A New Soil Structure Interaction Model for Moving Loads

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ABSTRACT

The study of time dependent response of beams on visco-elastic foundations subjected to dynamic loads has important applications in many engineering disciplines such as geotechnical, pavement and railway engineering. Most existing analyses of this class of problems consider the soil as a bed of Winkler springs. A difficulty with this approach is that the spring constants are often empirically determined and not accurate enough so that the resulting beam-soil responses are often not reliable. In this paper, a simplified continuum model for analysis of beams resting on a layered elastic soil subjected to a moving load is developed using the variational principles of mechanics. The coupled differential equations describing the beam vibration and soil displacements are obtained using the calculus of variations and solved following an iterative algorithm. The resulting differential equation for beam vibration resembles that of a beam interacting dynamically with a two-parameter foundation. However, it is shown that the two soil parameters can be mechanistically related to the soil Young's modulus and Poisson's ratio, thereby eliminating any need for empirical estimation of the soil parameters. Interestingly, the two soil parameters change with time even though the soil elastic constants remain constant – this is novel and verified with the results of equivalent finite element analysis. Examples illustrate the use of the method.

RÉSUMÉ
L'étude de la réponse en fonction du temps des faisceaux sur des fondations visco-élastiques soumises à des charges dynamiques a d'importantes applications dans de nombreuses disciplines d'ingénierie telles que la géotechnique, la chaussée et l'ingénierie ferroviaire. La plupart des analyses existantes de cette catégorie de problèmes considèrent le sol comme un lit de sources Winkler. Une difficulté avec cette approche est que les constantes de ressort sont souvent déterminées empiriquement et pas suffisamment précises pour que les réponses faisceau-sol résultantes ne soient souvent pas fiables. Dans cet article, un modèle de continuum simplifié pour l'analyse de poutres reposant sur un sol élastique stratifié soumis à une charge mobile est développé en utilisant les principes variationnels de la mécanique. Les équations différentielles couplées décrivant la vibration du faisceau et les déplacements du sol sont obtenues en utilisant le calcul des variations et résolues suivant un algorithme itératif. L'équation différentielle résultante pour la vibration du faisceau ressemble à celle d'un faisceau interagissant dynamiquement avec une base à deux paramètres. Cependant, il est montré que les deux paramètres du sol peuvent être reliés mécaniquement au module de Young du sol et au coefficient de Poisson, éliminant ainsi tout besoin d'estimation empirique des paramètres du sol. Il est intéressant de noter que les deux paramètres du sol changent avec le temps, même si les constantes élastiques du sol restent constantes - ceci est nouveau et vérifié avec les résultats d'une analyse par éléments finis sophistiquée équivalente. Des exemples illustrent l'utilisation de la méthode.

1 INTRODUCTION

The problem of beams on elastic or viscoelastic foundations is widely studied because of its wide range of applications in different fields of engineering (Avramidis and Morfidis 2005). In geotechnical engineering, the concept is widely used to analyze the behavior of flexible footings and structural elements, e.g., strip footings, grade beams, and concrete pavements, resting on the underlying soil (Barber 2010). Different models with different degrees of idealization have been proposed to simulate the behavior of the foundation (soil). The simplest and oldest idealization is to represent the soil as a bed of closely spaced linear springs, as proposed by Winkler (1867). The Winkler spring model (also often termed as the one-parameter model) is characterized by the spring constant k_s, which represents the compressive resistance of soil against applied vertical loads and can be related to the modulus of subgrade reaction of soil. The main drawback of the Winkler model is that the vertical springs are assumed to work in isolation with respect to each other because of which the resistance of soil obtained through shear stresses are neglected.

An improvement over the Winkler model was proposed by several researchers like Hetenyi (1946), Filonenko-Borodich (1945), Pasternak (1954), and Terzaghi (1955) by introducing a second parameter t_s, which essentially captures the shear interaction between adjacent Winkler springs (this model is often referred to as the two-parameter model).

Several studies on beams resting on one- and two-parameter foundations subjected to static and dynamic (vibrating and moving) loads have been performed (Timoshenko et al. 1974, Fryba et al. 1993, Teodoru & Musat 2010, Uzzal et al. 2012, Patil et al. 2013). In most of these studies, beam responses (e.g., deflection and bending moment) were investigated as functions of magnitude, frequency or velocity of applied loads and damping present in the system. The difficulty, however, in using these models is that the foundation parameters k_s and t_s cannot be reliably obtained from measurable soil properties and are often inaccurately determined from ad hoc, empirical equations (Bowles 1996). Further, for dynamic analysis, geometric damping cannot be explicitly considered using the one- or two-parameter models.

