Reliability-based design cost optimization of Laterally loaded drilled shafts considering spatial variability of clays

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ABSTRACT

A procedure for the reliability-based design cost optimization of laterally loaded drilled shafts considering spatial variability of soil properties is presented. The finite difference method implemented with *p-y* curves to predict the performance of laterally loaded drilled shafts. The spatial variability of soil parameters is modeled with random field combined with Monte Carlo simulation. For combinations of shaft diameter and shaft length, random finite difference modeling is conducted to compute the probability of serviceability failure (exceeding the limiting maximum lateral shaft deflection). Using recent bidding cost data, the reliability-based design cost optimization of laterally loaded drilled shafts demonstrates that although several combinations of shaft diameter/shaft length can meet the acceptable probability of failure, the final design decision can be readily determined by choosing the design with the minimum cost.

RÉSUMÉ

Cet article présenté l'optimisation des coûts de conception basée sur la fiabilité des puits forés chargés latéralement en tenant compte de la variabilité spatiale des propriétés du sol. La méthode des différences finies mise en œuvre avec les courbes p-y est adoptée pour prédire les performances des puits forés chargés latéralement. La variabilité spatiale des paramètres de sol est modélisée avec un champ aléatoire combiné à une simulation de Monte Carlo. Pour chaque combinaison candidate de diamètre et de longueur des puits, la modélisation par différences finies aléatoires est effectuée afin de calculer la probabilité de défaillance de service définie comme la probabilité de dépasser la déviation latérale maximale du puits. À l'aide des données récentes sur les coûts d'offres, l'optimisation des coûts de conception basée sur la fiabilité des puits forés chargés latéralement montre que, même si plusieurs combinaisons de diamètre et longueur du puits peuvent produire une probabilité acceptable de défaillance, la décision de conception finale peut être prise facilement en choisissant la conception avec le coût minimum.

1 INTRODUCTION

The use of laterally-loaded drilled shafts is very common in geotechnical engineering. In particular, the laterally loaded drilled shafts serve as foundations in practical applications in the form of single column support for bridges, foundation of overhead sign structures or transmission line, and abutment foundation (Reese 1984). The design of laterally-loaded drilled shafts is to ensure that the overall system can sustain the anticipated static and dynamic loads with stress levels in shafts and shaft displacements within the acceptable limits in the service life (Reese et al. 1984). In the past decades, significant contribution to the design of laterally loaded drilled shafts has been reported. For example, Hansen (1961) and Davidson et al. (1982) developed undrained models for lateral soil stress distribution. Broms (1964a, 1964b) investigated the lateral resistance of piles in both cohesive and cohesionless soils and the findings were referred as Broms method. Since 1950s, Reese and his colleagues have worked on laterally loaded piles and drilled shafts under various site conditions. Some popular finite difference method-based computer programs, e.g., COM624G, was developed for analyzing laterally loaded shafts using the *p-y* method (Reese 1984). Carter and Kulhawy (1992) worked on laterally loaded shafts socketed into the rock and studied the load displacement relations using the finite element technique and developed a close-form solution to the response behavior. Reese (1997) developed an extended *p-y* method for the analysis of laterally loaded piles in weak rock.

It is known that there has been a transition from deterministic geotechnical design to reliability-based geotechnical design in the past decades. As such, numerous efforts were dedicated to the probabilistic assessment of laterally loaded drilled shafts. For instance, Phoon and Kulhawy (2005) calibrated the model bias factors for nine different classical models for laterally loaded drilled shafts. Puła and Różański (2012) conducted the reliability analysis of rigid piles subjected to lateral loads using response surface method, and correlated the factor of safety with the probability of failure. Kozubal et al. (2013) also adopted response surface to study the effect of uncertain soil properties on the evaluation of pile reliability under lateral loads. In addition, the calibration work by federal agencies such as Federal Highway Administration (FHWA) and American Association of State Highway and Transportation Officials (AASHTO) also provided the resistance factors under the Load and Resistance Factor Design (LRFD) framework for laterally loaded pile and drilled shaft design. AASHTO (2012) provided the resistance factor (i.e., 1.0) for the strength limit design of laterally loaded drilled shafts, while FHWA (2010) provided the resistance factors for both strength limit design (i.e., 0.67 for p-y method and 0.40 for Broms simplified method) and serviceability limit design (i.e., 1.00).

