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Using CT-scanning to study internal erosion in soils

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ABSTRACT

The decision to select a granular material as a construction material depends, in part, on its internal erosion susceptibility, particularly for embankment dams and dikes. The internal erosion (suffusion) is commonly determined by the application of empirical criteria based on the shape of the soil grain size distribution curve. A new analysis method, the X-ray micro Computed Tomography, can be used in addition to the conventional methods in order to determine the susceptibility of the specimens to internal erosion. Micro-CT images are capable of producing the 3D microstructure of the specimens before and after flow tests. Furthermore, micro-CT scans can provide insight into the porosity structure, grain size distribution and particle migration. This research looks into how micro-CT images of granular material can be obtained and analyzed as well as the limitations of such scans. Procedures are presented in order to avoid and reduce noise in micro-CT images and to obtain repeatable and reliable data from the specimens.

RÉSUMÉ

Le choix d'un matériau de construction granulaire pour les digues et barrages en remblai dépend en partie de sa susceptibilité à l'érosion interne. La susceptibilité à l'érosion interne (suffosion) est souvent déterminée à l'aide de critères empiriques basés sur la forme de la courbe granulométrique. Cet article présente un nouvel outil permettant de déterminer la susceptibilité à l'érosion interne par la microtomographie aux rayons X. Cette méthode a l'avantage de reconstruire la microstructure 3D des spécimens granulaire avant et après des essais en perméamètre. Les microtomographies donnent aussi de l'information sur la structure des pores, la granulométrie et le mouvement des particules. Cet article montre comment les microtomographies peuvent être obtenues et analysées, ainsi que leurs limitations. Des procédures sont présentées pour diminuer ou éliminer les artéfacts dans les microtomographies et pour obtenir des résultats fiables et reproductibles.

1 INTRODUCTION

Internal erosion of soils is a critical part of design for earth structures where seepage occurs, namely: earth dams, dykes, drain filters and it includes cohesive and cohesionless soils. The internal erosion of granular material can be initiated by different mechanisms: concentrated leak erosion (concentrated flow through a crack in the soil due to construction defect or settlement); backward erosion piping (where erosion initiates downstream of the dam, forming a pipe which develops into the dam); and suffusion (a.k.a. internal instability), which is the loss of fine grained particles from a mixture of fine and coarse soil (ICOLD, 2017).

The current research deals with the selective internal erosion in granular material, namely the suffusion. Suffusion occurs when fine particles are transported by flow through the interconnected pore network formed by the larger particles without any overall volume change. This leads to a decrease in density and a variation in permeability and porosity in the concerned volume of soil.

In the literature, the words "suffusion" and "suffosion" are often used interchangeably, but refer to different mechanisms. Hence, for clarity, 'suffosion' will be defined as a special case of suffusion that occurs when there is a loss of fine material resulting in overall volume change (Fannin et al., 2015).

Numerous studies have looked into the suffusion process and many researches have proposed varying criteria in order to determine if a particular soil is susceptible to suffusion (Kezdi1979; Sherard 1979; Kenney and Lau 1985; Lafleur and al. 1989; Burenkova, 1993; Skempton and Brogan, 1994; Wan and Fell, 2008). These criteria are not unanimously accepted in the scientific community. Furthermore, review of research on suffusion indicates that soils considered "identical" based on their grainsize distribution can be labeled as stable or unstable for suffusion depending of the selected criteria (Wan and Fell 2008; Marlot et al. 2016). Moreover, according to Marot et al. (2016), there is no suitable assessment tool for the susceptibility of suffusion mainly due to the fact that it is a coupled processes of detachment–transport–filtration for the finest fraction of particles within the porous network. The current research program is aimed at exploring the widely used and accepted criteria in order to determine which ones are the most accurate and reliable. In order to do this, suffusion tests are performed in permeameters with different soils reconstituted with glass beads according to grain size distributions extrapolated from literature. The conventional methods used to determine suffusion in a granular specimen are: (a) the observation of hydraulic parameters during the flow test, (b) the comparison of the grain size distribution of different layers of the specimen from before and after the flow test and (c) the measurement of the mass of particles flushed out from the permeameter (Lafleur and al., 1989; Fannin and Moffat, 2006; Wan and Fell, 2008). A new non-invasive analysis method, the X-ray Micro Computed Tomography (CT) scan (referred to as CT-scan), can be used in addition to the conventional methods. CT-scan images may be obtained before and after a flow test on the same specimen and at a particular location of the specimen. Therefore, it allows reusing the same specimen to verify suffusion but with different initial conditions. Moreover, the CT-scan can confirm certain physical properties and provide additional insight about the specimen such as: porosity, presence of air and particle movement. However, CT-scan images are obtained following reconstruction and segmentation steps and these steps depend on the scanned material's density, dimension and heterogeneity. In this paper, the focus is to determine a methodology and limitations for using the CT-scan during suffusion tests.

