Correlations between Low and High Strain Stiffnesses of few Natural and Artificial Cohesive Soils

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ABSTRACT
Piezoelectric ring actuator-oedometer setup (PRA) was used to measure the shear wave velocity (V_s) and hence the dynamic shear modulus (G_{max}) of few natural/artificial cohesive soils at various applied pressures. The oedometric stiffness (from 1D consolidation data) were determined simultaneously with V_s measurements. The soils were subjected to isotropically consolidated undrained (CIU) triaxial tests to evaluate their undrained static stiffness at different confining pressures. Low stain stiffnesses obtained from PRA setup were correlated to the high strain stiffnesses obtained from oedometer and CIU tests. Some practical correlations (stiffness ratios) between low and high strain stiffnesses for the tested soils were proposed which can be used to estimate dynamic stiffness for similar soils.

RÉSUMÉ
La configuration actionneur-oedomètre à anneau piézoélectrique (PRA) a été utilisé pour mesurer la vitesse des ondes de cisaillement (V_s) et donc le module de cisaillement dynamique (G_{max}) de quelques sols cohérents naturels / artificiels à diverses pressions appliquées. La rigidité oedométrique (à partir des données de consolidation 1D) a été déterminée simultanément avec les mesures de V_s. Les sols ont été soumis à des essais triaxiaux non drainés à consolidation isotrope (CIU) afin d'évaluer leur rigidité statique non drainée à différentes pressions de confinement. Les faibles raideurs aux taches obtenues à partir de la configuration PRA ont été corrélées aux fortes raideurs de contrainte obtenues lors des tests oedometer et CIU. Certaines corrélations pratiques (rapports de rigidité) entre les rigidités de déformation faible et élevée pour les sols testés ont été proposées, qui peuvent être utilisées pour estimer la rigidité dynamique de sols similaires.

1 INTRODUCTION

Low strain shear modulus or dynamic shear modulus (G_o or G_{max} or $G_{dynamic}$) is a fundamental soil property, which depicts the deformation behavior of soils under small-strain conditions. It is an essential soil property for dynamic soil-structure interaction analyses that involve earthquakes, explosions, machine or traffic vibrations as well as small-strain cyclic loading situations such as those caused by wind or wave loading (Dyvik and Madshus, 1985). At very low strains (typically less than 0.001%), G_{max} can be related to V_s, i.e.

$$G_{max} = \rho V_s^2 \quad [1]$$

Where: ρ is soil's bulk density and V_s is the shear wave velocity.

Different field and laboratory tests are available to measure the dynamic soil properties. The laboratory tests employed to measure G_{max} include: resonant column (RC) test (Khan et al., 2008), ultrasonic pulse test (Khan et al., 2011) and the piezoelectric bender element (BE) test (Dyvik and Madshus, 1985) etc.

Many researchers including Gamal El-Dean (2007) showed that BE tests can lead to unreliable and erroneous results. The outcome of these such researches has motivated adopting a new laboratory technique that was pioneered at University of Sherbrooke [Université de Sherbrooke] for measurement of soil shear wave velocity (e.g. Gamal El Dean, 2007, Ethier 2009 etc.). This technique involved developing piezoelectric ring actuators (PRA) device incorporated in an oedometer equipment for measuring the soil shear wave velocity hence low strain shear modulus. A PRA device was developed for the oedometer test at The University of Western Ontario (Ahmad, 2015) with modifications to the original design proposed by Gamal El Dean (2007). The developed PRA setup was used to measure the low strain stiffnesses of the tested soils.

2 TESTED SOILS

Four different cohesive soils were tested in this study including three natural undisturbed soils and an artificial clay prepared by mixing kaolin (K) and silt (S).

This artificial clay, referred to as K-S soil, is widely used in laboratory and centrifuge studies (Ahmad et al., 2015). It was prepared by mixing kaolin and silt in equal proportions by weight, and was preconsolidated to a 300 kPa pressure.