In the reliably-based geotechnical design, it is important to consider the spatial variability of soil property. Recent research on other geotechnical problems indicates that neglecting the spatial variability can lead to either overestimation or underestimation of the predicted failure probability (e.g, Luo et al. 2011). Fan and Liang (2013) performed a reliability analysis of the performance of a laterally loaded drilled shaft considering one-dimensional (1D) spatial variability of undrained shear strength, and it was shown that the spatial variability has apparent influence on the failure probability. However, there is still a lack of research on the reliability-based design cost optimization of laterally drilled shafts to address the spatial effect for engineering practice.

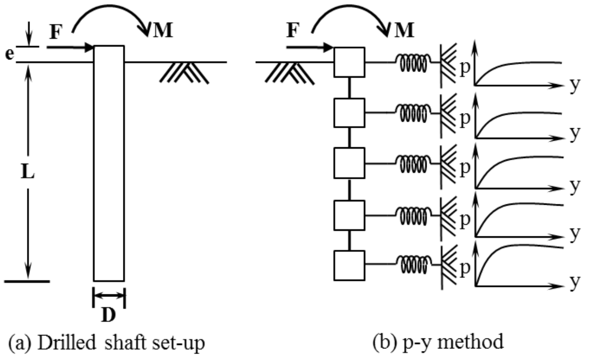
In this paper, a procedure for the reliability-based design cost optimization of laterally loaded drilled shaft is presented. The finite difference model implemented with *p-y* curves is adopted to predict the maximum lateral shaft deflection. The vertical spatial effect of soil property is modeled by random field theory. Probabilistic assessment is conducted by using Monte Carlo simulation for a series of candidate shaft designs. The candidate designs are first screened by choosing those with the acceptable failure probability. Based on the recent bidding cost data from the Ohio Department of Transportation, the final drilled shaft design is chosen as the one with the minimum cost.

2 GEOTECHNICAL MODEL FOR LATERALLY LOADED DRILLED SHAFTS

The geotechnical design of laterally-loaded drilled shafts with the consideration of spatial variability of soil properties is a complicated soil-structure interaction problem. There are several methods for modeling the soil-structure interactions of laterally-loaded drilled shafts such as Broms method (1964a, 1964b), *p-y* method implemented with finite different analysis (Reese et al. 1984) and finite element method (Carter and Kulhawy 1992, Yang 2006). The *p-y* curves (Reese 1977) are widely accepted by the practitioners due to the advantages in the analysis of laterally-loaded drilled shafts, including but not limited to the capability of modeling multiple soil layers, non-linearity in soil-structure interactions, and non-uniform flexural rigidity and soil property along the shafts (e.g., Fan and Liang 2013). Using *p-y* curves, the governing differential equation for soil-structure interaction is formulated as follows (Reese 1977):

$$EI\frac{d^4y}{dz^4}+Q\frac{d^2y}{dz^2}-p=0 \qquad [1]$$

where E is the Young's modulus, I is the moment of inertia, y is the lateral deflection at point "z" along the shaft, Q is the axial load acting on the shaft head and p is the lateral soil reaction per unit length of the shaft (Reese 1977). For this method, the pile is axially subdivided into a set of elements, and the lateral soil-pile interaction of each pile element is described by a set of independent *p-y* relationship, as shown in Figure. 1(b). The *p-y* curves are the functions of parameters including undrained shear strength (s_u), effective unit weight (γ'), and strain corresponding to 50% of the maximum principle stress difference (ε_{50}). The solution using *p-y* curves is realized using finite difference method through iteration. By solving Eq. 1 through iteration, the profiles for the shaft, such as lateral deflection, shear force and bending moment can be obtained.