2 LITERATURE REVIEW

2.1 Suffusion criteria

The study of suffusion can, in part, be traced back as far as Terzaghi (1931) at which time the focus was on the understanding and design of filters. Terzaghi proposed two geometric filter criteria to be used for soils in earth structures. In 1979, Kezdi proposed a geometric suffusion criterion based on Terzaghi's retention criteria for filters in which the grain size distribution is split into finer and coarser fractions and a representative diameter is selected for each. Sherard (1979) had a similar approach; the only difference was the threshold value. These criteria were based on a diameter ratio taken from the granulometric curves. In 1985, Kenney and Lau developed a criteria based on the shape of the grain size distribution curves, where a boundary separated the potentially unstable and stable soils. In general, these criteria have the following in common: (a) they are empirical, (b) they are limited to a particular soil gradation and (c) they are non-robust. The above-mentioned works and criteria have had their share of critical reviews and amendments (Semar et al., 2010).
Although only four suffusion criteria were mentioned in the preceding paragraph, the literature includes a multitude of criteria and combinations of these to identify suffusion in different soils (Marot et al., 2016).

2.2 Suffusion testing

Suffusion tests are currently not standardized mainly due to pending disagreements that can be classified into the following: (a) the preparation of the soil specimen, (b) the measurement of hydraulic parameters under a downward flow and (c) the confirmation or otherwise of the suffusion (Lafleur and al. 1989a, Fannin and Moffat 2006, Wan and Fell 2008). In this research the suffusion tests were setup to follow the recommendations of the flow test standard ASTM D2434-68 (2006). The first steps involve placing the specimen in the test cell, then compacting the specimen and saturating it with an upward flow of deaerated water combined with a vacuum (Chapuis et al. 1989a). During the test under a constant hydraulic gradient, the evolution of the hydraulic parameters (i.e. hydraulic gradient or hydraulic conductivity) are measured at different heights of the specimen thanks to peripheral piezometers (see Figure 1). In order to determine whether suffusion has occurred the specimen is subdivided into layers and each layer is sieved. Then the grain size distributions for each layer are compared with the initial grain size distributions. Additionally, this step can be coupled, for further insight, with the measurement of the mass of flushed out particles (Fannin and Moffat 2006). However, such methods raise concerns with respect to sample disturbance, accuracy and sampling quality. Furthermore, for particular methods, if an applied hydraulic gradient was not sufficient to cause movement of particles, this would only be evident once the specimen was removed from the cell and analyzed.

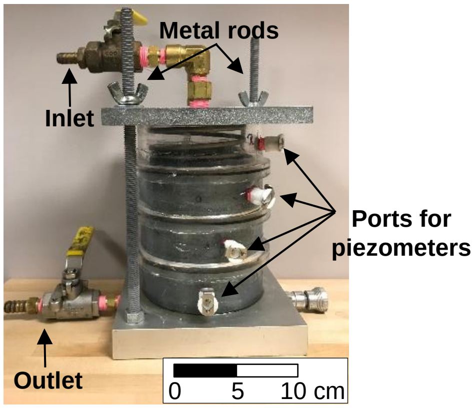


Figure 1. Test cell used for suffusion determination in granular materials; piezometers can be attached to ports located in each layer.

