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Burgers creep model used for describing and predicting the creep behaviour of a rock under uniaxial and triaxial compression test conditions

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ABSTRACT: Creep is usually associated with soft rock. Hard rock can also exhibit such behavior when the mine depth is large. A model is necessary to describe the creep behavior of rocks. In this paper, the Burgers creep model is used to describe the creep behavior of a rock under different stress conditions. Its ability of describing and predicting the time-dependent responses of rock was tested against experimental results available in the literature. The model parameters were obtained through the application of curve fitting technique on experimental results obtained under one confining pressure at one deviatoric stress level. The predictability of the calibrated model was tested against the experimental results under the same confining pressure, but at different deviatoric stress levels. The good agreements between the Burgers creep models and the experimental results indicate that the Burgers creep models are able to describe and predict the creep behavior of rocks under different stress states.

Key words: Rock; Creep; Rheology; Models; Description; Prediction

RÉSUMÉ: Le fluage est un comportement souvent observé avec des roches tendres. Il peut devenir significatif pour des roches dures lorsque la profondeur de mines est grande. Un modèle est nécessaire pour décrire le comportement rhéologique des roches. Dans cet article, le modèle de fluage de Burgers a été utilisé pour décrire le comportement rhéologique d'une roche sous différentes conditions. Sa capacité de décrire et de prédire les réponses évolutives des roches a été testée contre des résultats expérimentaux disponibles dans la littérature. Les paramètres du modèle ont été d'abord obtenus en appliquant la technique de régression sur des résultats expérimentaux obtenus sous une pression de confinement à un niveau de contrainte déviatorique. La capacité de prédiction du modèle calibré a été testée contre des résultats expérimentaux supplémentaires sous la même pression de confinement, mais à différents niveaux de contraintes déviatoriques. Les bonnes corrélations entre le modèle et les résultats expérimentaux indiquent que le modèle de fluage de Burgers est capable de décrire et de prédire le comportement rhéologique des roches sous différentes conditions de contraintes.

Mots-clés: Roches; Fluage; Rhéologie; Modèles; Description; Prédiction

1 INTRODUCTION

Creep is an important time-dependent behavior of rocks (Jaeger et al. 2009; Paraskevopoulou 2016). If a rock is submitted to a sufficiently high (but still lower than its peak strength) and constant load, it may demonstrate a long-term deformation, accompanied with micro seismic activities (Hardy Jr et al. 1969).

Figure 1 shows typical creep behavior of rock. When a constant load is applied, rock creep can generally be divided into three stages (Farmer 2012):

- Stage 1, increase of creep strains with declined rate. This stage is usually named as the primary stage or the transient creep;
- Stage 2, constant creep strain rate. This stage is known as the steady creep state;
- Stage 3, creep strain acceleration. This stage is usually named the tertiary stage.

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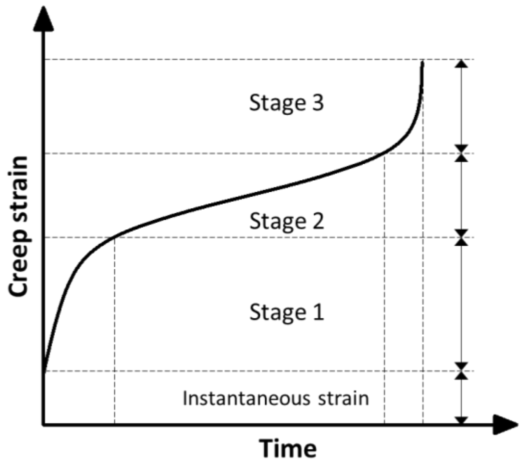

Figure 1. A schematic presentation of creep behavior of rock (after Farmer 2012).

Most rock and rock masses may have more or less degree of creep. Soft rocks usually demonstrate more obvious creep phenomenon (more creep strain) than hard rocks (Cristescu and Hunsche 1998). However, in hard rock mines, closure associated with creep around openings can also become significant and problematic when the mine depth is large. In mining engineering, an excessive deformation due to creep can significantly affect the function of the working spaces, resulting in unexpected high cost for the rehabilitation. It is thus important to characterize the creep deformation around rock infrastructures. This requires a model that is able to describe the creep behavior of rock under different stress conditions.