Improvements to the one- or two-parameter models have been proposed by some researchers in which the soil is idealized as an elastic continuum with simplified assumptions regarding its stress or displacement fields (Reissner 1958, Kerr 1964, Vlasov & Leont'ev 1966). Out of these simplified continuum approaches, the one by Vlasov and Leont'ev (1966) leads to the same differential equation as that of the two-parameter model and has the additional distinct advantage that the parameters k_s and t_s are rigorously related to the elastic constants of the soil without any ad hoc empiricism involved. Because of its ability to capture the continuum nature of soil, the model by Vlasov and Leont'ev (1966) has been widely used (Akoz and Ergun 2012, Omolofe 2013, Worku 2012, Limkatanyu et al. 2013). Notwithstanding, the model has a limitation that the value of a coefficient γ describing the rate of attenuation of vertical soil displacement with depth has to be assumed a priori without a very sound basis. Vallabhan and Das (1989) removed this limitation by developing an iterative approach such that the value of γ is determined as part of the solution and no a priori ad hoc assumption is necessary. Studies using the improved model have been mostly restricted to beams subjected to static loads (Vallabhan & Daloglu 1999, Liu & Ma 2013, Haldar and Basu 2016). Studies related to dynamic response of beams using the improved model were performed by Liang and Zhu (1995), Ayvaz and Ogzgan (2002), and Ogzgan (2012), but one of these studies is conceptually flawed and the others only focused on natural frequency of vibration of beams resting on single-layer soils. As far as the authors know, there is no study performed by using the improved model of Vlasov and Leont'ev (1966) that comprehensively considers both the free and forced vibration of beams resting on soils with explicit multiple layering, which is the focus of this paper.

In this paper, the improved model of Vlasov and Leont'ev (1966) is further developed for analysis of beams resting on multi-layered soil subjected to moving loads. The analysis considers both steady-state and transient vibration of beams under moving loads. A layered soil continuum under the beam is considered and the vertical soil displacement is expressed as a product of separable functions maintaining continuity and compatibility with the overlying beam. The differential equations describing the beam motion and soil displacement are obtained using Hamilton's principle and calculus of variations, and are solved following an iterative algorithm. The resulting differential equation for beam motion resembles that of a beam interacting dynamically with a two-parameter foundation with parameters k_s and t_s. These two soil parameters are mechanistically related to the soil Young's modulus and Poisson's ratio, and, interestingly, change with time even though the soil elastic constants remain constant – this is a novel feature of the model. The accuracy of the analysis is verified by comparing the results of the analysis with those of equivalent finite element analysis. Examples illustrate the application of the method for different moving loads.

2 ANALYSIS

2.1 Problem definition

A uniform Euler-Bernoulli beam of length L, width b, and depth (thickness) d, mass density ρ_b, and Young's modulus E_b is assumed to be resting on a layered elastic continuum (Figure 1). A dynamic load $P(x)$ is assumed to act on the beam (x = horizontal space coordinate) which moves from left to right (as x increases) with a velocity v. The beam is in full contact with the layered continuum at all times during the loading. The continuum (soil) beneath the beam is split into n layers with the bottom n^{th} layer resting on a rigid layer (e.g., bed rock). The i^{th} soil layer extends vertically downward to a depth H_i such that the thickness T_i of the i^{th} layer is $H_i - H_{i-1}$ ($H_0 = 0$). The total thickness of the soil deposit comprising of the n layers is $H_{\text{total}}\ (=\sum_{i=1}^{n} T_i)$. Each soil layer i is homogeneous and isotropic with mass density ρ_{si}, Young's modulus of E_{si}, and Poisson's ratio of υ_{si}.

A Cartesian x-z coordinate system is considered attached to the left end of the beam with x direction positive to the right and z direction positive vertically downward. For analysis, it is sometimes necessary to consider a domain extending beyond the two ends of the beam into the continuum in order to capture the displacements in the continuum (soil) that occurs beyond the loaded beam (Figure 1). Accordingly, the analysis domain is extended to a length βL in positive and negative x directions (where $\beta \geq 1$), respectively, from the right and left ends of the beams to produce accurate beam response and soil displacement, and eliminates boundary effects (β is typically determined by trial and error). A continuum strip of width b beneath the beam is considered as the analysis domain perpendicular to the x-z plane. This implies a plane-strain condition, similar to that assumed by Vlasov and Leont'ev (1966) and Vallabhan and Das (1989).

