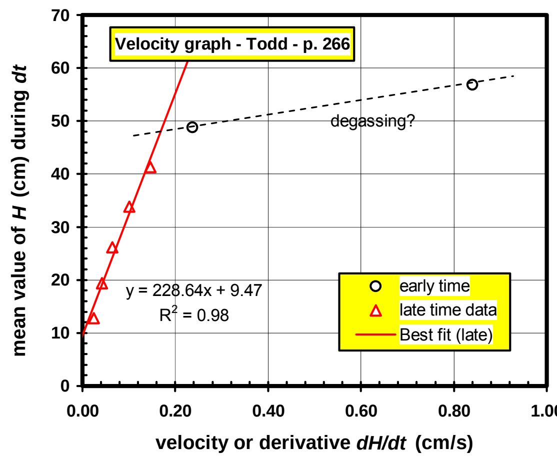

Figure 5. Third plot, derivative graph, for the 1st example. Note that all data form a straight line, with a H_0 value, after the inaccurate initial data.

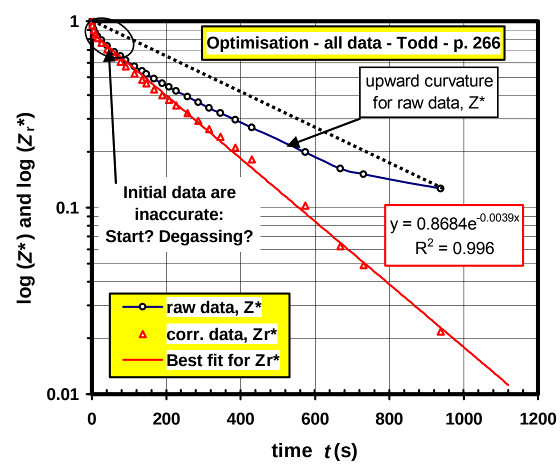

Figure 6. Best fit plot for the corrected data of the 1st example. Note that the uncorrected raw data, with an upward curve shape, appear in black.

2.3 Second example: Butler (1999, pp. 61-62)

The 2nd example is from Butler (1998). To illustrate the method of (Cooper, Bredehoeft and Papadopulos 1967), or CBP method, Butler (1998, pp. 61-62) used the data of a test performed in a MW of Lincoln County, for which the initial Z (t = 0) value was very large, exceeding 10 m.

Butler (1998) showed a doubtful fit for the CBP theory, and obtained $K = 3.69 \times 10^{-4}$ m/d. If you read rapidly, this seems to be a good number for an aquifer. However, Butler (1998) did not use the common unit of m/s. His result gives $K = 4.3 \times 10^{-9}$ m/s, which is a value for an aquitard, not an aquifer as written in Butler (1998). For this test, one could have used the group-2 theories to derive the geotechnical consolidation parameters; sadly, the group-2 theories are ignored in Butler (1998).

The common plot of ln (Z) vs. t, (Fig. 7) is curved upwards. The supporters of the group-3 theory incorrectly interpret this curve as an influence of S_s. The linear best fit seems good (R^2 = 0.97), but the curvature is obvious.

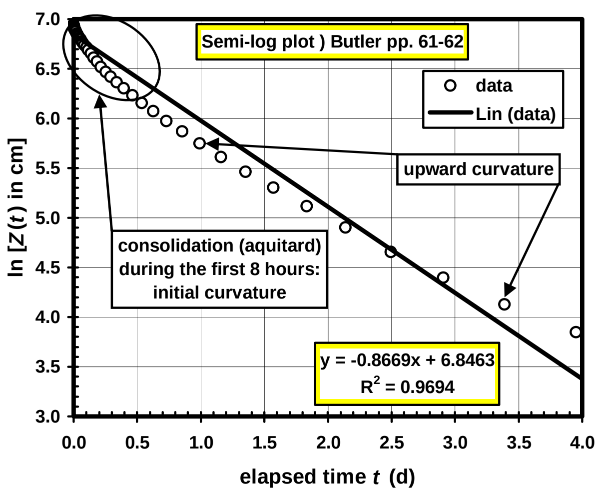

Figure 7. First plot, ln (Z) vs. t, for the 2nd example. Note the curvature, and the inaccuracy of initial data.