2.3 Granular material characterizations with CT-scan

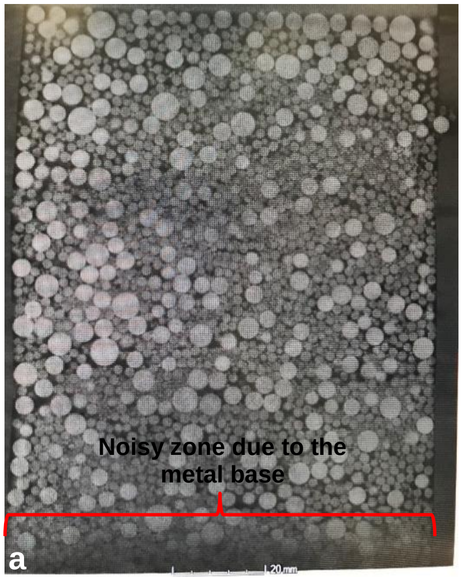
Although studies of suffusion with the aid of CT-scanned images are available, there is a lack of comprehensive calibration studies of those scans or a lack of discussion about possible sources of error due to the resolution constraints attributed to CT-scanning. Furthermore, published methods and algorithms developed for determining porosity or grain sizes have been typically validated by using computer-generated packings of particles with regular and irregular shapes (Vincens et al., 2015). Such packings are typically deprived of noisy voxels and artifacts contrary to what real CT-scanned images yield. Analysis by Périard et al. (2016) of the porosity and hydraulic conductivity distribution in a soil column performed with CT-scanned images was based on particles with a median diameter of 150 $\mu\mathrm{m}$, however their voxel (i.e. pixels multiplied by distance between two slices) size was 450 $\mu\mathrm{m}$. Humberg et al. (2008) prepared a gap-graded sample and analyzed it using micro-CT images, although the sample was prepared with grain size between 0.01 mm and 20 mm embedded in a resin, their results had to be limited to grains size diameter of 1 mm for analysis in order to be reliably extracted and segmented for analysis. Al-Raoush et al. (2006) used spherical glass beads with diameters from 0.1 up to 0.6 mm in a 5 mm inner diameter container and a scanning resolution of 6 to 11 $\mu\mathrm{m}$. This resulted in approximately 50 voxels per bead diameter but the specimen was very small. The articles above show some of the constraints and concessions that researchers had to accept in order to utilize their CT-scan images.

3 SUFFUSION TESTS COUPLED TO CT-SCANS

A grain size distribution called K from Kenney and Lau (1985) was reconstituted with glass beads (range 250 $\mu\mathrm{m}$ to 6 mm). The specimen should be stable according to the Kezdi (1979), Sherard (1979) and Kenney and Lau (1985, 1986) criteria and the suffusion test realized by Kenney and Lau (1985). The 100 mm diameter and 120 mm height specimen was placed in a PVC cell (see Figure 1) and compacted using a vibrating table.

This particular K grain size distribution was chosen for two reasons: (a) as mentioned previously the specimen should be stable and (b) the smaller grain diameter (250 m) of the specimen is in line with the resolution obtained with the ÉTS CT Scan (approx. 71 $\mu\mathrm{m}$) and the dimension of the tested cell. Indeed, particles with a diameter smaller than 4 voxels are difficult to identify and segment, therefore add significant error into the extracted grain size distribution (Homberg et al. 2008).

Nonetheless, preliminary CT scans on a fully assembled cell did not yield satisfactory results. The images were noisy and lacked contrast (see Figure 2a and 2b), although significant efforts were made in the reconstruction phase as well as in the segmentation phase. It was concluded that some efforts should be made on the selection of materials used to build the test cell. For example, the bottom part of the cell and the threaded rods could be in plastic, a material with a lower density. Moreover, the metal valves could be moved lower hence, away from the scanned region. These modifications would enhance the CT-scan images.