The three natural soils were Sombra, Ottawa and Brantford clay. Sombra and Ottawa clay are soft cohesive soils while Brantford is a stiff silty clay soil. The three natural soils are normally/lightly overconsolidated soils (overconsolidation ratio between 1 and 2). The index properties of the tested soils are given in Table 1.

3 V_S (G_{MAX}) MEASUREMENTS USING DEVELOPED PRA OEDOMETER SETUP

The tested oedometer/PRA soil specimens were trimmed from undisturbed Shelby tube samples. The oedometer/PRA specimens were 70 mm in diameter and 25 mm in height. The specimens were tested under different vertical pressures (σ_v') typically ranging from 25 kPa up to 500 kPa in nine (9) increments. Four specimens of each soil type were tested in oedometer for 1D-consoldiation and simultaneously for V_s measurements. The V_s measurements were made at each load increment after the primary consolidation using several input sine wave frequencies. Knowing the sample height and estimating the travel time t_s, V_s can be calculated as follows:

$$V_s = \frac{Sample\ Height}{Travel\ Time\ (t_s)} \qquad [2]$$

Table 1. Index Properties of tested soils

Index Properties (%)	Sombra	Ottawa	K-S	Brantford
Avg. Initial Void Ratio, e_0	1.07	0.96	0.85	0.51
Avg. Natural Water Content, W_N (%)	45.0	34.5	30.0	18.2
Specific Gravity, G_s	2.70	2.74	2.63	2.70
Unit Weight, γ_t (kN/m^3)	18.5	18.4	18.2	21.0
Liquid Limit (%)	51	32	38	25
Plasticity Index (%)	27	17	16	9

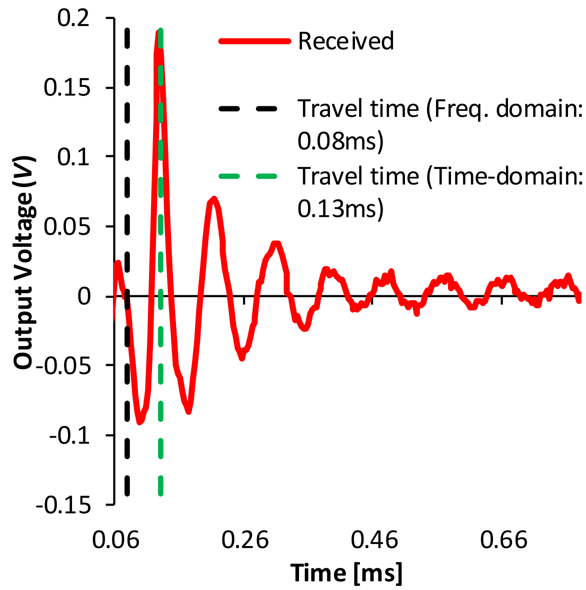

Figure 1. Time (cross-correlation) & frequency domain (group velocity method) arrival times

A typical output signal from the developed oedometric PRA setup is shown in Figure 1. Output signals were analyzed in time and frequency domains to obtain travel time (t_s). Cross-correlation technique (time domain analysis) was found to significantly overestimate t_s and hence underestimate V_s and, therefore, was not pursued further.

At a given effective pressure (σ'), knowing t_s (obtained from frequency domain analysis as shown in Figure 1) and height of the specimen, V_s was determined using Equation 2.

With the calculated V_s and knowing ρ at given σ', G_{max} was caluclated using Equation 1. Using the calculated G_{max} and an assumed dynamic Poisson's ratio (ν) of 0.42, Equation 3 was used to estimate low strain Elastic Modulus $E_{dynmaic}$.

$$E_{dynamic} = G_{max}\ 2(1+\nu) \qquad [3]$$

The V_s - σ' power relationship (Cha et al. 2014) in its general form is given by:

$$V_s = \alpha\sigma'^{\beta} \qquad [4]$$

α and β in Equation 4 are curve fitting parameters.