The fundamental creep formulations can generally be divided into two categories: empirical (Betten 2008) and rheological model-based formulations (Fahimifar et al. 2015; Zhao et al. 2017). All of these formulations need experimental results to obtain the required model parameters. The literature review showed that this is usually done by applying the curve fitting technique to all of the available experimental results to obtain different sets of model parameters. One set of model parameter is then used to describe the creep behavior of the rock at the stress level. Good agreements were obtained between the experimental results and model formulations. But the established model cannot be used for stress conditions different than the tested conditions.

In this paper, the Burgers creep model is briefly described. Its ability of describing and predicting the creep behavior of rock is then tested against some experimental results available in the literature.

2 BURGERS CREEP MODEL

The Burgers creep model is composed of Maxwell model and Kelvin-Voigt model in series (see Figure 2). The former is a combination of a spring and a dashpot in series whereas the latter is a parallel combination of a spring and a dashpot (Jaeger et al. 2009). The spring element is used to represent the instantaneous response of elastic behavior and the dashpot element is for the time-dependent viscosity behavior. In the Burgers creep model, the Kelvin model is adopted to describe the transient (i.e. the primary creep) stage while the Maxwell model is mainly for describing the instantaneous strain and secondary creep stage.

In the following sub-section, the one-dimensional and three-dimensional Burgers creep model will be recalled. Their predictive ability is tested later against some experimental results available in the literature.

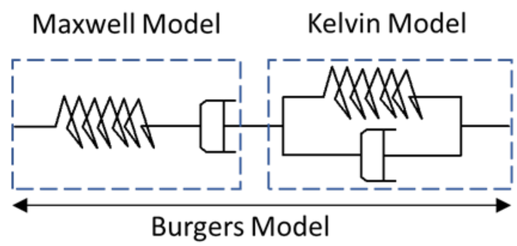

Figure 2. Schematic illustration of the Burgers creep model composed of Maxwell model and Kelvin-Voigt model (after Goodman 1989).

2.1 One-dimensional Burgers creep model

Since the Burgers creep model is a combination of the Maxwell and Kelvin-Voigt models in series, the total axial strain can then be obtained by summing their axial strains.

The one-dimensional strain ε_M of Maxwell model is given as follows (Jaeger et al. 2009):

$$\varepsilon_M = \frac{\sigma}{E_M} + \frac{\sigma \cdot t}{\eta_M} \quad [1]$$

where σ is an axial stress; E_M and η_M are the elastic modulus and viscosity coefficient of the Maxwell model, respectively; t denotes time.

The one-dimensional strain ε_K of Kelvin-Voigt model is given as (Jaeger et al. 2009):

$$\varepsilon_K = \frac{\sigma}{E_K} \cdot \left[1 - exp\left(-\frac{E_K \cdot t}{\eta_K}\right)\right] \quad [2]$$

where E_K and η_K are the elastic modulus and viscosity coefficient of Kelvin-Voigt model, respectively.

The one-dimensional strain ε_B of the Burgers creep model can then be written as:

$$\varepsilon_B = \frac{\sigma}{E_M} + \frac{\sigma \cdot t}{\eta_M} + \frac{\sigma}{E_K} \cdot \left[1 - exp\left(-\frac{E_K \cdot t}{\eta_K}\right)\right] \quad [3]$$

2.2 Three-dimensional Burgers creep model

The three-dimensional Burgers creep model has been given through a generalisation of the one-dimensional model by considering the l Hooke’s law in Eq. (4).

The total strain of an isotropic, homogeneous and linear elastic material can generally be decomposed in volumetric strain ε_m associated with the mean (spherical) stress σ_m and shear strain ε_{ij} associated with the deviatoric stress S_{ij}. Their relationships are expressed as:

$$\sigma_m = 3K\varepsilon_m$$
$$S_{ij} = 2G\varepsilon_{ij} \qquad [4]$$

where K and G are the bulk and shear moduli, respectively. The mean (σ_m) and deviatoric (S_{ij}) stresses are expressed as:

$$\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$
$$S_{ij} = \sigma_{ij} - \sigma_m \cdot \delta_{ij} \qquad [5]$$

where σ_1, σ_2 and σ_3 denote the major, intermediate and minor principle stresses, respectively; δ_{ij} is Kronecker's delta (δ_{ij} = 1 if $i = j$ or δ_{ij} = 0 if $i \neq j$).

In conventional triaxial compression tests, the applied stress condition is $\sigma_1 > \sigma_2 = \sigma_3$ (where). In addition, the confining pressure σ_3 is normally applied before the deformation measurement. Therefore, the volumetric strain in the three-dimensional strain equation of the Burgers creep model resulted from confining pressure is not considered.