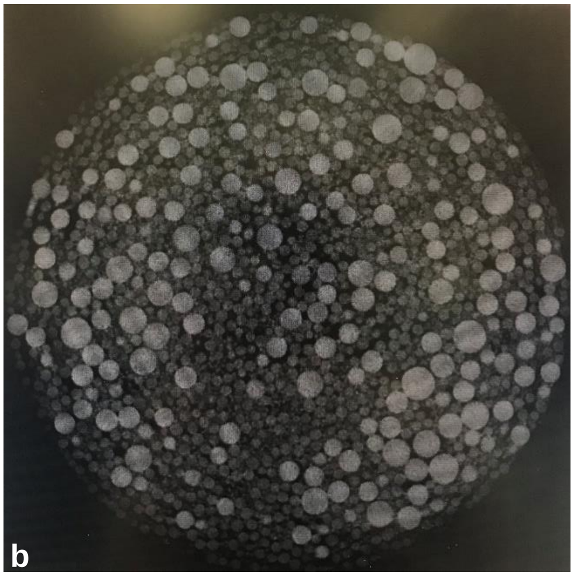


Figure 2: CT-Scan images of the K grain size distribution of Kenney and Lau (1985) soil column (a) side view and (b) sectional view.

4 MATERIAL AND METHODS

4.1 Experimental program

Since, the preliminary results obtained from CT-scans on the fully assembled testing cell were not satisfactory; a testing program was put in place to determine the possible causes and potential methodologies to address the encountered issues. It was decided that some calibrations tests must be performed using a simpler specimen. These calibrations were done on mono-sized beads placed in a PVC container in a dry and saturated state. The first step was to find the most appropriate settings for the CT-scan, determine the sources of error and confirm the measured porosity.

4.2 Glass beads and cell

The specimen was composed entirely of uniform sized glass spheres (density: 2.5 $\mathrm{g/cm^3}$) in a PVC cylindrical container (37.1 mm height and 101.2 mm diameter, see Figure 2a), identical in material to the cell used for testing suffusion. The glass beads were measured with a digital caliper at an average diameter of 7.99 mm.

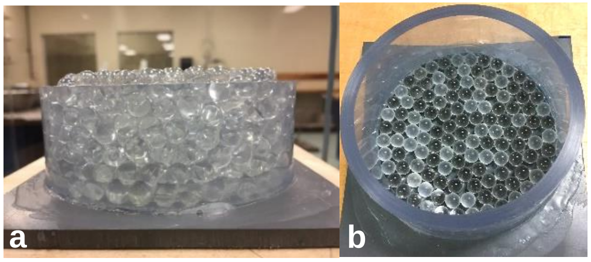


Figure 2. PVC container: a) filled with 8 mm beads; b) first row filled with beads to quantify edge effect on porosity.

In total eight laboratory tests were performed using the water saturation technique in order to assess the effective porosity of the specimen. The beads were slowly poured into the container filled with water in order to avoid the introduction of air, and were gently leveled every two rows with a circular tamper. Porosity was determined by the volume of water contained in a saturated sample of known volume. The mass of saturated sample less the oven-dry mass of the beads, divided by the density of water, gave the volume of water. This volume divided by the PVC container volume gives porosity. The porosity was measured at 40%, which is close to the theoretical porosity of 36% for a random closing packing of spheres and 39% for a poured random packing (Dullien, 1992). The difference between measured and theoretical value was attributed to the finite size of the container, the size of the beads and the edge effects that increase the overall porosity. The bottom edge effect on porosity was assessed by filling the container with only one row of beads (see Figure 2b). The obtained porosity for the bottom layer of half height of a bead was determined to be 39.2%, which is 0.8% lower than the overall porosity.

