Seismic deformation analysis for a caisson wharf on liquefiable soils

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ABSTRACT
A new marine terminal including an offshore embankment with a caisson wharf structure is proposed to be constructed on a thick deposit of silt/sand soils in the Vancouver area. The caissons will be installed on a rockfill mattress placed on a dredged seafloor. A 2-D dynamic analysis with FLAC, using the PM4Sand model for liquefiable soils and UBCHYST model for non-liquefiable soils, was performed. The purpose of the analyses was to estimate the movement of the caisson and retained soils induced by the seismic motions and liquefaction of soils beneath the caisson and rockfill mattress.

The results of the analysis indicated a limited lateral translation of the caisson and rockfill mattress, with much of the shearing occurring in the liquefied and/or softened Silt/Sand layers beneath the rockfill mattress. Some downward movement of the rockfill beneath the toe of the caisson is also indicated, with a resulting rotation of the caisson. The translation and rotation of the caisson with respect to the embankment fill tends to the formation of a shear wedge behind the caisson.

Several parameters were modified during trial analyses including the model geometry, constitutive model, grid zone spacing, rockfill parameters, boundary conditions, input motions, drainage conditions, and extent of ground improvement. A qualitative review of the sensitivity of the FLAC analysis to some of these parameters is also presented in this paper.

RÉSUMÉ
Un nouveau terminal maritime au large de la côte, comprenant un remblai avec des quais à caissons, sera construit sur un épais dépôt de sols limoneux et sableux dans la région de Vancouver. Les caissons seront installés sur un remblai d'enrochement placé à même le fond marin préalablement dragué. Une analyse dynamique 2D avec FLAC en utilisant le modèle PM4Sand pour les sols liquéfiables et le modèle UBCHYST pour les sols non liquéfiables a été réalisée. Le but des analyses était d'estimer le déplacement des caissons et des sols induit par les mouvements sismiques et la liquéfaction des sols présents sous les caissons et l'enrochement.

Les résultats de l'analyse ont indiqué une translation latérale limitée des caissons et de l'enrochement, la plus grande partie du cisaillement se produisant dans les couches de limon / sable liquéfiées et / ou ramollies sous l'enrochement. L'analyse a également rapporté un mouvement descendant de l'enrochement sous le caisson résultant d'une rotation de ce dernier. La translation et la rotation du caisson par rapport à l'enrochement produiraient ainsi une zone de cisaillement derrière le caisson.

Plusieurs paramètres ont été modifiés au cours des analyses, notamment la géométrie et la constitution du modèle, l'espacement des zones quadrillées, les paramètres d'enrochement, les conditions aux limites, les conditions de drainage et l'amélioration des sols en place. Une revue qualitative de la sensibilité de l'analyse FLAC à certains de ces paramètres est également présentée dans cet article.

1 INTRODUCTION

The proposed project entails development of a marine terminal (1,500 m by 700 m) to expand the existing terminal complex. Additionally, the existing causeway will be widened to accommodate increased truck and rail traffic for the new terminal. Figure 1 shows the proposed site layout for the new terminal.

The geotechnical investigation program including several offshore mud-rotary and sonic boreholes, and CPTs and SCPTs, was carried out.

In summary, the results of the investigations indicated general soil conditions comprising deltaic sands and silts from the Holocene epoch to depths more than 80 m below the mudline (up to EL. -145 m at the site). These deltaic sands and silts comprise primarily of cohesionless soils with occasional cohesive layers of limited thickness and were underlain by cohesive soils from the Holocene epoch (primarily low plasticity soil), which, in turn, were underlain by till-like soil from the Pleistocene epoch extending to the maximum investigated depth of 175 m below mudline.

As part of the reference design for the new terminal a non-linear numerical analysis was carried out using the computer program, FLAC (Itasca, 2011, version 7) to make a preliminary estimate of the seismic induced deformation at the caisson wharf location (southwest edge of the new terminal) for a limited number of selected design earthquake records.

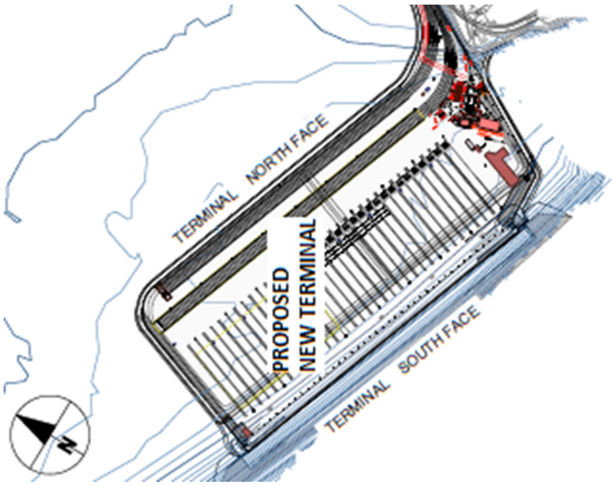


Figure 1. New terminal site layout.