2.2 Soil displacements, strains and stresses

For the plane-strain problem considered, it is assumed that the soil displacement u_x in the horizontal direction caused by the vertical forces are negligible and that the vertical soil displacement u_z can be expressed as a product of separable functions (Figure 1):

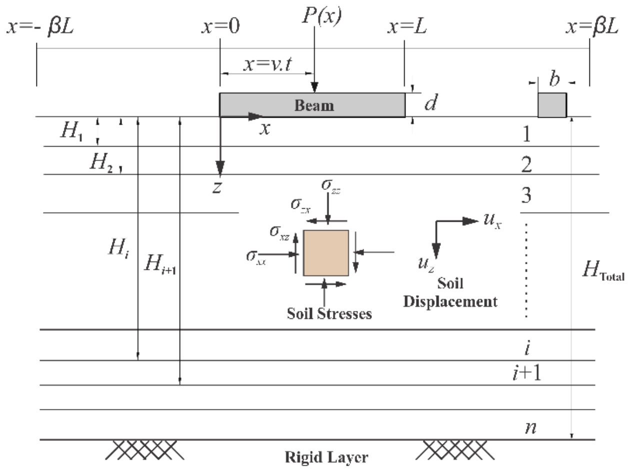


Figure 1. Dynamic Beam-Soil interaction model

$$u_z = w\left(x,t\right)\phi\left(z\right) \quad [1]$$

where $w(x, t)$ is the displacement of the top surface of the continuum (t = time), which is the same as the beam displacement for $0 \le x \le L$, and $\phi(z)$ is a dimensionless displacement function varying with depth. It is assumed in the analysis that $\phi(0) = 1$, which ensures perfect contact between the beam and the underlying continuum and that $\phi(H_{\text{total}}) = 0$, which ensures that vertical displacement in the continuum arising from applied forces decreases with increase in depth and becomes zero at the interface with the rigid layer.

The displacement field described in Eq. [1] can be used to obtain the strain tensor within the soil, and elastic constitutive relationship relates the strain tensor at any point within the soil to the stress tensor (Figure 1):

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{Bmatrix} = \frac{E_s}{\left(1+\upsilon_s\right)\left(1-2\upsilon_s\right)} \begin{bmatrix} 1-\upsilon_s & \upsilon_s & 0 \\ \upsilon_s & 1-\upsilon_s & 0 \\ 0 & 0 & 0.5-\upsilon_s \end{bmatrix} \begin{Bmatrix} 0 \\ -w\left(x,t\right)\dfrac{d\phi\left(z\right)}{dz} \\ -0.5\dfrac{\partial w\left(x,t\right)}{\partial x}\phi\left(z\right) \end{Bmatrix} \quad [2]$$

2.3 Differential Equations for Beam and Surface-Soil Displacements

The extended Hamilton principle of least actions is used to obtain the differential equations of motion of beam and the continuum under dynamic equilibrium:

$$\delta\int_{t_1}^{t_2}\left(T - U + W_{nc}\right)dt = 0 \quad [3]$$

where K and U are the kinetic and strain energies of the beam-soil system participating in the vibration, W_{nc} is the work done by the non-conservative forces acting on the system, t_1 and t_2 are any arbitrary times at which the equilibrium configuration of the beam-soil system is known, and δ is the variational operator.

Using Eqs. [2] and [3], applying the variational operator δ on the resulting equation, and considering the variation of w gives the differential equations respectively for the beam and surface soil displacements adjacent to the beams as

$$\begin{aligned} &E_b I_b \frac{\partial^4 w}{\partial x^4} - 2t_s \frac{\partial^2 w}{\partial x^2} + k_s w + c\frac{\partial w}{\partial t} \\ &\quad + \left(\eta_s + \rho_b A_b\right)\frac{\partial^2 w}{\partial t^2} = P\delta_d\left(x_0 - vt\right) \end{aligned} \quad [4]$$

$$-2t_s \frac{\partial^2 w}{\partial x^2} + k_s w + c\frac{\partial w}{\partial t} + \eta_s \frac{\partial^2 w}{\partial t^2} = 0 \quad [5]$$

with the associated boundary and continuity conditions

$$w_{\text{Right}}\Big|_{x=0} = w_{\text{Left}}\Big|_{x=0} \quad [6a]$$

$$w_{\text{Left}}\Big|_{x=L} = w_{\text{Right}}\Big|_{x=L} \quad [6b]$$

$$\left(-2t_s\frac{\partial w}{\partial x}\right)_{\text{Left}}\Bigg|_{x=0} = \left(E_b I_b \frac{\partial^3 w}{\partial x^3} - 2t_s \frac{\partial w}{\partial x}\right)_{\text{Right}}\Bigg|_{x=0} \quad [6c]$$

$$\left(E_b I_b \frac{\partial^3 w}{\partial x^3} - 2t_s \frac{\partial w}{\partial x}\right)_{\text{Left}}\Bigg|_{x=L} = \left(-2t_s\frac{\partial w}{\partial x}\right)_{\text{Right}}\Bigg|_{x=L} \quad [6d]$$

$$E_b I_b \frac{\partial^2 w}{\partial x^2}\Bigg|_{x=0\ \&\ x=L} = 0 \quad [6e]$$

and initial conditions $w = 0$ and $\partial w/\partial t = 0$ at $t = 0$ (δ_d in Eq. [4] is the Dirac's delta function).