4.3 CT-scanning

4.3.1 Apparatus

For the tomographic analysis of the specimens, the Nikon XT-H-225 microfocus X-ray source system was utilized, located in the Laboratoire Institutionnel at ÉTS. The main characteristics of the system are: 225 kV microfocus X-ray source with 3 µm focal spot size and a Variant 2520Dx detector. The specific parameters used for the scanning of prepared specimens were: effective pixel size of 71 µm, a copper filter of 4.5 mm to reduce the effect of beam hardening, a gain of 24.0 dB, energies of 200 kV and 170 µA, exposure time of 4 seconds with 2634 projections.

In the current research, the least noisy scans were obtained, if the entire cell was rotated in the field of view of the scanner. Therefore, due the size of the testing cell a significant distance was allowed between the source of the X-ray and the testing cell. This led to a decrease in resolution (approx. 71 μm) for the images, because only a fixed number of pixels were available to cover the full extent of the specimen. The resolution in turn limits the size of the fine particles that can be used for preparing the specimen if one wants to have clear post-analysis images.

4.3.2 Energy levels

Increasing the energy levels during micro CT-scanning allows to scan denser materials with a deeper penetration of the X-rays. However, this comes at the expense of additional noise in the image. The study by Ketcham and Carlson (2001) shows that energy levels above 100 kV will produce Comton scattering, a form x-ray by-production from electron being ejected and causing random noise in the image. The micro CT-scanner produces an X-ray beam that is said to be polychromatic due to a range of energy levels produced (i.e. range of frequencies). In the current study the parameters were balanced to avoid overexposure and to allow taking advantage of the maximum range of intensities (i.e. 10 000 and 60 000). The usage of the full range of intensities gives better contrasts between different materials (i.e. between their densities). Given the input parameters for the CT-scanner, the entire scan of the beads specimen took about 6 hrs.

4.3.3 Image Analysis

Once the scanning was completed, for one saturated and one dry specimen, the images were reconstructed by applying the beam hardening and noise reduction filter to the images in CT Pro 3D (ver. 3.5.1) software. This initial step in image reconstruction was definitively user influenced as parameters were adjusted according to the preference of the user in order to obtain clear, sharp and noise-free image. After the reconstruction was completed the images were uploaded to a segmentation image processing software (Dragonfly 3.5 by ORS inc.). Again, in this software, there was user input that can be subjective. However, in order to limit the bias, steps were taken to minimize the interpretation. First, the median filter was applied to reduce the amount of noise in an image. Afterwards, the Otsu thresholding method was used. This method determines the optimal thresholding value that minimizes the weighted within class variances of two basic classes (Otsu, 1979). In other words, the Ostu method transforms a greyscale image into a monochrome (i.e. binary) image by finding the optimal thresholding value (i.e. the valley) in an intensity histogram in order to separate the objects from the background (see Figure 3) without any user input. After these image-processing methods were applied a region of interest was selected and the voxels in that region were investigated. Although numerous filters and thresholding methods were tried, the ones presented above gave the best images and results, with minimal user input and fast processing times.

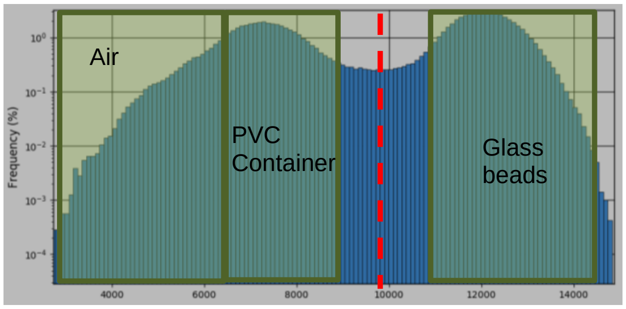


Figure 3. Intensity histogram showing the different intensities attributed to different materials as well as the Otsu optimum thresholding value shown as a vertical dotted line.