2 INPUT PARAMETERS

Geotechnical strata and soil parameters including cyclic resistance ratio (CRR), earthquake input motions, and preliminary design drawings of the proposed caisson wharf structure and ground improvement were used as inputs for the FLAC analysis and adjusted by the reviewer team (i.e. owner's engineer) during the review process of the analysis results. The input parameters are briefly reviewed in this paper for reference.

2.1 Proposed Structure

A simplified section of the caisson wharf structure (preliminary design - southwest edge of the new terminal) used for the deformation analysis is shown on Figure 2.

The geometry of the model was set to include 250 m on either side of the caisson, the base of the model was set to Chart EL. -90.0 m, and the ground surface offshore of the caisson was assumed horizontal.

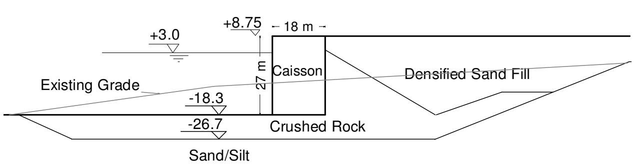
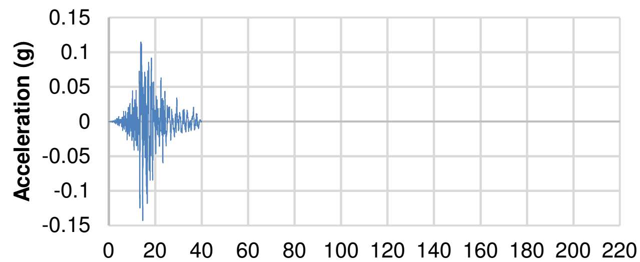


Figure 2. Caisson wharf section used for FLAC analysis

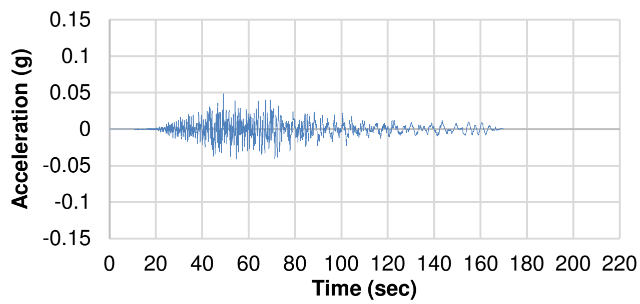
2.2 Earthquake Input Motions

The earthquake input motions provided were derived using the Uniform Hazard Spectrum (UHS) from the probabilistic 5th Generation Seismic Hazard model developed by the Geological Survey of Canada (GSC) and adapted by the National Building Code of Canada (2015 NBCC) as well as 2014 Canadian Highway Bridge Code (CSA-S6-14). The motions were developed for outcropping Site Class C soil conditions using the spectral matching technique. The motions were modified such that the response spectrum of each motion would match the corresponding Scenario Response Spectrum (i.e.: Crustal, In-slab and Interface).

A select number of input motions developed for return periods of 975 years and 475 years were used for the numerical analysis. 'Within' ground motions at Chart Elevation -90.0 m were developed using the computer program, SHAKE and used as input for FLAC. Figure 3 presents acceleration time histories for two of the input motions used for the FLAC analysis. A summary of the results of the analysis of these motions are presented in this paper.



a) 975-year Hector Mine (crustal)



b) 475-year Tokach-oki (interface subduction)

Figure 3. 'Within' motions at EL. -90.0 m for FLAC analysis.

2.3 Material Parameters

Soil parameters used in the analysis are shown in Table 1.

Table 1. Material Parameters

Soil Unit	γ (kN/m³)	Φ' (°)	C' (kPa)	ν' / ν_u	K (cm/sec)
Crushed Rock	21	45	15	0.3/0.4	5×10^{-1}
Densified Sand Fill	19	40	15	0.3/0.4	5×10^{-2}
Sand/Silt [2]	18.5 to 19.5 [1]	29	-	0.3 / -	5×10^{-3} to 1×10^{-4} [1]

[1]value varied with depth.
[2]same values used for Un-densified Sand Fill.

The design shear wave velocity profile for the Sand/Silt unit was derived from the site-specific shear wave velocity measurement data (as shown on Figure 4 for one of the test holes), and approximated with following equation:

$$V_s = 175\,(\sigma'_v/P_a)^{0.25} \quad [1]$$

In the above equation σ'_v is the vertical effective stress and P_a is the atmospheric pressure. A shear wave velocity of 350 m/s was assumed for the Rockfill and Densified Sand Fill units.