The parameters in the above equations are given by

$$\eta_s = \sum_{i=1}^{n}\rho_{si} b\int_{H_{i-1}}^{H_i}\phi_i^2 dz \quad [7a]$$

$$k_s = \sum_{i=1}^{n} b\int_{H_{i-1}}^{H_i}\bar{E}_{si}\left(\frac{d\phi_i}{dz}\right)^2 dz \quad [7b]$$

$$t_s = \sum_{i=1}^{n}\frac{b}{2}\int_{H_{i-1}}^{H_i} G_{si}\phi_i^2 dz \quad [7c]$$

where subscript i corresponds to the i^{th} layer, and $\bar{E}_{si}$ and G_{si} are given by

$$\bar{E}_{si} = \frac{E_{si}\left(1-\upsilon_{si}\right)}{\left(1+\upsilon_{si}\right)\left(1-2\upsilon_{si}\right)} \quad [8a]$$

$$G_{si}=\frac{E_{si}}{2(1+\upsilon_{si})} \quad [8b]$$

Considering the variation of the function ϕ in Eq. [3], the Euler-Lagrange equation (the differential equation) of $\phi(z)$ within the i^{th} layer can be obtained as:

$$\frac{d^2\phi_i}{dz^2}+\left(\frac{\bar{\gamma}_i}{H_i}\right)^2\phi_i=0 \quad [9]$$

with

$$\left(\frac{\bar{\gamma}_i}{H_i}\right)^2=\left(\frac{\zeta_{si}-n_{si}}{m_{si}}\right) \quad [10]$$

where

$$m_{si}=b\int_{-L}^{2L}\bar{E}_{si}w^2dx \quad [11a]$$

$$n_{si}=b\int_{-L}^{2L}G_{si}\left(\frac{\partial w}{\partial x}\right)^2dx \quad [11b]$$

$$\zeta_{si}=b\rho_{si}\int_{-L}^{2L}\left(\frac{\partial w}{\partial t}\right)^2dx \quad [11c]$$

which make the dimensionless parameter

$$\left(\frac{\bar{\gamma}_i}{H_i}\right)^2=\frac{\rho_{si}\int_0^{3L}\left(\frac{\partial w}{\partial t}\right)^2dx-G_{si}\int_0^{3L}\left(\frac{\partial w}{\partial x}\right)^2dx}{\bar{E}_{si}\int_0^{3L}w^2dx} \quad [12]$$

For the boundary conditions corresponding to differential equation [9], it is assumed that $\phi(0)$ = 1, which ensures perfect contact between the beam and the underlying soil. It is also assumed that $\phi(H_{Total})$ = 0, which ensures that the displacement in soil decrease with depth and become zero at the boundary with the rigid bedrock. The continuity across the soil layers is also ensured with the continuity condition that $\phi_i=\phi_{i+1}$ at $z=H_i$.

3 SOLUTION OF THE DIFFERENTIAL EQUATIONS

Solutions of the differential equations [4] & [5] of w are obtained by using the finite element method. Two-noded rod (bar) elements with linear Lagrangian shape functions $\{N_L\}_{2\times1}$ are used to discretize the domains $-\beta L \le x \le 0$ and $L \le x \le \beta L$ (i.e., the domains in x direction with no beam), and two-noded beam elements with cubic Hermitian shape functions $\{N_H\}_{4\times1}$ are used to discretize the domain $0 \le x \le L$ (i.e., domain in x direction in which the beam is present) to obtain a set of algebraic equations of the form $\sum_e[m]^e\{\ddot{w}\}+\sum_e[c]^e\{\dot{w}\}+\sum_e[k]^e\{w\}=\sum_e\{f\}^e$ where $[m]^e$, $[c]^e$ and $[k]^e$ are the elemental mass, damping and stiffness matrices, respectively, $\{f\}^e$ is the elemental force vector, $\{w\}$ is the global degrees of freedom vector (consisting of the unknown displacements w and slope $\partial w/\partial x$ for the portion of the beam) at the nodes, $\{\ddot{w}\}$ is the global acceleration vector, $\{\dot{w}\}$ is the global velocity vector, and $\sum_e$ represents assembly. The Wilson-θ method (Bathe 1996) is used to perform the dynamic time integration scheme required for obtaining solution.