5 RESULTS

5.1 Porosity

To the best of knowledge of the authors, the 8 mm spherical beads were scanned under the most optimal parameters in the micro CT-scanner. Furthermore, the reconstructed images were carefully analyzed through a series of different filters and thresholding methods to arrive at the most optimal and accurate results with the least input from the user. However, from the obtained results it is clear that the dry beads present a sharper and clearer image versus the saturated beads (see Figures 4a and 4b). The intensity histograms for both saturated and dry case show the lack of clear peaks and valleys in the saturated condition (see Figures 4c and 4d). This leads to difficulties during segmentation of the images.

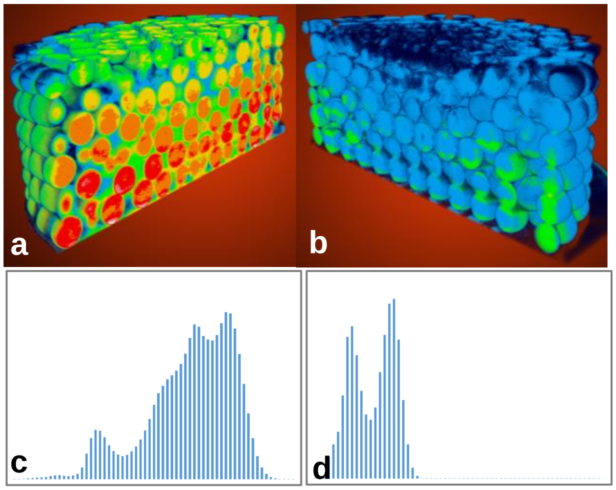


Figure 4. Distribution of intensities in a 8 color palette: a) the saturated beads and b) the dry beads; histograms for frequency vs intensity in c) saturated and d) dry condition.

This additional noise introduced by the presence of pore-water makes the determination of porosity problematic. For visual assessment of the noise, the entire data sets of images for dry and saturated specimens which were filtered and thresholded with the Otsu method are shown for a particular section in Figures 5a and 5b.

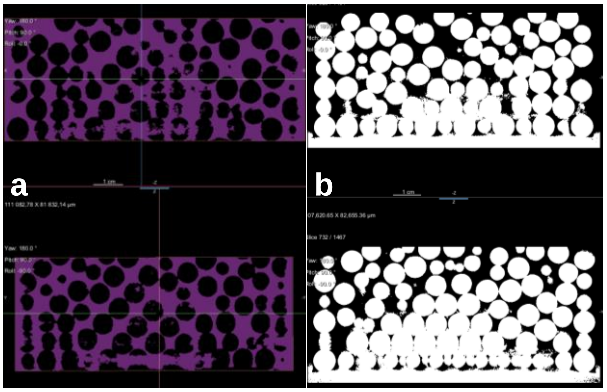


Figure 5. Distribution of intensities in a median filter and Otsu: a) for the saturated beads and b) for the dry beads.

The average computed porosity from these data sets was 35.4% and 36.9% for saturated and dry beads respectively. When comparing those results to the experimental porosity of 40.0%, the difference is around 4%.

A similar difference, of around 4.5%, between porosities obtained from CT-scanned images versus measured values in glass beads was as reported in the research by Al-Raoush et al. (2006).

However, such difference is deemed unacceptable for the current research program when testing for suffusion in soils, since according to Skempton and Brogan (1994) and Andrianatrehina et al. (2016) a 4-5% difference in grain size distribution curves was proposed as a limit to distinguish stable from unstable soil during suffusion tests.

5.2 Sources of error

A source of error that can be introduced into the results during the image analysis can be attributed to the thresholding techniques. A comprehensive review of different thresholding methods is presented by Sezgin and Sankur (2004). They conclude that the Ostu method gives satisfactory results when the numbers of pixels in each class (i.e. foreground and background) are close to each other. Furthermore, additional errors are introduced with calculations of porosity, sphere size, and local void ratio, in part, because of digitization (pixelization) effects (Al-Raoush et al., 2006).

Although, the scanned specimen in this research was prepared with uniform size and density beads for simplicity, it is clear from the image analysis that the size of specimen is problematic. The amount of secondary X-ray emission due to electron ejection for such a large specimen is significant and leads to non-uniform distribution of intensities on the edges as well as towards the center of the specimen (see Figure 6), which results in difficulties during segmentation and thresholding.