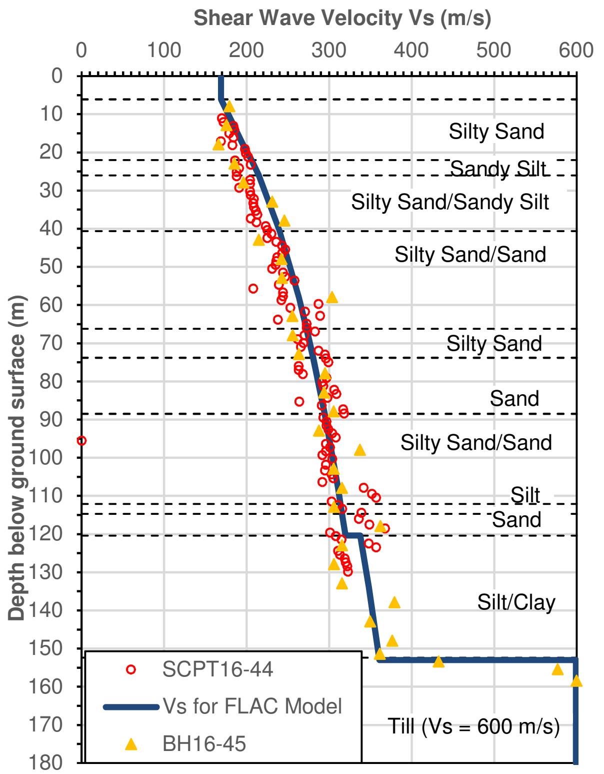


Figure 4. V_s measurement for selected soundings and V_s profile adopted for FLAC model.

The CRR profiles were for Existing, Fill and Dredged conditions. The Existing condition represents soils that will remain at or near current stress conditions, and the Fill conditions assume soils beneath the placement of fill to EL. +8.75 m. The Dredged condition represents soils that have developed an over-consolidation ratio (OCR) of about 1.7 due to removal of overburden soils.

Average and 40th-percentile trends of the CRR versus σ'_{vo} profiles were developed for the 3 site conditions. Simplified trend lines were developed to approximate the CRR profiles. The 40th- percentile profiles were considered because the average profile may overestimate the equivalent CRR in the analysis (see research of Boulanger and Montgomery (2016) on the effect of soil variability on liquefaction-induced deformations estimated by 2D non-linear deformation analyses). Figure 5 presents the 40th-percentile profile, average profile, and the simplified equivalent CRR trend for Existing condition.

The post-liquefaction undrained strengths of the Sand/Silt soil, S_r, were assumed to be represented by the mean estimate of undrained strength for 15% shear strain as shown on Figure 6. This strength was used only in the post-earthquake stability/deformation evaluation and was assigned to elements that liquefied during the earthquake shaking. For the liquefied soils a post-earthquake liquified shear modulus, G_{liq}, equal to the maximum of $S_r/0.15$ or P_a was used.

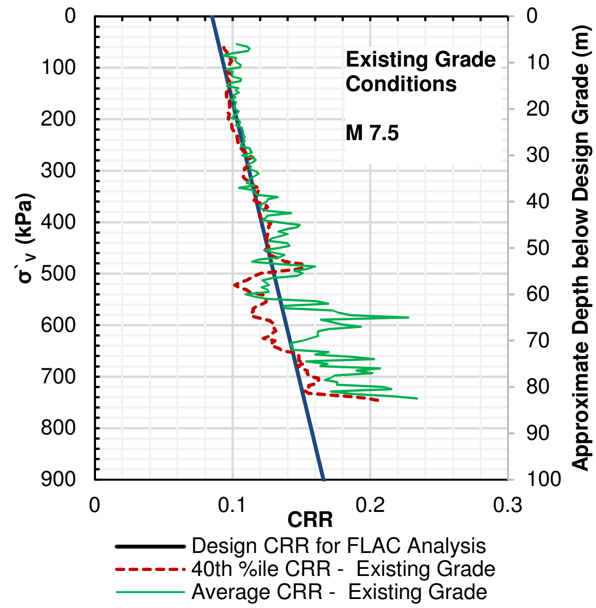


Figure 5. CRR profile for Existing conditions.

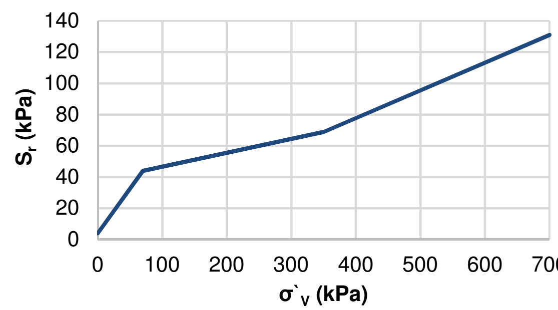


Figure 6. Post-liquefaction undrained shear strength.

A total density of 21 kN/m^3 was used for the filled portion of the caisson. An equivalent shear modulus of 500 MPa was estimated for the caisson. Friction angles of 26° and 30° were assigned for the vertical and horizontal interfaces between the caisson and adjacent granular fill materials.

3 CONSTITUTIVE MOLDELS AND CALIBRATION

The linear elastic model was used to model the caisson structure, and the Mohr-Coulomb model was used to model soil strata for initial static and post-earthquake analyses. UBCHYST (Naesgaard et al. 2015) model was used to model non-liquefiable soils and PM4Sand (Boulanger and Ziotopolou, 2015) was used for liquefiable soils (i.e. Sand/Silt unit) during dynamic analysis.

The UBCHYST model was calibrated at many stress states to simulate the modulus reduction and damping lower-bound curves of Seed and Idris (1970).