Solution of the differential Eq. [9] of ϕ is obtained analytically, and is given by

$$\phi_i(z)=A^{(i)}e^{\sqrt{\frac{\zeta_{si}-n_{si}}{m_{si}}}z}+B^{(i)}e^{-\sqrt{\frac{\zeta_{si}-n_{si}}{m_{si}}}z} \quad [13]$$

where the integration constants A and B are obtained from the corresponding boundary conditions. For a single layer problem (i = 1), ϕ is given by

$$\phi_1(z)=\frac{\sinh\left[\bar{\gamma}_1\left(1-\frac{z}{H_1}\right)\right]}{\sinh(\bar{\gamma}_1)} \quad [14]$$

4 ITERATIVE SOLUTION ALGORITHM

The soil parameters k_s, t_s and η_s must be known in order to solve the differential equations for w (Eqs. [4] & [5]) and these parameters depend on ϕ. At the same time, the parameters m_{si}, n_{si} and ξ_i must be known in order to obtain ϕ (from Eqs. [13] or [14]) and these parameters depend on w. Therefore, the equations of w *and* ϕ are coupled and are solved simultaneously following an iterative scheme. The function $\phi(z)$ is first determined iteratively for static loading condition (ϕ_{static}) with the same magnitude of load placed at the mid-span of the beam.

In order to obtain $\phi_{static}(z)$, an initial guess for the spatial distribution of ϕ is made by assuming a linear distribution of ϕ. With this assumed distribution, k_s and t_s are calculated (note that η_s is zero for static loading as inertia forces are negligible under static conditions) using Eqs. [7a], and [7b], and these parameters are then used to calculate the displacement w and its slope $\partial w/\partial x$. Using the calculated w and $\partial w/\partial x$, parameters m_{si} and n_{si} are calculated from Eqs. [11a], and [11b] (note that ζ_{si} is zero under static loading). The parameters m_{si} and n_{si} are then used to obtain a new $\phi_{static}(z)$. These newly calculated $\phi_{static}(z)$ is compared with the assumed $\phi_{static}(z)$ and, if the difference is greater than a prescribed tolerance (10^{-5}), the calculations are repeated with the calculated ϕ_{static} as the new guess. The iterative calculations are continued until the assumed and calculated ϕ_{static} values fall within a tolerable limit.

For dynamic beam response, iterations similar to that described above are performed to obtain solutions. For the first time integration step, an initial position x_i of the applied moving load, geometry, material properties, and $\phi_{static}(z)$

are given as inputs and the soil parameters k_s, t_s and η_s are calculated. Using these parameters, the transverse displacement w, slope $\partial w/\partial x$, velocity $\partial w/\partial t$, and acceleration $\partial^2 w/\partial t^2$ are calculated. These values are used to calculate m_{si}, n_{si} and ζ_{si}, which are used to calculate a new distribution $\phi_{\text{dynamic}}(z)$ of the function ϕ. The newly obtained ϕ_{dynamic} is checked against the previous ϕ (which is ϕ_{static} for the first iteration of the first time increment), and iterations are continued until the difference between two consecutive distributions of $\phi_{\text{dynamic}}(z)$ are within tolerable limits (10^{-5}). At this point, the calculated w, $\partial w/\partial x$, $\partial w/\partial t$, and $\partial^2 w/\partial t^2$ are the final values for the given time step, and the next increment of time is then applied and the whole iterative process is repeated with each subsequent time steps until the final time increment is complete to reach the final time t_{final}.

5 RESULTS

Four numerical examples are considered to assess the accuracy of the proposed model and to demonstrate the use of the model. The accuracy of the proposed model is evaluated by comparing the results obtained from the analysis presented here with the results of equivalent finite element (FE) analysis in which the same constitutive relation and boundary conditions are used. The FE analysis is performed using PLAXIS 2D software.

Example 1: Time dependent response of a finite beam subjected to a moving load at constant velocity

A 6 m long strip footing of width 1 m and thickness 1m, and with a Young's modulus of 150 MPa is assumed to be resting on 5 m thick continuum soil layer. The Young's modulus and the Poisson's ratio of the soil are 1.8 MPa and 0.3, respectively. The beam is considered to be free to displace and rotate at both ends. A single point load of 35 KN is assumed to move with a constant speed of 10 m/sec from the left to the right side of the beam. The resulted beam displacement along the span of the beam is shown in Figure 2 for two load positions: the load at the mid-span of the beam, and the load just about to exit the beam. It is clear that the results from the proposed model are in good agreement with those from the FE analysis.