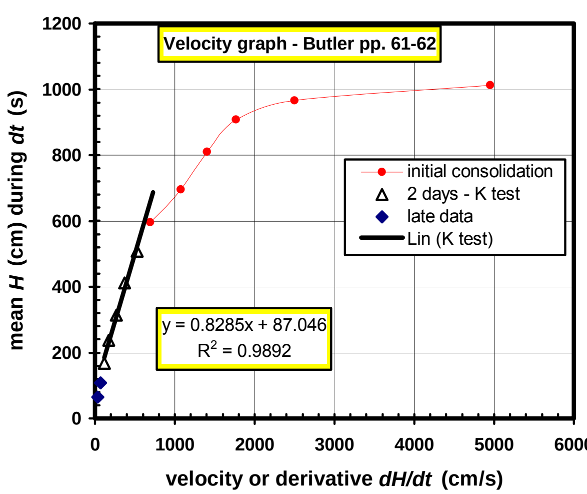

Figure 8. Third plot, derivative graph, for the 2nd example. Note that all data form a straight line, with a H_0 value, after the inaccurate initial data.

The derivative plot (Fig. 8) is not typical of an aquifer. It is rather typical of an aquitard (Chapuis 2009a). Data plot as a curve during the first hours and then as a straight line for the next 3 days with a high H_0 value. This is a usual shape for a test in an aquitard. This plot does not look like those in Fig. 3, and the K value is much too low for an aquifer and a testing time of 4 days.

The H_0 value of this second example is high. This is "normal" for an aquitard where a MW never gives the piezometric level (Chapuis 2009a; Chapuis et al. 2012), and also for a test that started with an initial Z(t = 0) exceeding 10 m = 1000 cm.

Finally, the test data, when plotted as ln (Z_r) vs. t, yield a small curve for the first 4 h, and then a central straight line (Fig. 9). This central part is the part to be used to derive the K value for this aquitard as confirmed by using numerical modeling and other independent verifications (Chapuis et al. 2012). Note that the late data, after 2 days, show a divergence from the straight central portion (red dotted line), which is also typical of late time effects for slug tests in aquitards.

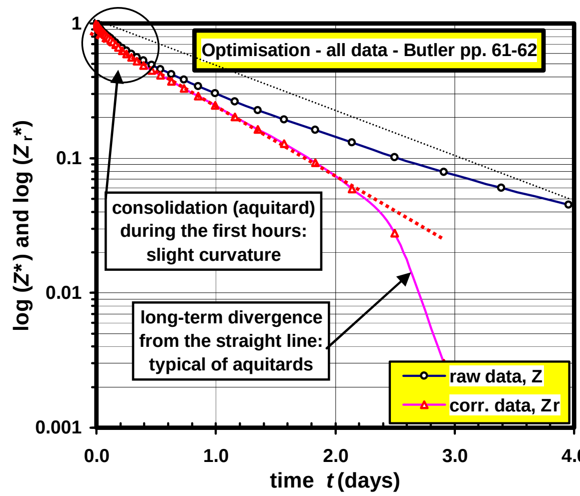

Figure 9. Best fit plot for the 2nd example. The uncorrected data, with an upward curve shape, are also plotted.

For this test, Butler (1998) found $S_s = 4.38 \times 10^{-4}$ m^{-1}, and estimated that this value was "*reasonable for the material through which the well is screened*." However, the material is mudstone and very fine sandstone, thus soft rock. Considering the correspondence between S_s and E_s, the elastic modulus of the solid matrix (Chapuis 2017), it appears that the previous S_s-value corresponds to an E_s-value typical of peat, thus at least three orders of magnitude too low for the tested soft rock.

Thus, this second example from a textbook did not use m/s for K, but m/d, which may confuse any reader. The textbook selected this test to illustrate the CBP method for slug tests in aquifers, whereas the tested material was clearly an aquitard, not an aquifer. In addition, the CBP method severely underestimated the Es values. By not providing the value of elastic modulus, and the mechanical context, which is needed to understand, this second example has simply hidden how poor the results obtained with the CBP method are.

3 GENERAL RESULTS

The following textbooks provided twenty-one examples of overdamped slug tests: Butler (1998), Dawson and Istok (1991), Fetter (2001), Hudak (2004), Schwartz and Zhang (2003), Todd and Mays (2005), and Weight (2008). Not all examples given in textbooks were solved in textbooks. All examples in textbooks ignored the derivative or velocity graph method, and therefore assumed that the observed water level before the test was a piezometric level.

First, it is possible to compare two H_0 values: one is obtained using the derivative or velocity graph, and the other is obtained using the optimization process. The comparison appears in Fig. 10. The two values are very close for most cases, except for two cases for which the field data were very inaccurate. This similarity of H_0 values for correctly performed slug tests was also shown in Chapuis (2015).

Fig. 10 also indicates that many data sets used as textbook examples were selected to have a nearly straight plot of log (Z) vs. t, as in the theory of Hvorslev (1951) or Bouwer and Rice (1976). This is obvious because many H_0 values are very small and form a pack around the axes origins in Fig. 10. By selecting such data, the example is "well chosen" to show that the group-1 theory works.