The PM4Sand was calibrated to capture the soil behavior consistent with the CRR profiles provided for various conditions. The calibration was based on simulating the behavior of a single element in a series of cyclic DSS tests with stress-controlled loading. The calibration was repeated at many stress levels for different locations to develop a smooth variation of the calibration parameters with stress.

The three primary input parameters for PM4Sand are relative density (D_R), shear modulus coefficient (G_o), and contraction rate parameter (h_{po}). D_R was estimated from the design CRR profiles in a manner consistent with the CRR correlation recommended by Idriss and Boulanger (2008) for $CRR_{M=7.5,\ \sigma`=1atm}$. G_o was selected to match the site-specific V_s estimates. h_{po} was varied to achieve the target CRR for cyclic loading with conditions of no static shear stress bias (i.e., 3.75% shear strain in 15 cycles of CSR = $CRR_{M=7.5}$). In addition to the primary parameters, three secondary parameters (i.e. n_b, C_{kaf} and φ_{cv}') were used for calibration. The estimated profile for D_R is shown on Figures 7.

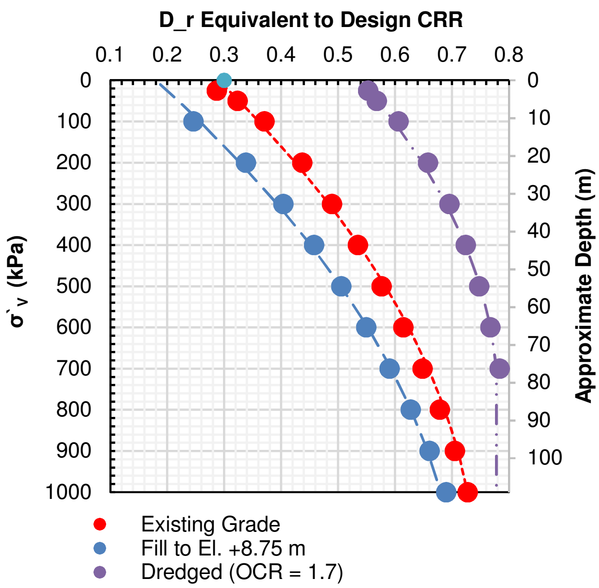


Figure 7. Selected profiles of D_r for calibrated model.

Figure 8 shows shear stress – shear strain behavior of the calibrated PM4Sand for a single element at a select stress state with no static bias.

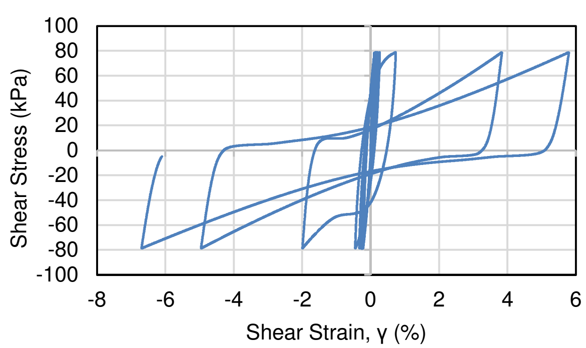


Figure 8. Shear stress – shear strain behavior from a single element FLAC simulation for a select stress state.

4 MODELING AND ANALYSIS SEQUENCE

The grid mesh was developed with zones having a horizontal dimension of approximately 3.0 m and a vertical dimension of approximately 1.5 m as shown on Figure 9 for the caisson area. The vertical spacing was checked for its ability to adequately pass 10 Hz motions.

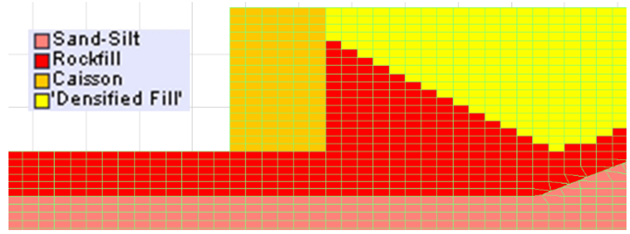


Figure 9. FLAC mesh near caisson showing relative grid spacing (note: background grid at units of 10 m).

Static analysis was carried out to establish initial state for dynamic analysis. The analysis stages started with the existing conditions followed by construction stages including dredging, placement of the rockfill mattress, placement of the empty caisson, and placement of backfill materials inside and behind the caisson. Figure 10 shows static shear stress ratio (α) at the end of the static analysis.

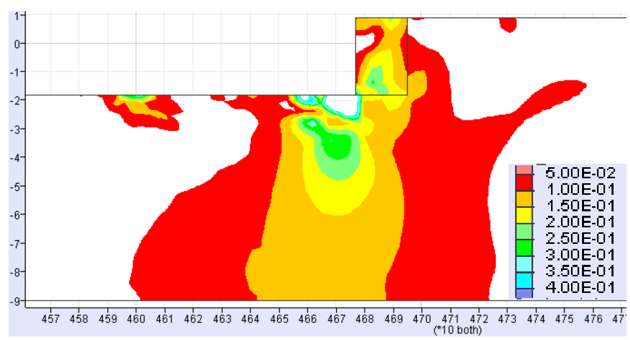


Figure 10. Initial shear stress and static shear stress ratio (α) at end of static analysis.