The axial strain ε_M^T of Maxwell model under conventional triaxial compression test condition can be written as:

$$\varepsilon_M^T = \frac{\sigma_1 - \sigma_3}{9K_M} + \frac{\sigma_1 - \sigma_3}{3G_M} + \frac{(\sigma_1 - \sigma_3)\cdot t}{3\eta_M} \qquad [6]$$

where K_M, G_M, η_M are the bulk modulus, shear modulus and viscosity coefficient of Maxwell model, respectively.

The axial visco-elastic strain ε_K^T of Kelvin-Voigt model due to the deviatoric stress is written as

$$\varepsilon_K^T = \frac{\sigma_1 - \sigma_3}{3G_K}\left[1 - exp\left(-\frac{G_K \cdot t}{\eta_K}\right)\right] \qquad [7]$$

where G_K, η_K are the shear modulus and viscosity coefficient of Kelvin-Voigt model, respectively.

The axial strain ε_B^T of the Burgers creep model in three-dimensional stress state is obtained by adding Eq. (6) and (7) as follows:

$$\varepsilon_B^T = \frac{\sigma_1 - \sigma_3}{3}\left\{\frac{1}{3K_M} + \frac{1}{G_M} + \frac{t}{\eta_M} + \frac{1}{G_K}\left[1 - exp\left(-\frac{G_K \cdot t}{\eta_K}\right)\right]\right\} \qquad [8]$$

Eq. (8) constitutes the three-dimensional Burgers creep model.

The Burgers creep model has been used as a base model in many more elaborated rheological models to describe the time-dependent behavior of rocks. However, most previous studies are limited to describe the creep of rocks through the application of curve-fitting technique to all of the available experimental results. Different sets of model parameters were obtained for the different stress conditions. The almost perfect agreements obtained between the model and available experimental results showed the versatility of the models to describe the creep behavior of the rock. Nevertheless, the required model parameters thus change as the stress state changes. The ability of the calibrated model to predict or describe the creep responses of the rock under untested stress conditions remains unknown. As the induced stress state can be non-uniform and unknown before numerical modeling (especially for complex geometry with nonlinear elastic behavior), the model established in such way cannot be used in field conditions with a reel mining project.

In the following section, we will show a methodology through the application of the Burgers creep model, by which the required model parameters are first obtained by applying the curve fitting technique on a set of experimental results at one stress level. These model parameters are then fixed and taken as constant. The model with these constants is then called calibrated model. The predictability of the calibrated model is then tested against other experimental results, which were obtained under different stress conditions and not used in the previous curve-fitting process.

3 APPLICATION OF THE BURGERS CREEP MODELS

3.1 Uniaxial creep tests

Wang et al. (2017) conducted uniaxial compression creep test on a green sandstone, which has an average uniaxial compression strength (UCS) of 29.8 MPa.. Constant loads were manually applied using dead weight of masses at 45, 55, 65 ,75, and 85 % of the UCS. The axial creep strain was monitored by strain gauges glued on the lateral surface of the specimen.

At the final load level equaling to 85% of the UCS, tertiary creep stage occurred. The consideration of this state is beyond the scope of this study and will not be discussed further. Subsequently, only the experimental results of the first four stress levels will be used to compare with the one-dimensional Burgers creep model (Eq. 3).

The experimental data under the stress level at 65% of the UCS are chosen to obtain the required model parameters through the application of curve fitting technique. These model parameters are then taken as the constants of the calibrated model. The predictability of the model is tested by comparing the calibrated model with the experimental results obtained under the stress levels equaling to 45, 55, 75, and 85 % of the UCS, respectively.

Table 1 shows the obtained model parameters through the application of the curve fitting technique on the experimental results obtained under an axial stress of 19.5 MPa, which corresponds to a stress level at 65% of the UCS of the tested rock.

Figure 3 shows the comparison between the experimental results and the Burgers creep model using the obtained model parameters shown in Table 1. It can be seen that the model description of the experimental results at the stress level equaling to 65% of the UCS is perfect while the model prediction of the experimental results at other stress levels are quite good.

Table 1. Material parameters obtained by applying the curve-fitting on the experimental results obtained at the stress level equaling to 65% of the UCS.