Example 2: Steady-state response of an infinite beam subjected to a moving load at constant velocity

In order to simulate the steady state response, a very long (theoretically infinite) strip footing of 1 m width and 1 m thickness, and with a Young's modulus of 20000 MPa is assumed to be resting on a 4 m thick continuum soil layer. The Young's modulus and the Poisson's ratio of the soil are chosen as 30 MPa and 0.25, respectively. The beam is considered to be free to displace and rotate at both ends. A single point load of 10 KN is considered to move with constant speed of 106 m/sec from the left to the right of the beam. The resulting beam displacement at the steady state is shown in Figure 3.

Example 3: Time dependent response of a finite beam subjected to a moving load at variable speed

A 20 m long strip footing of 1 m width and 0.5 m thickness, and with a Young's modulus of 125 MPa is assumed to be resting on 4 m thick soil. The Young's modulus and Poisson's ratio of the soil is assumed to be 22 MPa and 0.2, respectively. The beam is considered to be hinged at both ends. In order to simulate the effect of variable speeds, four cases are considered. In Case 1, a 5 KN point load moves a constant speed of 5 m/sec and traverses the entire beam from left to right. In Case 2, a 5 KN point load enters the beam from the left at an initial speed of 5 m/sec and decelerates uniformly at 0.625 m/sec^2 such that the load stops at the right end of the beam. In Case 3, a 5 KN point load starts to move from the left end of the beam with zero initial velocity, and accelerates uniformly at 0.625 m/sec^2 such that it attains a speed of 5 m/sec at the right end of the beam. In Case 4, a 5 KN point load enters the beam at the left end with an initial velocity of 5 m/sec and decelerates uniformly at 1.25 m/sec^2 so that the load stops at the mid-span of the beam.

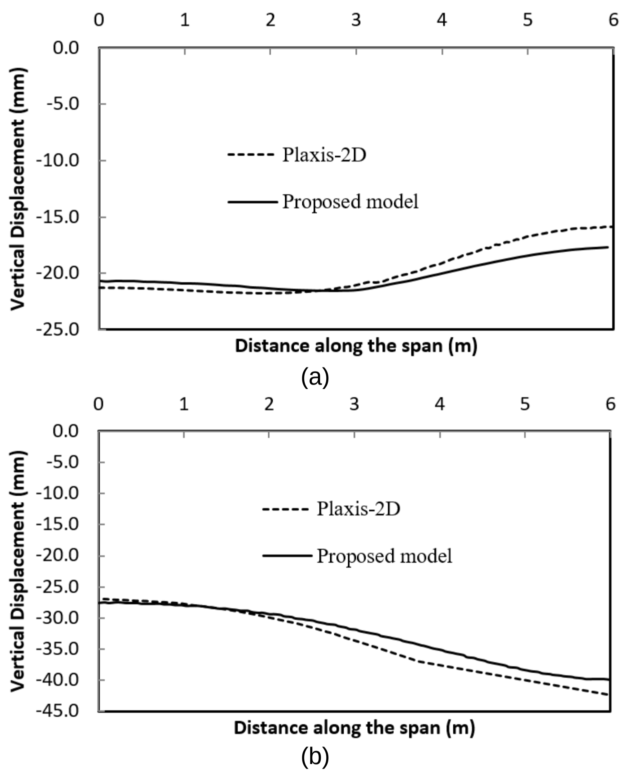


Figure 2. Time dependent response of a 6 m long beam when (a) the load is at the mid-span of the beam, and (b) the load is about to exit the span

Figure 4 shows the dynamic amplification factor ψ_D versus the normalized load position for the four cases mentioned above. The dynamic amplification factor ψ_D is calculated as the ratio between the mid-span dynamic deflections to the maximum mid-span static deflection caused by a 5 kN load acting at the mid span. The normalized load position is calculated as the ratio between the load position for which the dynamic mid-span deflection is calculated ($v.t$) to the span length (L).

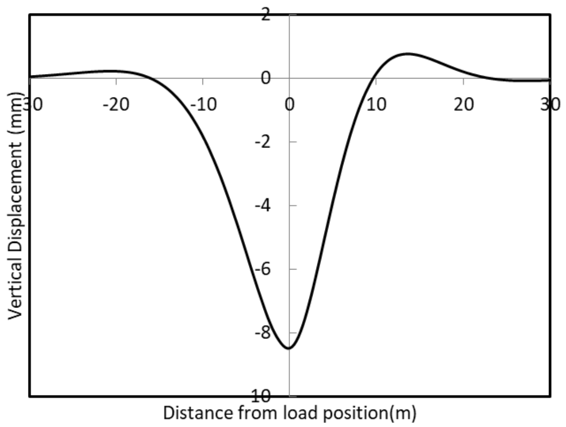


Figure 3. Steady state response of infinite beam subjected to moving load.