For dynamic analysis, the UBCHYST and PM4Sand models were applied to the relevant zones of the model. A stiffness/mass-proportional Rayleigh damping of 1% at a center frequency of 1.4 Hz was used for all materials to provide a more reasonable dissipation of energy during small-strain vibrations.

To simulate an infinite medium extending beyond the lateral model boundaries, free-field boundary conditions were specified at the vertical boundaries. In addition to the free-field boundaries, zones located within 40 m from each free-field boundary were constrained to behave as simple shear beams with the displacement response controlled by the response of the elements just inside and adjacent to the shear beams. Given the proximity of the boundaries to the caisson, the shear-beam approach was used to provide smoother response estimates near the boundaries.

A rigid base boundary was adopted since a compliant base approach (Mejia and Dawson, 2006) was not suitable as the prescribed elevation for the base of the model was within the potentially liquefiable Sand/Silt soil. The input 'within' motion was directly applied to the rigid base of the model as a horizontal acceleration history.

The groundwater transient flow and large-strain options were used for dynamic analysis.

As the PM4Sand model cannot be directly calibrated to simulate an empirically-derived residual strength, a post-earthquake deformation analysis was carried out using post-liquefaction parameters for liquefied soils after completion of the dynamic analysis.

5 ANALYSIS RESULTS

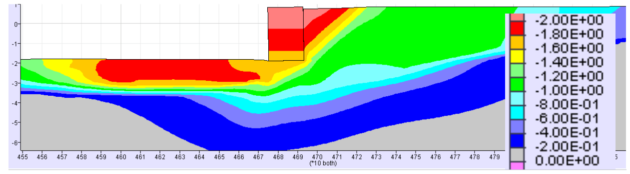
A summary of the permanent shear induced displacement estimates for the caisson is provided in Table 2. As presented in the table, horizontal displacements of 2.2 m and 0.7 m and vertical displacements of 0.7 m and 0.2 m (downward) were estimated at the top/front of the caisson for the 975-year and 475-year input motions analyzed.

Table 2. Shear-induced displacement estimates at caisson

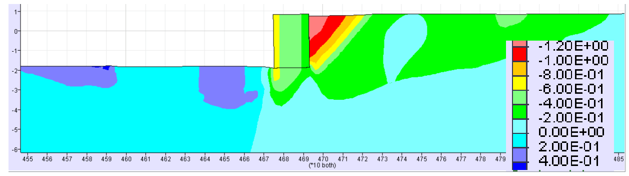
Input Motion	Analysis State	Horizontal Displacement (m)			Vertical Displacement (m)	
		Top Front	Bottom Front	Difference	Top Front	Behind Grade
Hector Mine (975-yr)	End of EQ	2.1	1.6	0.5	-0.6	-1.2
	Post-EQ	2.2	1.7	0.5	-0.7	-1.3
Tokachi-Oki (475-yr)	End of EQ	0.7	0.7	0.05	-0.2	-0.3
	Post-EQ	0.7	0.7	0.05	-0.2	-0.3

Figure 11 and 12 present shear-induced displacement estimates (including post-earthquake displacements) and peak excess pore water pressure ratio ($R_{u\text{-peak}}$) generated during the earthquake analysis for the 975-year and 475-year input motions. As can be observed on the figures, peak excess pore water pressure ratios $R_{u\text{-peak}} > 0.7$ developed within the Sand/Silt unit at the seafloor and underneath the rockfill mattress in front of the caisson and extended to approximately EL. -50 m for the 975-year input motion. Liquified zones also developed beneath the terminal embankment extending to within approximately 60 m of the caisson for one of the 975-year input motions. The analysis for the 475-year motion yielded liquefied zones at the seafloor extending to approximately EL. -30 m and beneath the far end of the rockfill mattress.

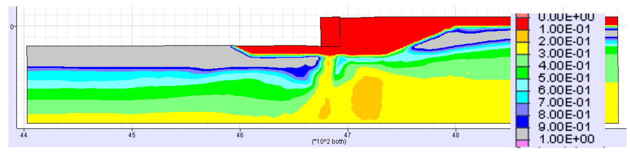
The displacements generally indicate a lateral translation of the caisson and offshore rockfill mattress, with much of the shearing occurring in the liquefied and/or softened Silt/Sand layers beneath the rockfill mattress. Some potential for downward movement of the rockfill beneath the waterside face of the caisson is indicated, with a resulting rotation of the caisson. The translation and rotation of the caisson in relation to the terminal island fill materials leads to the formation of a shear wedge and graben behind the caisson.