(a) Drilled shaft set-up (b) p-y method

Figure 1. Finite difference model implemented with *p-y* curves for a laterally loaded drilled shaft.

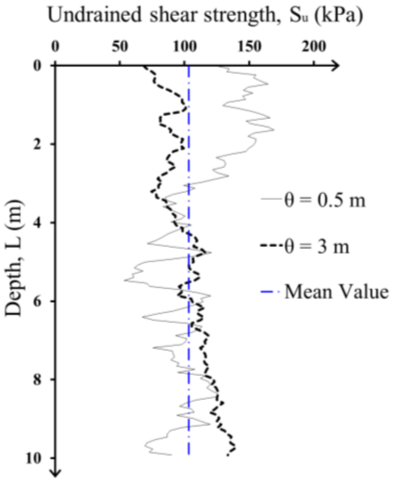


Figure 2. Examples of 1D random fields with different correlation lengths.

3 RANDOM FIELD MODELING OF SPATIAL VARIABILITY OF SOIL PROPERTY

Similar to other geotechnical design problems, the design of laterally loaded drilled shafts also involves various uncertainties. In particular, the inherent variability of soils can have significant influence on the geotechnical analysis of laterally loaded piles. The effect of spatial variation of soil properties is generally dealt with random field modeling. The spatial soil parameter such as the undrained shear strength (s_u) is described by the mean, coefficient of variation (COV) and correlation length (θ) under lognormal distribution. This study adopts the local averaging subdivision (LAS) technique (Fenton and Griffiths 2008) to generate the one-dimensional (1D) random field of undrained shear strength of clays. For a given mean value of 103.4 kPa and COV of 40% for S_u, Figure 2 shows the generated 1D random field of S_u for two levels of θ: 0.5 m and 3 m. As shown in Figure 2, smaller θ indicates larger spatial variation in the field.

Next, the generated random field such as the one shown in Figure. 2 can be mapped onto the finite difference model for a laterally loaded drilled shaft as in Figure 1. By solving a set of Eq. 1 through iteration, the solutions to shaft lateral deflection, shear force and bending moment can be obtained. It is noted that the generated spatial variation of undrained shear strength in Figure 2 is just one realization of various spatial variations in the field. In order to address all possible spatial variations, Monte Carlo simulation (MCS) is generally combined with random field modeling. After a sufficient number of MCS is performed (N), the resulting distribution such as the shaft head lateral deflection can be statistically analyzed. Given a specified maximum acceptable shaft deflection y_a, the probability of serviceability failure of the laterally loaded shaft is determined with:

$$P_f \approx \frac{1}{N}\sum_{i=1}^{N} I_i\left(y \geq y_a\right) \qquad [2]$$

where N = sample size, I[.] = Indicator function, and y_a = limiting allowable displacement as a performance criterion.

4 DESIGN EXAMPLE

Figure 1(a) shows the design example of a laterally loaded drilled shaft in this study. The detailed parameters of this design example are summarized in Table 1. This drilled shaft is to be constructed in a site with very stiff clays. The design horizontal load is 111.2 kN and the bending moment applied at the top of drilled shaft is 677.9 kN·m. The undrained shear strength of the stiff clays is 103.4 kPa and the unit weight is 19.0 kN/m^3. The strain corresponding to 50% maximum principle stress difference is 0.005. The p-y curves used in this study follow those for stiff clays proposed by Reese et al. (1975).

The laterally loaded drilled shaft design involves shaft structural design and geotechnical design, respectively. The geotechnical design typically consists of the ultimate limit state (ULS) design and serviceability limit state (SLS) design. Considering the geotechnical performance is a key index, this study only focuses on the SLS design by meeting the acceptable maximum lateral shaft deflection. The AASHTO (2012) specified that the maximum allowable deflection of the topmost point of shaft is 12.7 mm (i.e., half an inch), which is used in this study.

Table 1. Design input parameters in drilled shaft design.