5.1 Effect of the dynamic action on the soil parameters and soil displacement function

The same geometry and material properties as that of Example 2 are used in the study presented in this section. Figure 5(a) shows the variation of the soil parameters k_s and t_s with time as obtained using the model presented in this paper for the infinite beam. Further, comparisons are made with the results obtained by using the traditional Vlasov foundation model (Vlasov and Leont'ev 1966). In the Vlasov model, the two model parameters k_s and t_s are obtained for a homogeneous foundation as

$$k_s = \frac{E_s b\gamma(1-\upsilon_s)}{2(1+\upsilon_s)(1-2\upsilon_s)} \qquad [15]$$

$$t_s = \frac{E_s b}{8\gamma(1+\upsilon_s)} \qquad [16]$$

where γ is a parameter ranging between 1 and 2 (with γ =1.5 for the presented comparison).

Interestingly, the two soil parameters in the proposed model change with time even though the soil elastic constants remain constant. Initially, the new model produce a greater value of k_s compared with the k_s of the Vlasov model, but it stabilizes at a lower value when the steady state response is reached. An opposite trend is observed for the parameter t_s.

The soil displacement function ϕ changes with time, as shown in Figure 5(b) for the problem of Example 2. The function ϕ decreases with depth high at a greater rate for up to about 0.1 sec after which the change stabilizes and eventually attain a equilibrium state when the beam vibrates at the steady state. change in mode accompanied with a low rate fluctuation; progressing to some kind of steady state by the end of the time-dependent response duration.

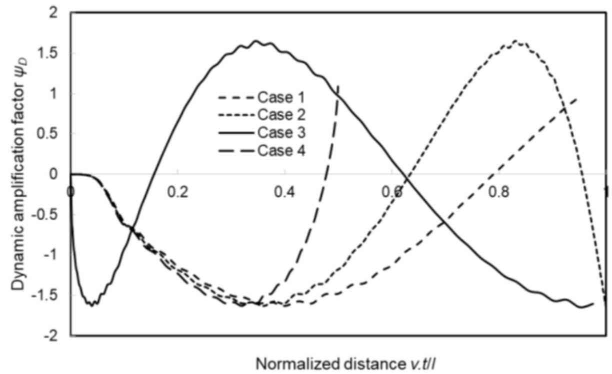
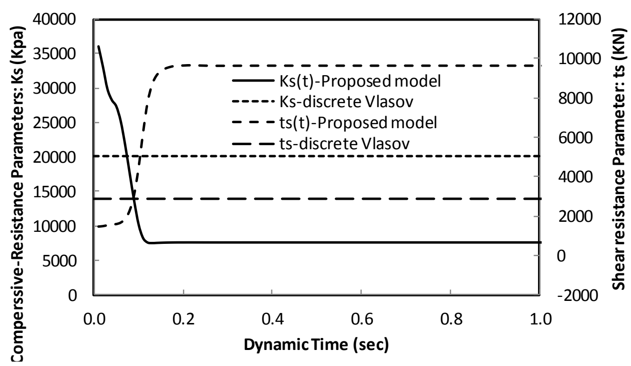
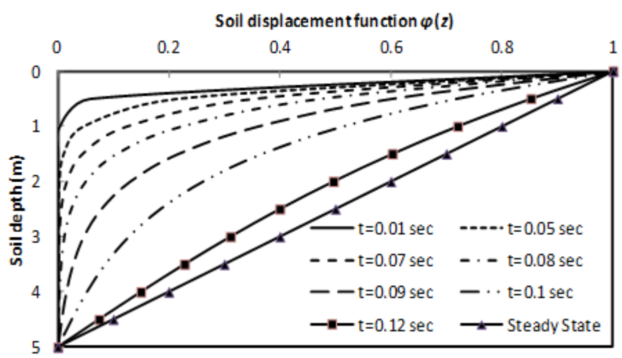


Figure 4. Time variation of mid-span deflection of a finite simply supported beam traversed by a load at variable speeds.