a) Horizontal displacements

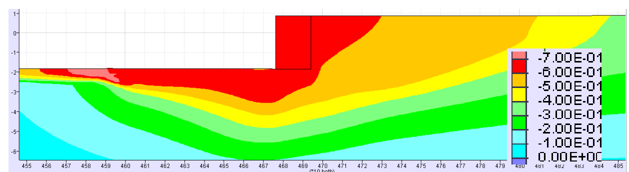


b) Vertical displacements

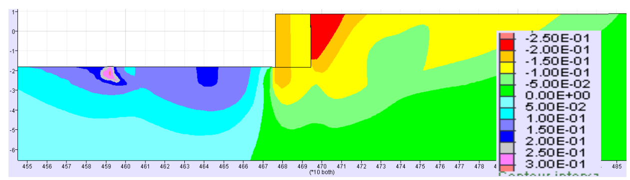


c) $R_{u\text{-peak}}$

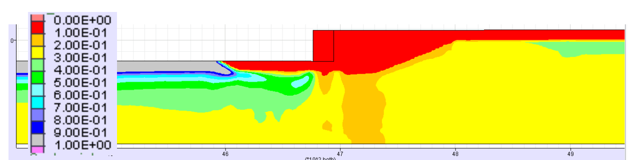
Figure 11. Displacements (in m) and peak excess pore water ratio ($R_{u\text{-peak}}$) contours for 975-year Hector Mine motion.



a) Horizontal displacements



b) Vertical displacements



c) $R_{u\text{-peak}}$

Figure 12. Displacements (in m) and peak excess pore water ratio ($R_{u\text{-peak}}$) contours for 475-year Tokachi-Oki motion.

Figure 13 presents contours of shear strain for the 975-year input motion with the largest shear strain within the liquefied soils beneath the rockfill mattress and in front of the caisson (end of earthquake). Figure 14 shows strain distributions in the vicinity of the caisson.

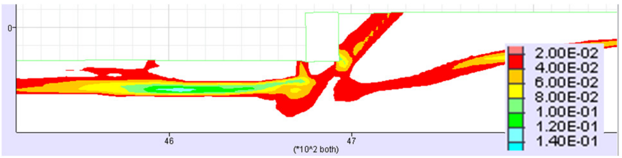


Figure 13. Estimated shear strain (γ_{xy}) for 975-year Hector Mine motion. Multiply legend by 200 to convert γ in %.

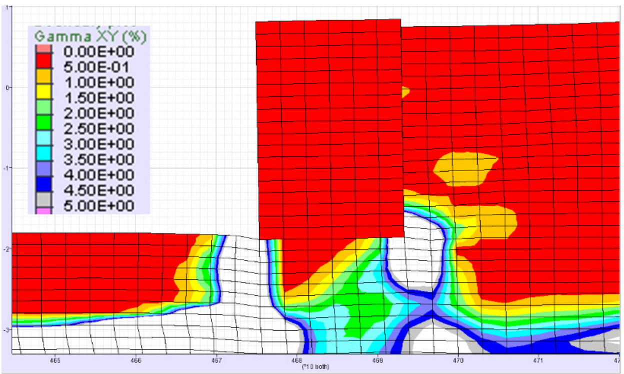


Figure 14. Estimated shear strain (γ_{xy} in percent) in vicinity of caisson for 975-year Hector Mine motion (end of earthquake). (note: white contours indicate shear strains in excess of 5%).

Figure 14 illustrates displacement profiles at select locations (i.e. caisson centerline, 15 m in front of caisson and 15 m behind caisson). Figure 15 shows horizontal and vertical displacement histories for the top/front and bottom/front of the caisson and the grade immediately behind the caisson for the 975-year motion. Permanent displacements develop after approximately t = 13 second and tend to increase until the end of the motion. The histories also include the estimates of post-earthquake displacement (i.e. after 45 second).

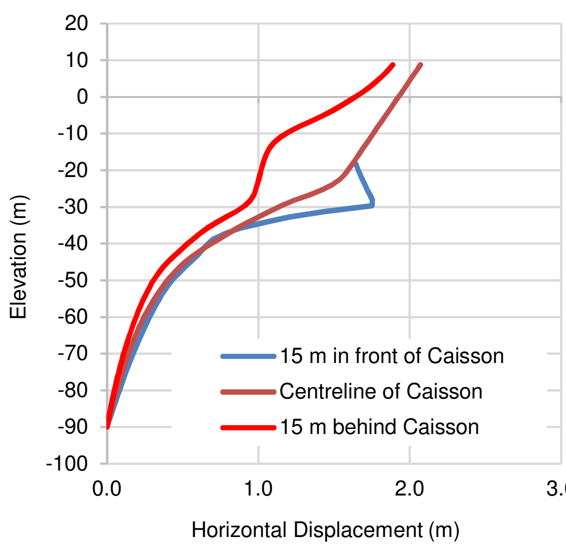
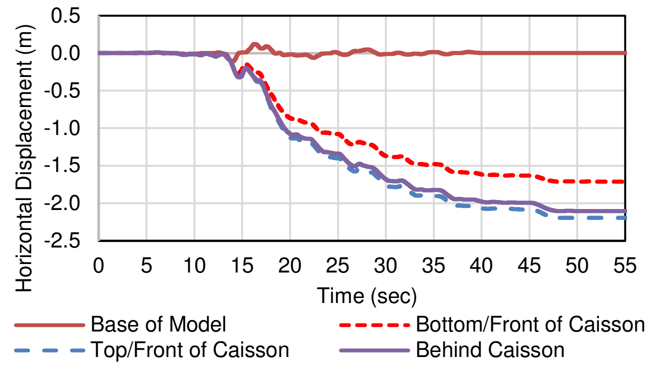
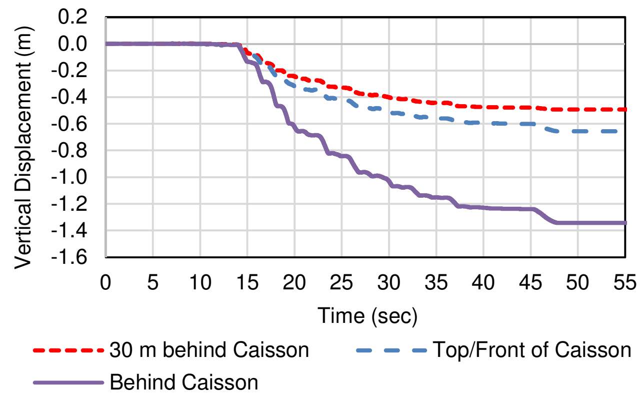