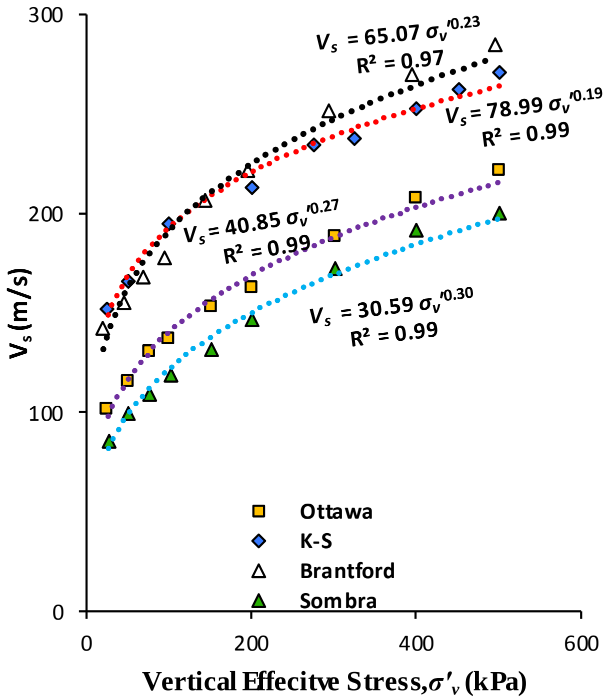

Figure 2. V_s – σ_v' models for the tested soils

Figure 2 presents the V_s – σ_v' models for all the tested soils covering the full applied pressure range (i.e. both overconsolidated and normally consolidated phases).

Figure 2 shows that Sombra clay exhibited the highest stress exponent, β (0.3) and lowest constant, α (30.6) among the tested soils. This may be attributed to its soft consistency, high natural water content, high e_o, high plasticity and being normally consolidated at most of the applied pressure increments. Sombra was also the most compressible (highest compression index) among the tested soils.

The Brantford soil exhibited the highest α (65) and lowest β (0.23) among all the natural soils, which represents the typical behavior of stiff silty lightly overconsolidated clay soil with low plasticity (Cha et al. 2014). V_s – σ_v' behaviour of Ottawa clay is in between that of the Sombra and Brantford clays. Plasticity index and compression index of Ottawa clay were similarly in between those measured for Sombra and Brantford clays.

The K-S soil displayed overconsolidated behavior during most of the applied pressure (i.e. up to 300 kPa), and displayed normally consolidated soil behavior afterwards. K-S being overconsolidated at most of the applied pressure increments showed a relatively high α and low β values as compared to other tested soils, which is attributed primarily to the effect of its overconsolidation and low to medium plasticity.

4 UNDRAINED ELASTIC STATIC STIFFNESS (E_{STATIC}) FROM STATIC TRIAXIAL TESTING

The tested triaxial samples were 50 mm in diameter and 100 mm in height. Undisturbed samples were used to prepare triaxial samples for all the natural soils. The samples of each soil type were tested under different isotropic confining pressures. Depending upon the availability of the samples, 3-4 CIU triaxial tests were conducted on specimens of each soil type. The strain rate during undrained shearing stage was usually kept around 0.0081 mm/min.

The slope of the initial/linear part of deviator stress (q) vs. axial strain (ε) curve obtained from CIU tests as shown in Figure 3 is defined here as an undrained static Elastic Modulus E_{static}.