Design input parameters	Notations	Parameter values	
		Mean	Coefficient of variation
Lateral load on shaft head	F	111.2 kN	0.001
Moment on shaft head	M	677.9 kN·m	-
Angle of internal friction of soil	φ'	0	-
Undrained shear strength	s_u	103.4 kPa	0.40^1
Unit weight of soil	γ'	19 kN/m^3	0.04^1
Strain corresponding to 50% of the maximum principle stress difference	ε_{50}	0.005	0.20^1
Maximum allowable lateral deflection at the shaft head	y_a	12.7 mm^2	-
Compressive strength of concrete	f'_c	31,027.5 kPa	-
Yield strength of reinforcing steel	f_y	413,700 kPa	-
Elastic modulus of steel	E_s	200 GPa	-
Number of elements in shaft	t	100	-

^{1}COV from Fan and Liang (2013); ^{2}Recommendation by AASHTO (2012)

The geotechnical serviceability design of laterally loaded drilled shafts refers to the determination of shaft diameter and shaft length that satisfy the maximum probability of failure requirement and minimum cost requirement simultaneously. For a single drilled shaft, the suggested reliability index requirement is 3.0 and the equivalent maximum acceptable probability of failure (P_f) is 10^{-3} (AASHTO 2012).

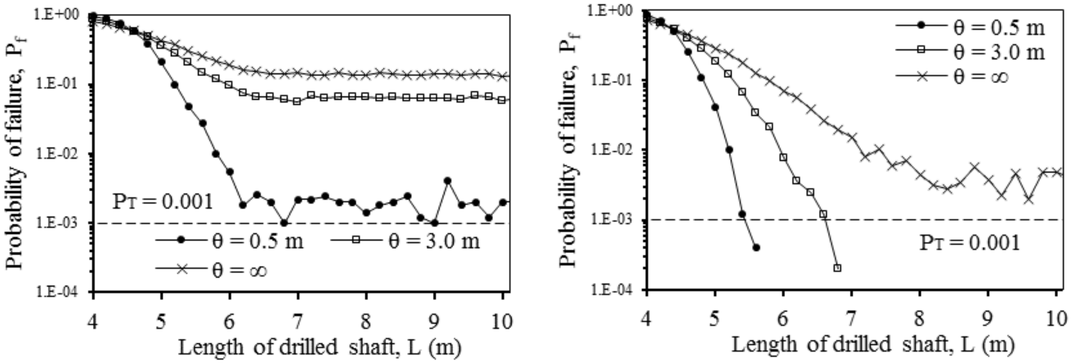
For drilled shafts, the diameter typically ranges from 0.6 m (2 ft) to 3.7 m (12 ft), and the shaft can be constructed up to a depth larger than 91 m (300 ft) (Garder et al. 2012). Each candidate design is a pair of shaft diameter and shaft length. The shaft diameter is generally dominated by the auger size, and the typical auger diameters include but are not limited to the following values: 1.06 m (42 in.), 1.2 m (48 in.), 1.37 m (54 in.), 1.5 m (60 in.), 1.67 m (66 in.) and 1.8 m (72 in.). Although other auger sizes can be used, this study adopts these candidate shaft diameters in geotechnical SLS design. The candidate shaft length ranges from 2 m to 25 m with an increment of 0.2 m.

As shown in Figure 1(b), the shaft is subdivided into a set of elements for finite difference method solution. In this study, the length of each element depends on the total shaft length, with 100 equally subdivided elements in the vertical direction. The generated random field of S_u is mapped onto the shaft finite difference model, followed by