(a) Soil resistance parameters

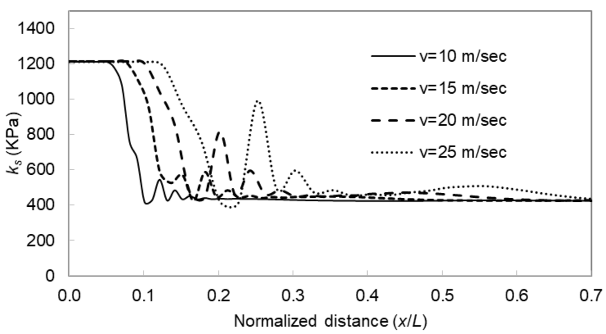


(b) Soil displacement function

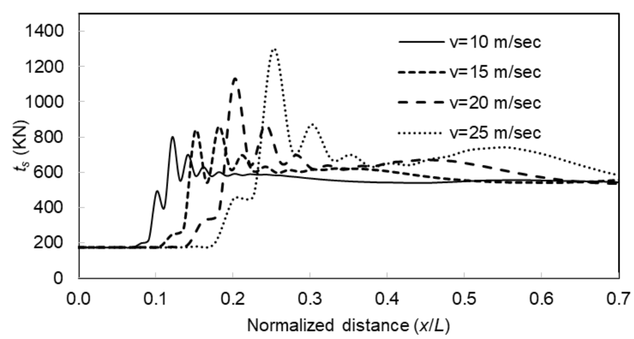
Figure 5. Effect of dynamic action on infinite beam-soil system

5.2 Effect of the velocity of the moving load on the soil parameters

A 10 m length strip footing of 1 m width, 1 m thick, and elastic modulus of 15000 MPa is assumed to be rested on single soil layer. The Young's modulus and the Poisson's ratio of the soil are chosen as 18 MPa and 0.3, respectively. Two end support conditions are chosen: simply supported, and free-free. A single point load of 40 KN is considered to move with constant speed from the left to the right side of the beam. The time step is selected to be 0.01 sec, and the initial positon was chosen as 0.1 m from the left end of the beam. The velocity of the moving load *v* is considered to vary from 10 to 25 m/sec. The time history of the soil parameters k_s and t_s resulted from the proposed model are plotted in Figures 6, and 7 for the both support conditions. In the case of free-free end support conditions, it's noted that the change of the moving load velocity does not affect the maximum values of the soil parameters. However, phase differences are observed. In contrast, no behavior trend can be identified in case of the simply supported conditions. As a general observation, changing the end support conditions significantly affects both the values and the profiles of the time histories of the soil parameters k_s and t_s.



(a) Time history of k_s



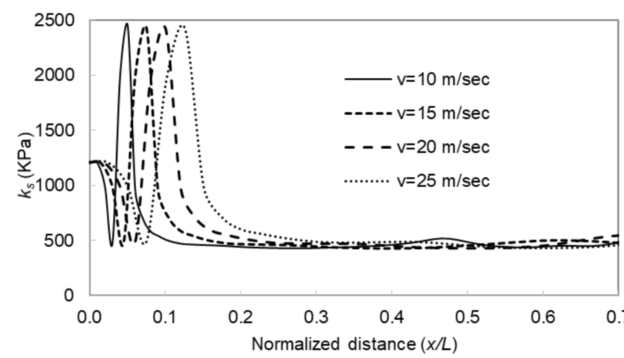
(b) Time history of t_s

Figure 6. Effect of the moving load velocity on the soil parameters for simply supported beam

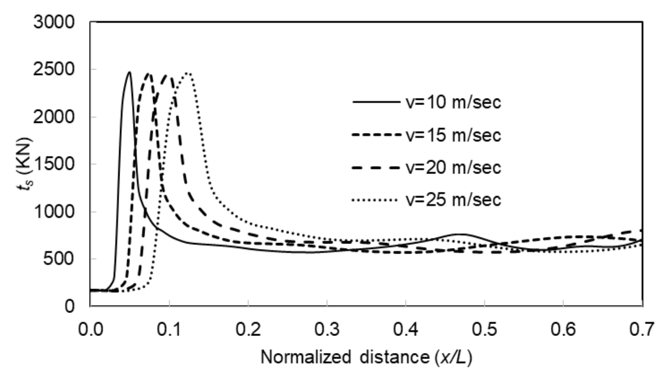
6 CONCLUSIONS

A new continuum-based soil structure interaction model is presented for beams on elastic foundations subjected to moving loads. The governing differential equations for the beam and the soil (continuum) are obtained using Hamilton's principle and calculus of variations. The soil displacement field is assumed to be a product of separable functions. The obtained differential equations of beam and soil are coupled and solutions are obtained iteratively at every time step.

It is shown through comparison with equivalent finite element analysis that the new model presented in this paper produces accurate beam response. It is further observed that the mechanical behavior of the soil can be described by two soil parameters which are similar to the two-parameter foundation model. However, for the present analysis, these two parameters can be mechanistically related to the soil Young's modulus and Poisson's ratio, thereby eliminating any need for empirical estimation of the soil parameters. Further, the two soil parameters change with time even though the soil elastic constants remain constant – this is novel and presented for the first time in the literature. Examples of infinite and finite beams subjected to loads with constant and variable speeds are presented to illustrate the use of the method.



(a) Time history of ks



(b) Time history of t_s

Figure 7. Effect of the moving load velocity on the soil parameters for freely supported beam

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