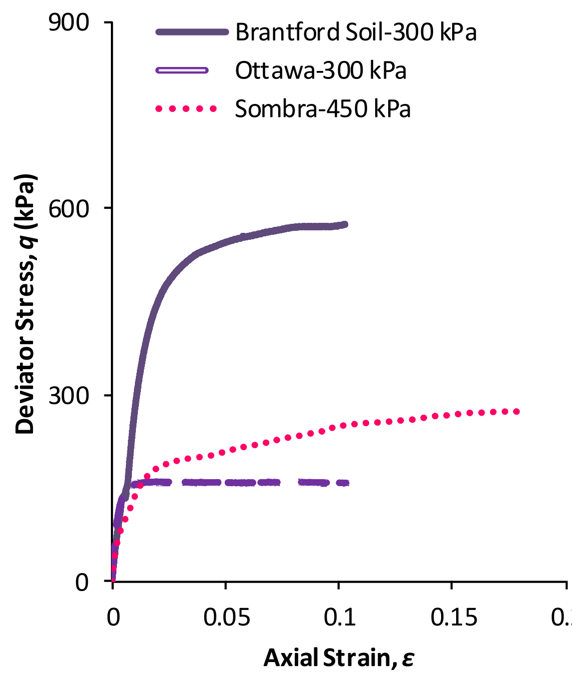

Figure 3. Stress-Strain Curves of the natural clays at different isotropic confining pressures

5 OEDOMETRIC STIFFNESS ($E_{OEDOMETER}$)

The soil's constrained modulus ($E_{oedometer}$) was determined from oedometer test measurements. The drained oedometric tangent constrained modulus for the normally consolidated loading (along the virgin compression line) was calculated from the oedometer tests results using the following relationship (Karlsrud and Hernandez-Martinez 2013; Janbu (1963, 1967)).

$$E_{oedometer} = \frac{2.3(1+e)}{C_c}\sigma'_v = m\sigma'_v \quad [5]$$

Where m is the modulus number, e is void ratio, C_c is compression index and σ'_v is the applied vertical effective stress. The modulus number, m = 2.3/$C.R$, where $C.R$ is the compression ratio and $C.R$ = C_c/ (1+e).

6 VERTICAL AND MEAN EFFECTIVE STRESS CONCEPTS

It is important to mention that the stiffness measurements from triaxial tests (E_{static}) were made at isotropic confining conditions while $E_{oedometer}$ and G_{max} measurements were made under vertical effective stress or anisotropic or K_o conditions. The vertical effective stresses or anisotropic or K_o conditions were converted into to isotropic (mean effective stress conditions) using K_o - OCR relationships (Jaky (1948) and Mayne & Kulhawy (1982)) and soil friction angles obtained from triaxial tests.

After the conversation mentioned above, G_{max} (and hence $E_{dynamic}$) as well as $E_{oedometer}$ values were found at each applied mean effective pressure at which E_{static} measurements were made.

7 KEY RESULTS

7.1 Effect of Confining Pressure on Stiffness Ratio (G_{max}/G_{static})

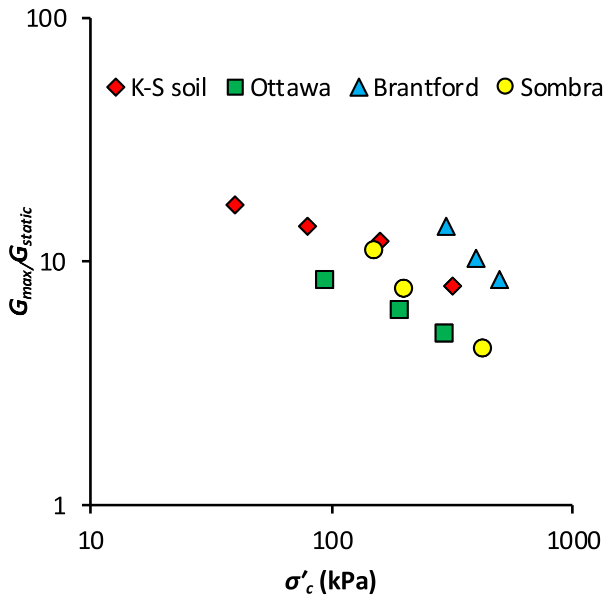

Figure 4. Variation of ratio of dynamic and static shear moduli with confining pressure

Figure 4 displays the variation of G_{max}/G_{static} with confining pressure (σ'_c). As discussed above, G_{max} is obtained from PRA and G_{static} is obtained from Figure 3 using the linear portion of stress strain curve and undrained Poisson ratio of 0.5. It can be noted from Figure 4 that G_{max}/G_{static} ranges from 5 to 13 for natural clay soils for the applied confining pressures and the ratio decreases as confining pressure increases. For example, G_{max}/G_{static} for Ottawa clay increased from 5 to 8 as the confining pressure decreased from 293 kPa to 93 kPa. The same figure shows that G_{max}/G_{static} varied from 8 to 13 for stiff lightly overconsolidated silty soil (Brantford) for the change in confining pressure from 500 kPa to 300 kPa while it increased from 5 to 13 for soft Sombra clay when the confining pressures decreased from 425 kPa to 150 kPa.