Figure 14 Horizontal displacement profiles at selected points for updated 975-year Hector Mine motion (end of earthquake).



a) Horizontal displacement



b) Vertical displacement

Figure 15. Selected displacement histories for 975-year Hector Mine motion (including post-earthquake).

Figure 16 presents response spectra for the selected acceleration histories, input motion ('within' motion at Elevation -90) and Site Class C outcrop motion. The response spectra for the FLAC estimates are somewhat approximate as the timestep changes through the analysis. The deep soil site is deamplifying the higher frequency motions (T < 0.7 seconds) and amplifying longer period motions (1 < T < 2 seconds). Figure F17 shows response spectra amplification ratios for the selected acceleration histories with respect to the Site Class C outcrop motion.

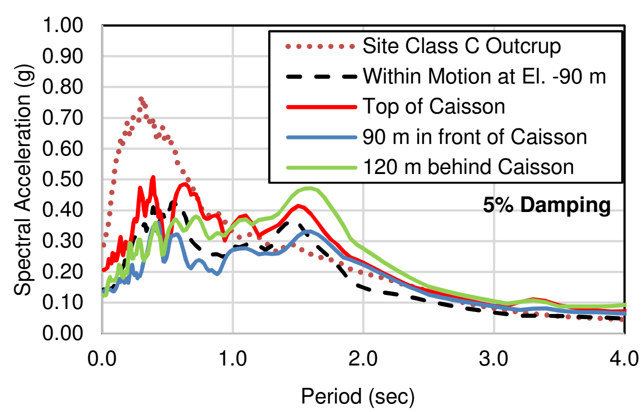


Figure 16. Response spectra for selected acceleration histories for 975-year Hector Mine motion.

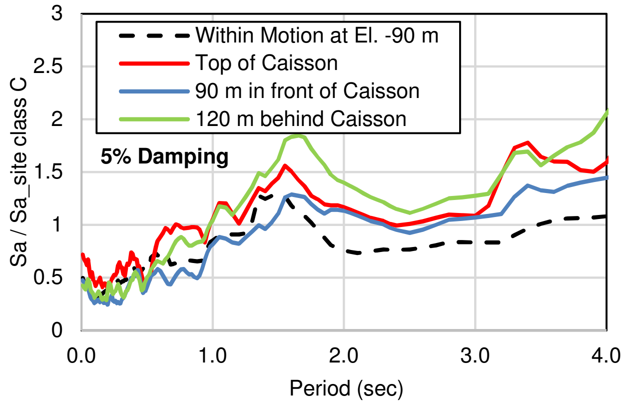


Figure 17. Response spectra amplification ratios for selected acceleration histories for 975-year Hector Mine motion.

6 TRIAL ANALYSES

Many trial analyses were performed as part of the development of the FLAC analysis. The parameters modified during these trial analyses included the model geometry, constitutive model, element size, material parameters, boundary conditions, input motions, drainage conditions, and extent of ground improvement. A brief qualitative summary of the trial analyses is provided. The trial analyses are presented in order of their execution.

6.1 Initial Trial Analyses

The FLAC model initially developed was approximately 2 km long to include the entire width of the proposed embankment and approximately 1 km of the seafloor slope. The model was also extended to the underlying till (e.g. EL. -160 m or deeper) to allow direct application of the Class C input motion with a compliant base approach. The grid spacing for the model was generally 2.5 m x 1.25 m for the upper strata.

The UBCSAND model was used to model the liquefiable soils. The UBCSAND model was calibrated using a similar approach to that described above for the calibration of the PM4Sand model.

As with the preliminary analysis, the UBCHYST model was used for the Rockfill, Densified Fill and a clay stratum present between the Silt/Sand and the underlying till. As compared to the final model, the strength and stiffness values used for the Rockfill and Densified Fill were less than those presented herein.

The elastic model was used to simulate the infinite half-space of the till beneath the model. The elastic model was also used to model columns adjacent to the lateral boundaries to reduce the potential for disturbance at the boundaries during dynamic analysis.