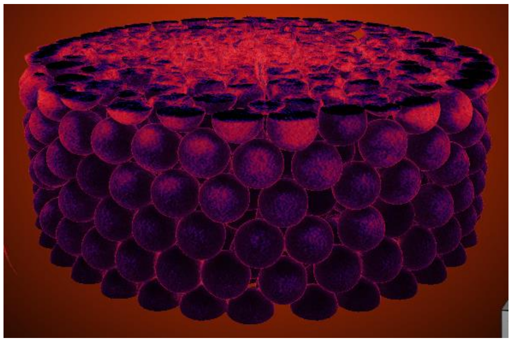


Figure 6. Distribution of intensities

Furthermore, cropping the image during segmentation leads to the problematic of determining where the inner boundary of the container touches the beads. This operation is left for interpretation by the user and in the current study, given the size of the specimen, has led to a variance in porosity of 0.5% if the cropping boundary is moved slightly (see Figure 7).

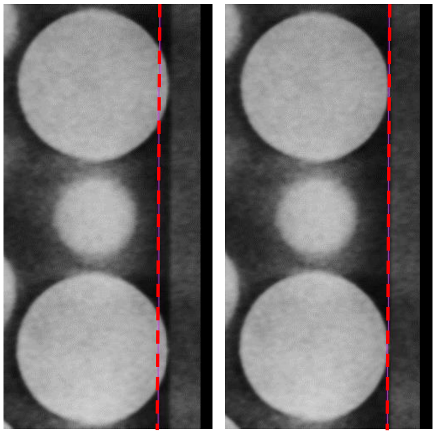


Figure 7. Zoomed in section of the data set presenting the dotted cropping line at two different locations.

6 CONCLUSIONS

The results for porosity show a difference of 4% between the measured porosity from laboratory tests and the porosity obtained from the CT-scanned images. This discrepancy will cause doubts for the planned suffusion tests when calculating the grain size distribution and porosities. Since, according to the results by Skempton and Brogan (1994) a difference between 4 to 12% in the grain size distribution curves, in the fines, can initiate suffusion in susceptible soil. Furthermore, the sensibility in cropping the image leads to significant fluctuations in calculated results. In order to avoid cropping problems and to decrease the noise a solution would be to use a smaller region of interest (i.e. representative volume element) selected from a well-segmented location within the specimen. Furthermore, a parallel technique utilizing sieve analysis and CT-scans can be used to accommodate such shortcomings with deep learning methods.

The use of watershed method for image processing and segmentation to identify individual particles has been shown to provide reasonable results with sands and glass beads (Taylor et al, 2017). However, such research, which focused on the internal erosion (i.e. suffusion) with the aid of a micro CT-scanner was limited to small specimen dimensions (i.e. diameter $\approx 1\,\mathrm{cm}$). Whether such small specimens are representative is up to debate.

Furthermore, it remains unclear as to how much noise is introduced into the scans by the size of the specimen and its corresponding additional amount of electron scattering. In this article, the problems associated with the separation of overlapping and touching particles was omitted. However, in order to determine particles diameters and then reconstruct grains size distribution curves such problems need to be addressed.

If a reliable quantitative analysis of specimen with the CT-scan is not possible, the scans can provide a qualitative assessment of the suffusion, by identifying loss of material at a particular location in a specimen. Furthermore, it is possible to introduce tracers (i.e. particles with engravings or iodide tracer) which can provide insight as to how far a particular grain was displaced given a specified hydraulic gradient (Anderson et al, 2003; Luo et al. 2008).

This work and results presented herein show that the use of CT-scan for the study of internal erosion has to overcome several difficulties associated with segmentation and resolution of images as well as the accuracy of results. Nonetheless, the CT-scanning approach is however promising and should become more available considering rapid technological developments. In the meantime, the technique is surely useful at least from a qualitative point of view.

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