For K-S clay, G_{max}/G_{static} increased from 8 to 17 when the confining pressures decreased from 320 kPa to 40 kPa.

7.2 Stiffness Ratios, $E_{Dynamic}/E_{Oedometer}$, $E_{Static}/E_{Oedometer}$ and $E_{Dynamic}/E_{Static}$

Figure 5 to 8 were plotted for the tested soils which shows the correlations between the three moduli.

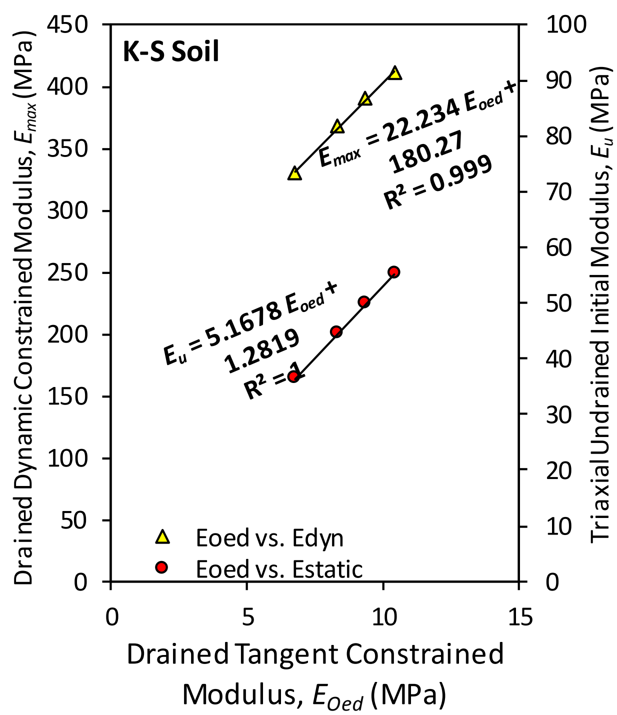

Figure 5. Correlation between various moduli for K-S Soil

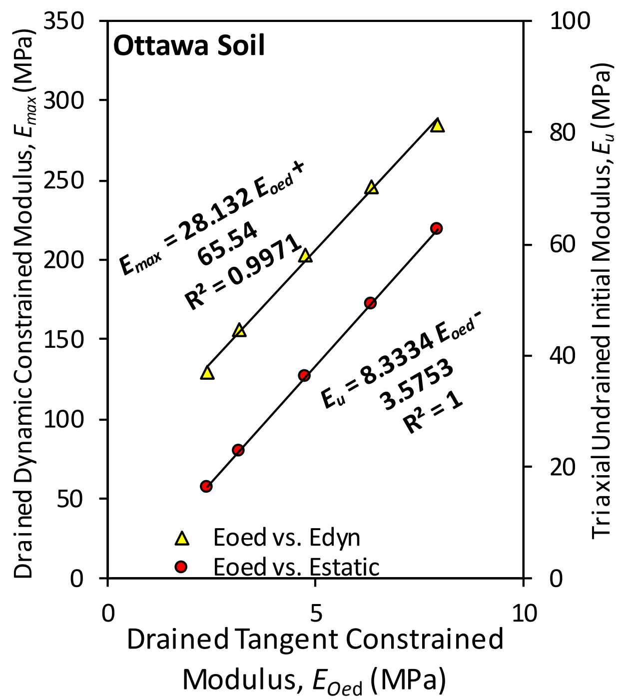

Figure 6. Correlations between various moduli for Ottawa Soil

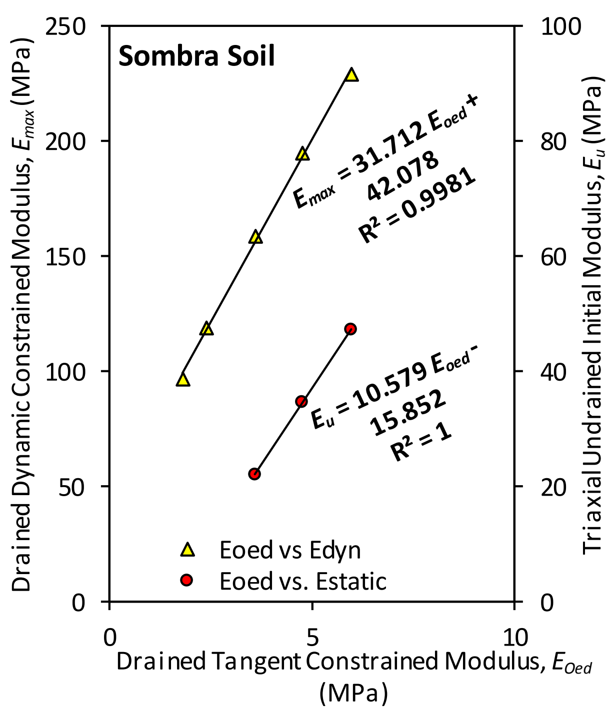

Figure 7. Correlations between various moduli for Sombra Soil

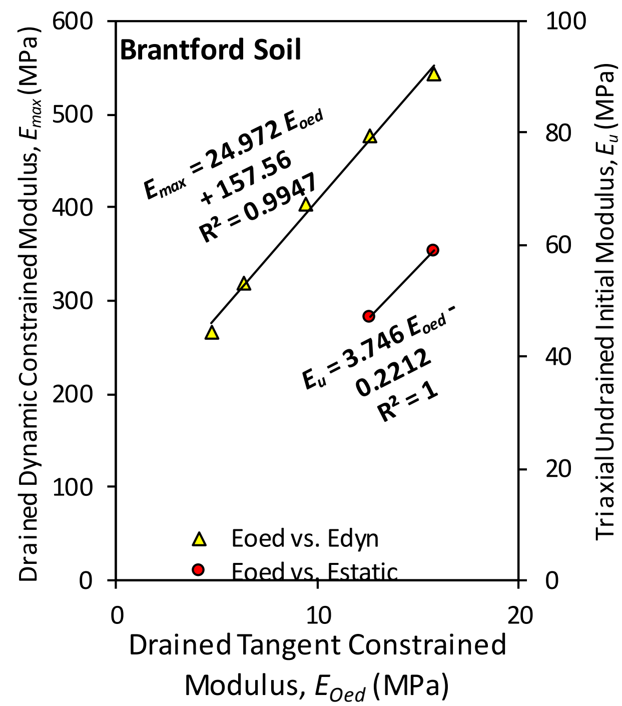

Figure 8. Correlations between various moduli for Brantford Soil

Figures 5 to 8 compares different measured/computed moduli, i.e. constrained oedometric) stiffness ($E_{oedometer}$), dynamic Elastic Modulus ($E_{dynamic}$) and undrained static modulus (E_{static}). Forcing the intercept to zero in Figures 5 to 8 results in a lower correlation R^2 especially for K-S soil but yields interesting results. For the cohesive soils tested within the normally consolidation loading, $E_{dynamic}/E_{oedometer} \approx 40$, $E_{static}/E_{oedometer} \approx 5$ and $E_{dynamic}/E_{static} \approx 8$.

8 CONCLUSIONS

Based on the obtained results, it was found that the ratio of dynamic and static stiffness decreases with increase in applied/confining pressures. Dynamic stiffness is a key soil property and generally difficult to measure. For practical purposes and for the soils similar to the ones tested in this research and subjected to pressures similar to the ones applied in this current study, dynamic stiffness can approximated as 8 times the static stiffness and 40 times the oedometric stiffness.

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