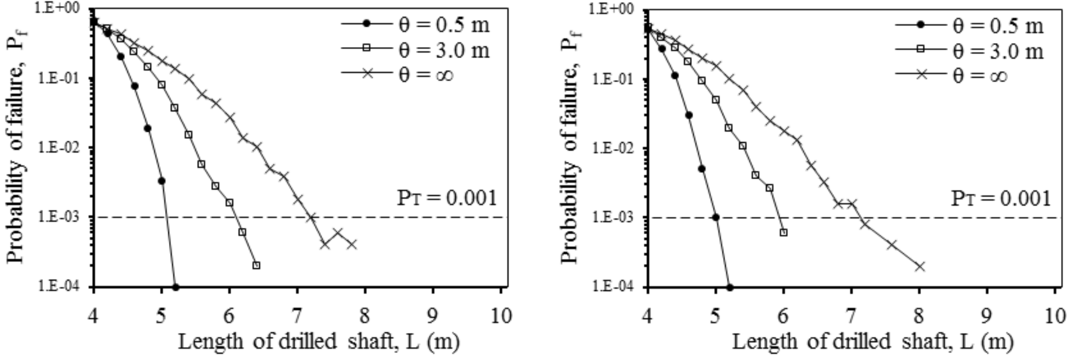
MCS. In this study, the S_u, γ' and ε_{50} are assumed to follow lognormal distributions. The coefficients of variation of the random variables are shown in Table 1. For simplicity, the longitudinal reinforcement ratio is 1% for all candidate designs.

5 RESULTS OF RANDOM FIELD-BASED MONTE CARLO SIMULATION

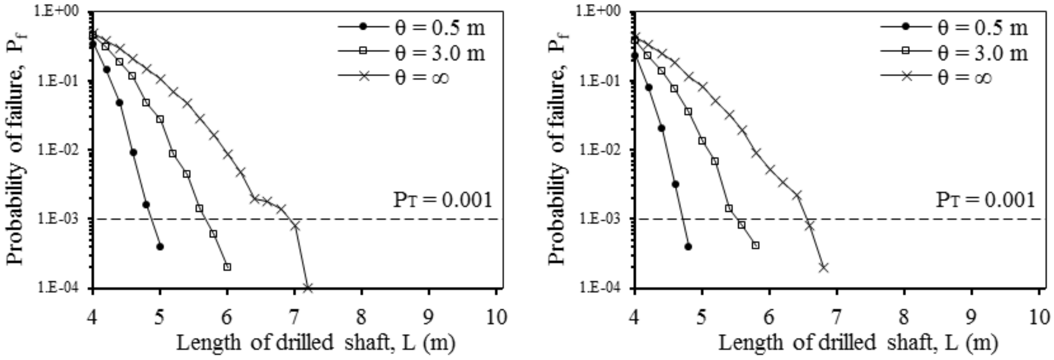
For each candidate design (i.e., a pair of shaft diameter and length, denoted by *D* and *L*, respectively), a total of 5000 MCS is performed. Three levels of spatial variability are simulated (θ = 0.5 m, 3 m, and ∞). The probability of exceeding the maximum acceptable shaft deflection (y_a = 12.7 mm) for each candidate design is computed and shown in Figure 3 for *D* = 1.06 m, 1.2 m, 1.37 m, 1.5 m, 1.67 m and 1.8 m, respectively. Figure 3 shows that larger θ leads to higher probability of serviceability failure (P_f) for the same shaft diameter and shaft length. It indicates that higher spatial variability of soil parameter S_u (i.e., smaller θ) has larger spatial averaging effect, which causes smaller variation in the predicted maximum lateral shaft deflection.



(a) *D* = 1.06 m (b) *D* = 1.20 m



(c) *D* = 1.37 m (d) *D* = 1.50 m



(e) *D* = 1.67 m (f) *D* = 1.80 m

Figure 3. Effect of scale of fluctuation on probability of serviceability failure.

Figure 3(a) shows that for *D* = 1.06 m, all candidate designs fail to meet the maximum acceptable probability of serviceability failure (P_T) of 0.001. For these scenarios, further increase the design shaft length *L* has no contribution to the design reliability. Figure 3(a) indicates that for the smallest shaft diameter considered in this study, the performance criterion is hardly satisfied no matter how long the shaft is designed. For D = 1.20 m, figure 3(b) shows that no design satisfies the required P_T for $\theta = \infty$, while for θ = 0.5 m and 3 m the design shaft lengths that can meet the required P_T are larger than or equal to 5.6 m and 6.8 m, respectively.