A few slug test data sets used as examples gave a clear upward curvature in a plot of log (Z) vs. t, but none gave a clear downward curvature, whereas this happens statistically in 20-30% of cases (Chapuis 2015).

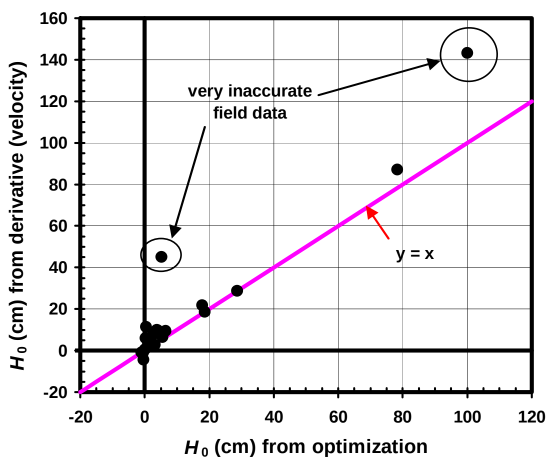

Figure 10. Comparison of the systematic piezometric error, H_0, as derived using the optimization method and as obtained with the velocity graph method.

Concerning the group-3 or CBP theory, no example found in textbooks could clearly fit one of the theoretical master curves (see Fig. 2). All test data sets cut through the master curves, or could be plotted outside to the right of them. This confirmed one of the statistical findings of Chapuis (2015) that field data sets follow the group-1 theory and not the CBP theory based upon the misunderstanding of stress-strain relationships, and the misuse of an oversimplified equation with S_s.

Second, it is possible to compare the K values given by textbooks (only for those cases that were analyzed in textbooks), versus the K value obtained using the three-diagnostic-graphs method. The results appear in Fig. 11. They indicate that most often, the K value given by textbooks was between 0.1 and 10 times the real K value for the tests, which is not a good result. The true K value was obtained after taking into account the piezometric error, initial inaccurate readings due to dynamic effects and gas bubble release, and other possible phenomena such as ignoring the initial consolidation (irreversible strains), thus confusing an aquifer with an aquitard.

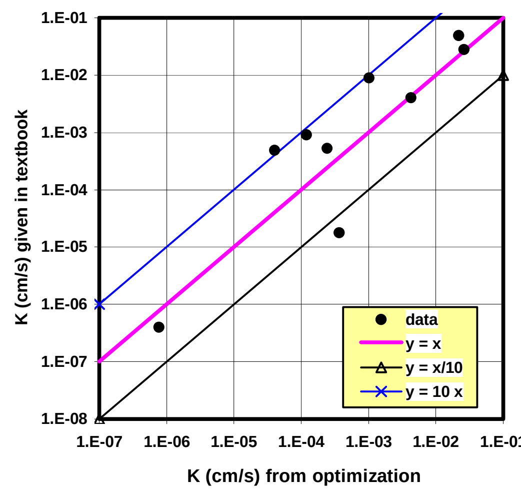

Figure 11. Comparing the K values given in textbooks (not all examples are solved) and the K values obtained with the optimization method.

4 DISCUSSION AND CONCLUSION

The examples presented in textbooks reveal a variety of the features that are found with slug tests in monitoring wells. However, they are biased, for three reasons: (1) they ignore the cases of downward curvature in a plot of log (Z) vs. t; (2) they ignore the derivative or velocity plot; and (3) they use visual matching instead of scientific methods. A few features are discussed hereafter.

One textbook mistakenly presents a test in an aquitard as an example of a test in an aquifer, while providing the K value in m/d instead of m/s. The velocity plot presents the classical features of a variable-head test in an aquitard: (1) initial curve (irrecoverable strains during consolidation), then (2) a straight-line portion, and then (3) an irregular shape for the late portion, which is influenced by hydraulic head fluctuations caused by several external factors during the long duration slug test (Chapuis et al. 2012).

In addition, for the group-1 theory the textbooks do not question the shape factor values proposed by Bouwer and Rice (1976). These values were shown to be incorrect, chiefly because the water table was confused with a constant-head recharge, which leads to a systematic underestimation of the K value by about 30 to 50% (Chapuis 2009b).

For the group-3 theory, many persons have tried to obtain field data looking like the CBP predictions of

Cooper et al. (1967). Many practitioners have spent time and put their professional liability at stake by using the CBP theory to interpret their field test data. Because the data are merely superposed (see Fig. 2), these persons may have had some feeling that the theory seemed to be correct. Alas, a simple superposition is not a proof, and the theory is physically incorrect (Chapuis 2017). These losses of time and energy are unfortunate.

Acknowledgments

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