E_M (GPa)	E_K (GPa)	η_M (10^3 GPa h)	η_K (GPa h)
6.48	175.14	5.46	25.49

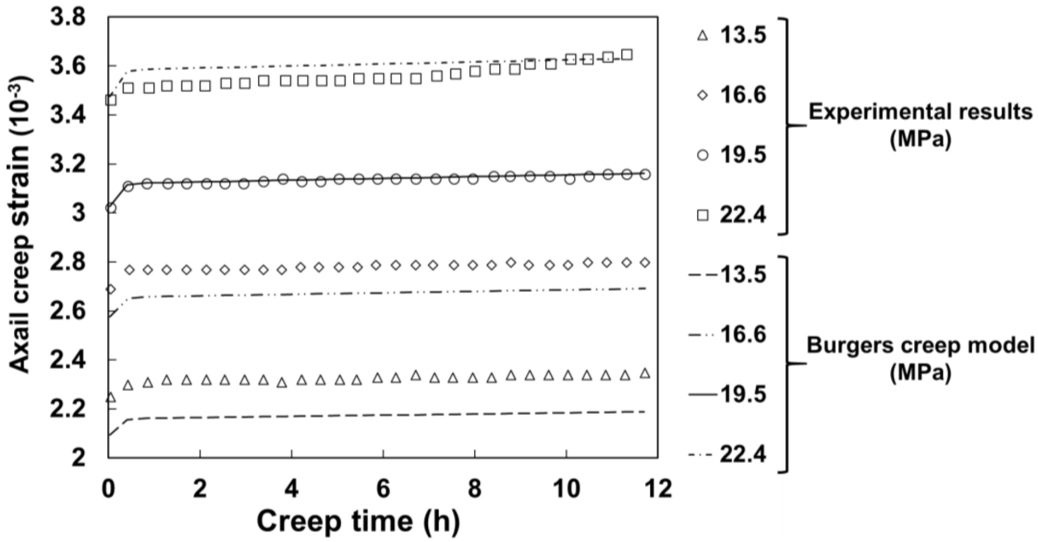

Figure 3. Application of the Burgers creep model for one description and three predictions of the strain evolutions of a cylinder rock, submitted under different constant axial stresses (experimental data taken from Wang et al. 2017); Burgers creep model parameters are given in Table 1.

3.2 Creep test under triaxial compression loading

Zhao et al. (2017) conducted a series of triaxial loading and unloading creep tests on an iherzolite rock. The samples were cylindrical, having a diameter of 50 mm and a height of 100 mm in accordance with the suggestion from ISRM. Fifteen specimens divided into five groups have been tested. A servo-controlled rheological test machine was used. The axial deformation was measured by a linear variable displacement transducer (LVDT). The applied confining pressures were 0, 3, 6, 9 and 12 MPa, respectively. The creep tests on each sample were made at seven stress levels (or steps). The maximum loads under different confining pressures were approximately 80% of the peak values. The loading and unloading rates were 0.03 MPa/s. The test at each stress level lasted 90 hours while each unloading period lasted 20-30 hour. Table 2 shows the test program of Zhao et al. (2017).

Table 2. The triaxial compression creep test program of Zhao et al. (2017).

Groups	Confining pressure (MPa)	Deviatoric stress (MPa)						
		1st	2nd	3rd	4th	5th	6th	7th
1	0	4.07	10.1	16.2	22.3	28.4	34.5	40.6
2	3	4.84	12.1	19.3	26.5	33.7	41.2	48.4
3	6	5.37	14.8	22.1	29.5	36.9	44.2	53.1
4	9	6.30	15.8	25.2	34.6	44	53.4	63
5	12	7.85	19.6	31.4	43.2	55	66.8	78.5

In this study, the experimental results of Group 3 with the confining pressure of 6 MPa will be used to test the ability of description and prediction of the Burgers creep model. The required model parameters of Eq. 8 are obtained by applying the curve-fitting technique on the experimental results of the 4th stress level at a deviatoric stress of 22.3 MPa.

Table 3 shows the obtained model parameters for the Burgers creep model. The calibrated Burgers creep model is then used to predict the strain evolutions during the primary and secondary creep stages of Group 3 at other five deviatoric stress levels.

Figure 4 shows the comparisons between the three-dimensional Burgers creep model and some experimental results of Group 3 from Zhao et al. (2017). One sees that the model description for the 4th stress level is excellent while the model predictions for the other five stress levels are quite good by using the same material parameters given in Table 3. The Burgers creep model can thus be considered capable of describing and predicting the creep behavior of rocks under different deviatoric stress level.