If a larger shaft diameter D is adopted, a design that satisfies the required P_T can be reached. For example, Figure 3(c) shows that the minimum design shaft lengths L for θ = 0.5 m, 3 m, and ∞ are 5.2 m, 6.2 m and 7.4 m, respectively. As a summary, all the designed D and L that meet the P_T requirement for three levels of θ are listed in the first three columns in Table 2. Therefore, the candidate designs are screened by the reliability-based SLS design of laterally loaded drilled shafts through meeting the maximum acceptable probability of serviceability failure (P_T).

Table 2. Final design based on minimum cost requirement.

θ (m)	D (m)	Minimum required L (m)	Unit cost[1] (USD/ 0.3 m in length)	Total cost (USD)	Final design
0.5	1.06	-[2]	405.12	-	D = 1.67 m L = 5.0 m
	1.20	5.6	613.75	11,456.7	
	1.37	5.2	641.60	11,121.1	
	1.50	5.2	692.86	12,009.6	
	1.67	5.0	658.40	10,973.3	
3.0	1.06	-[2]	405.12	-	D = 1.67 m L = 5.8 m
	1.20	6.8	613.75	13,911.7	
	1.37	6.2	641.60	13,259.7	
	1.50	6.0	692.86	13,857.2	
	1.67	5.8	658.40	12,729.1	
∞	1.06	-[2]	405.12	-	D = 1.67 m L = 7.0 m
	1.20	-[2]	613.75	-	
	1.37	7.4	641.60	15,826.1	
	1.50	7.2	692.86	16,628.6	
	1.67	7.0	658.40	15,362.7	

[1]Unit cost data from Ohio Department of Transportation (ODOT 2017); [2]unsuitable design.

6 RELIABILITY-BASED DESIGN COST OPTIMIZATION

For a given level of spatial variability, several candidate shaft designs, expressed by the auger-dependent design shaft diameter and the minimum required design shaft length, can be determined by meeting the maximum acceptable probability of serviceability failure (P_T). Next, the final design is reached by selecting the design with the least cost. In this study, for demonstration purpose, the average awarded cost for drilled shaft project contracts of Ohio Department of Transportation (ODOT) is adopted in the cost analysis. The most recent data of unit cost for each shaft diameter are shown in Table 2 (ODOT 2017).

Based on the unit cost information for each shaft diameter, the total cost for each candidate design screened by the previous reliability analysis is estimated and shown in Table 2. For this design example, it is illustrated in Table 2 that the diameter D = 1.67 m is most economical for all levels of spatial variability of undrained shear strength. The minimum required shaft lengths L for θ = 0.5 m, 3 m, and ∞ are 5.0 m, 5.8 m and 7.0 m, respectively. It is concluded that higher level of spatial variability of undrained shear strength requires smaller drilled shaft length, indicating that neglecting spatial effect may lead to an over design. It should be noted that in this design example the unit cost information from ODOT (2017) is adopted in the demonstration, other local cost information can be readily implemented in this reliability-based design cost optimization procedure for laterally loaded drilled shafts.

7 CONCLUSION

In this paper, a procedure for the reliability-based design cost optimization procedure for laterally loaded drilled shafts considering spatial variability of soil properties is presented. The performance of laterally loaded drilled shafts is evaluated with finite difference method implemented with *p-y* curves. Random field theory combined with Monte Carlo simulation is adopted to model the spatial variability of undrained shear strength for clays. The effect of spatial variability on the reliability-based design cost optimization is demonstrated through a design example. It is shown that smaller scale of fluctuation leads to lower probability of serviceability failure for the same shaft diameter and shaft length.

The most recent bidding cost data from the Ohio Department of Transportation is adopted to demonstrate the reliability-based design cost optimization procedure for laterally loaded drilled shafts. First, the candidate designs are screened by the maximum acceptable probability of serviceability failure. Next, the final design decision is reached by choosing the design with the minimum cost.-

In this study, the spatial variability of the undrained shear strength of soil is only considered. It should be noted that the spatial variability of other soil parameters (e.g., soil unit weight) can also be readily included in this procedure for the analysis and design of laterally loaded drilled shafts.

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