The seismic analyses were carried out assuming undrained conditions (i.e. transient flow was not allowed).

The input motions used in the trial analyses were related to different earthquakes developed to match the site-specific Uniform Hazard Response Spectrum (UHRS) with linear scaling.

The analyses for 975-year motions yielded displacements significantly larger than anticipated and the analyses were terminated due to excessive distortion. The following were considered as potentially contributing to an overestimation of seismically-induced displacements:

- The selected input motions were not representative of the hazard level and site conditions (i.e.: all input motions were linearly scaled to the UHRS);
- The behavior targets developed for the Silt/Sand soils required a set of calibrated model parameters outside of typical ranges. These parameters, coupled with the presence of significant initial shear stress ratios in front of the caisson, produced a post-liquefaction response in critical UBCSAND zones that was not considered reliable; and,
- The potential effect of deformations estimated with the UBCSAND model for the seafloor slope located beyond the dredged zone.

6.2 Revised Trial Analyses

During the review process, the review team recommended the following modifications to the initial trial analyses:

- develop a smaller mesh (e.g. 500 m long x 90 m deep) with refined grid spacing (note: the stability of the offshore slope was to be evaluated separately);
- use PM4Sand to model the liquefiable soils in addition to UBCSAND;
- assume higher stiffness/strength parameters for the Rockfill and Densified Sand;
- allow pore pressure flow to occur during the dynamic analysis;
- adjust the post-earthquake, post-liquefaction strength limit to the mean estimate of post-cyclic strength at 15% shear strain; and
- use the 975-year Hector Mine motion initially developed (linearly scaled).

The FLAC model was updated with the above changes. The dynamic boundary condition at the base of the model was converted from a compliant base to a fixed base. The input motion was applied as a 'within' motion estimated from SHAKE analysis.

The initial analyses of the revised trial model revealed that the proximity of the lateral boundaries to the caisson would impact the caisson response due to unrealistic boundary effects. The boundary influence was investigated by developing alternative options. The boundary conditions discussed in this paper (i.e. shear-beams) were adopted.

The analyses of the revised model with UBCSAND and the 975-year Hector Mine motion initially developed resulted in excessive displacements before the analyses was terminated due to excessive distortion.

The PM4Sand model was calibrated as discussed in this memorandum and utilized in the revised FLAC model for the liquefiable soils. The analysis of the revised model with the PM4Sand yielded less but still excessive displacements for the 975-year Hector Mine motion initially developed.

The analysis of the revised model with the refined grid spacing (i.e., approximately 1.5 m x 0.75 m) and the PM4Sand model was found to be very time-consuming (e.g., more than a week of computer time for a 60-sec input motion). To reduce the analysis time, the grid spacing was revised to approximately 3 m x 1.5 m. A comparison of the results of the two models (i.e. fine mesh versus coarse mesh) for the 975-year Hector Mine motion initially developed revealed that the estimated displacements at the caisson were within approximately 10%. The revised model with the coarse mesh was used for subsequent analyses (i.e. final model).

6.3 Revised Input Motions

Revised input motions discussed in Section 2.2 were adopted for subsequent analyses, and for the preliminary FLAC analyses. The analysis for the revised input motions yielded significantly less displacements as discussed in the paper.

6.4 Additional Ground Improvement

Several conceptual ground improvement options were analyzed with the final model to study their effect on the estimated displacements for the 475-year input motions. The options included using a shear key beneath the rockfill mattress immediately in front of the caisson (e.g. 30 m long x 20 m deep) and using a thicker rockfill mattress (e.g. extending to Elevation -32.0 m). The same soil parameters and model used for the rockfill were assigned to the improved ground. The effect of using a larger friction angle in the rockfill (e.g. 50 degree) and the use of Mohr-Coulomb model for the rockfill and shear key were also studied in separate analyses.

The results of the analyses revealed that utilizing a shear key or thickening the rockfill mattress could eliminate or reduce the extent of the estimated liquefiable zone in the vicinity of the caisson but would not substantially reduce the displacements as estimated in the preliminary FLAC analyses. Relatively large shear strains were still possible beneath the ground improvements concepts that were analyzed.

7 CONCLUSIONS

The results of the preliminary analysis indicated a limited lateral translation of the caisson and rockfill mattress, with much of the shearing occurring in the liquefied and/or softened Silt/Sand layers beneath the rockfill mattress. Some downward movement of the rockfill beneath the toe of the caisson is also indicated, with a resulting rotation of the caisson. The translation and rotation of the caisson with respect to the embankment fill tends to the formation of a shear wedge and graben behind the caisson.

With the presence of significant initial shear stress ratios in the native soils beneath the front of the caisson, calibration of the model to static shear stress bias was found critical to produce reliable results.

The trial analyses revealed that the “shear-beams” lateral boundary mitigated unrealistic boundary effects.

The selection and scaling method of the input motions can have a significant influence of the results.

The use of ground improvement shear key beneath the caisson may not significantly reduce or eliminate displacements as relatively large shear strains are still possible beneath the ground improvements.

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