Table 3. Material parameters obtained by applying the curve-fitting on the experimental results of Group 3 under the confining pressure of 6 MPa at the 4th stress level with the deviatoric stress of 22.3 MPa.

K_M (GPa)	G_M (GPa)	G_K (GPa)	η_M (10^3 GPa h)	η_K (GPa h)
5.75	3.62	7.39	9.33	21.65

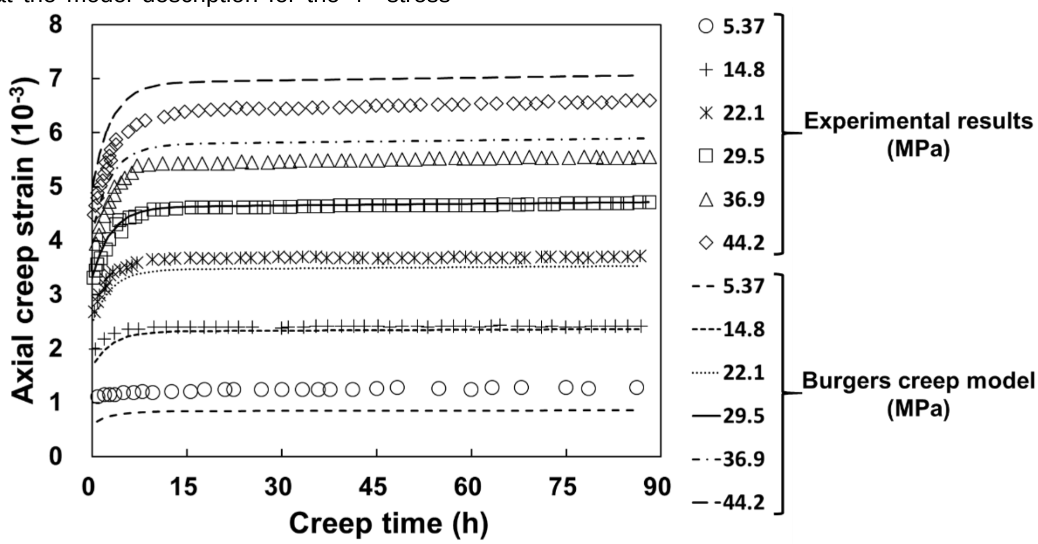

Figure 4. Strain evolutions measured with a cylindrical rock submitted to a confining stress of 6 MPa at different deviatoric stress levels (experimental data taken from Zhao et al. 2017); on the figure are plotted one description and five predictions of the experimental results using the Burger creep model with the model parameters shown in Table 3.

4 DISCUSSIONS

In this study, the ability of description and predictability of the Burgers creep model has been tested against experimental results available in the literature. The results showed that the Burgers creep model can be used to describe and predict the creep behavior of rocks under different stress conditions. However, the Burgers creep model is a visco-elastic model. It does not take into account the plastic deformation. In many case, rocks may exhibit partly irreversible deformation. In such case, a visco-elasto-plastic model is necessary to better represent the creep behavior of the rocks.

In the Burgers creep model (see Eq. 3 and Eq. 8), the strain rate of the secondary creep stage depends on the deviatoric stress, not on the confining pressure. Implicitly, the model consider a value of zero for the friction angle. More work is necessary to take into account a non-zero friction angle in the model.

The tertiary creep stage can be of critical importance to an infrastructure. The Burgers creep model is unable to describe or predict this stage. More work is necessary in the future on this aspect.

This study was based on limited experimental results taken from the literature. More work is required by considering more experimental results obtained on a wider range of rocks in the future.

Most theoretical and experimental studies have been devoted to intact rocks. More work is necessary on rock mass in the future

5 CONCLUSIONS

In this paper, the one and three dimensional Burgers creep models have been briefly recalled. Its ability of description and prediction of the creep responses has been tested against some experimental results available in the literature. The model parameters were first obtained through the application of curve fitting technique on a part of the experimental results. The Burgers creep model along with the obtained model parameters is then called calibrated model. The predictability of the calibrated model was then tested against other experimental results, which were not used to obtain the model parameters. In all cases, good agreements were obtained between the Burgers creep model and experimental results. The Burgers creep model can thus be considered as able to describe and predict the creep behavior of rocks under different deviatoric stress state conditions. Nevertheless, more works are necessary to overcome several limitations included in the Burger creep model.

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