Statistical Correlations between effective angle of internal friction (ϕ) and SPT- N value for cohesive glacial tills

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ABSTRACT:



This paper presents a statistical analysis of the correlation between the effective angle of internal friction (ϕ') and standard penetration test blow count (SPT-N) for cohesive glacial tills in the city of Toronto. The (ϕ') values were derived from the consolidated undrained triaxial test. This study is based on the results of a comprehensive geotechnical investigation for the Eglinton Crosstown Light Rail Transit (LRT) project in Toronto. This study focused primarily on the statistical correlations between (ϕ') and SPT-N value for cohesive glacial tills with different textures, such as silty clay, silty clay till and clayey silt till. In this paper, the correlation equations between SPT – (*N*) ₆₀, (ϕ') for cohesive glacial tills is suggested.

RÉSUMÉ:

Cet article présente une analyse statistique de la corrélation entre l'angle effectif de frottement interne (ϕ [']) et le nombre de coups par essai de pénétration standard (SPT-N) pour des tills glaciaires cohésifs dans la ville de Toronto. Les valeurs (ϕ [']) ont été dérivées du test triaxial non drainé consolidé. Cette étude s'appuie sur les résultats d'une enquête géotechnique complète réalisée dans le cadre du projet de train léger sur rail Eglinton Crosstown à Toronto. Cette étude a porté principalement sur les corrélations statistiques entre (ϕ [']) et la valeur de SPT-N pour des tills glaciaires cohésifs de différentes textures, tels que l'argile limoneuse, le till argileux limoneux et le till limoneux argileux. Dans cet article, les équations de corrélation entre les valeurs SPT - (N) 60 et (ϕ [']) sont suggérées pour les tills glaciaires cohésifs est suggérée.

1.0 INTRODUCTION

Statistical correlations between in-situ soil testing results have become growingly more and more poplar during the site investigations especially for being practical and economical. Hence, estimations of geotechnical parameters from in – situ test results hold a significant place in the geotechnical design practice. Keep that in mind, in this study also statistical correlation between standard penetration test (SPT) and consolidated undrained (CU) triaxial test was performed.

The SPT is a well-established method for soil investigation. As many forms of the test are in use worldwide, standardization is essential in order to facilitate the comparison of results from different investigations, even at the same site (Thorburm 1986). In this paper, SPT was performed in accordance with the ASTM D 1586 method. This means that the test was standardized using a 50 mm O.D. split spoon sampler, driven into the soil with a 64 kg weight having a free fall of 760 mm auto hammer was used exclusively on the project. The blows required to drive the split –barrel sampler a distance of 305 mm, after an initial penetration of 152 mm, is referred to as the SPT – N value. This method has been accepted internationally and is useful in field investigation.

The CU triaxial test is a common laboratory testing method widely used for obtaining shear strength parameters for a variety of soil types under undrained condition. It is a conventional laboratory test to determine the shear characteristics under undrained conditions and is applicable to field conditions where soils that have been fully consolidated under one set of stresses are subjected to a change in stress without time for further consolidation to take place (undrained condition), and the field stress conditions are similar to those in the test method. Therefore, it is widely used and much more common in geotechnical investigations. The CU triaxial test is performed in accordance with the ASTM D4767 -11 methods. This means that the specimen is allowed to consolidate under the confining pressure in the initial stage. In this consolidation stage σ_3 is the same as σ_1 . Then the axial load is applied without drainage. This stage of the test is also commonly referred to as load stage, compression stage or shearing to failure stage. During the CU test, the pore pressures are typically measured so that both total and effective stress strength parameters such as cohesion (C') and internal angle of

friction (ϕ ') can be determined using Mohr circle and stress path plots. In this study the effective angle of internal friction (ϕ ') is considered to correlate with SPT - N values. The typical triaxial cell is shown in Figure 1.



Figure 1 Schematic of a triaxial compression test

Normally CU triaxial tests are carried out for long term engineering problems such as slope failure, cut slope failure, earth dams and tunnel lining.

In this study, an attempt was made to develop correlations between SPT - N values with internal angle of friction (ϕ ') were performed for cohesive glacial tills based on the extensive site investigation program and laboratory test conducted for the Eglinton Crosstown LRT Project in the city of Toronto. As emphasized by Phoon and Kulhawy (1999), local correlations that are developed within a specific geologic setting generally are preferable to generalized global correlations because they are significantly more accurate.

2.0 LITERATURE REVIEW

The literature review was conducted on statistical correlation between SPT - N and internal angle of friction (ϕ ') values in this paper. Information available from specific research studies on statistical correlation between SPT - N and internal angle of friction (ϕ ') values are few, as only a few researchers have

studied for clay, silty clay and sand even rare for Toronto cohesive glacial tills. Such information, as it was considered very valuable, is presented in this section

2.1 The literature review on statistical correlation between SPT - N values and internal angle of friction (ϕ ')

Approximate ranges of (b) and corresponding SPT -N values for cohesionless soils proposed by Meyerhoff (1956) are given in Table1. Range of angle of friction of soil with SPT - N value has been given by Terzhagi and Peck (1967) along with soil condition representing various ranges of cohesion as shown in Table 2. Further Tolia (1971) suggested approximate empirical equation which can be used directly for predicting angle of internal friction (ϕ ') for sandy soils. After Tolia (1971), many studies have been done in this area to express the correlation equations between SPT - N values and soil friction angle (ϕ). Peck et al. (1974) suggested correlation between N and (ϕ') in the graphical form which was approximately by Wolff (1989). Schmertmann (1975) suggested correlation equation for friction angle with SPT – N_{60} . Shioi and Fukui (1982) are presented equations for different structures such as for roads and bridges (ϕ') = $\sqrt{N_{70}}$ + 15, for building (ϕ ') = 0.36 N_{70} + 27 and in general (ϕ ') = 4.5 N_{70} + 20. Jon w et al. (1989) suggested range of friction angle for well graded gravel, sandy gravel, with little or no fines 33 to 40 degree and for poorly graded sands, gravelly sands, with little or no fines 30 to 39 degree. Japan road association (1990) presented correlation equation for SPT - N >5 and $(\phi') \leq 45^{\circ}$. The values of effective friction angle (ϕ') observed by Carter, M. and Bentley, S. (1991) for fine soils fall in a wide range from 18[°] to 42[°]. Further they suggested effective friction angle (ϕ ') for compacted clays are provided in Table 3. Friction angle of granular soils has also been correlated to SPT - N values by Hatanaka and Uchida (1996). Obrzud, R and Truty, A. (2012) suggested specific value for soil friction angle for compacted clayey silt 25 degree. Salari et al. (2015) presented correlation equation for internal friction angle for well graded gravels with sand (ϕ ') = 0.474 SPT + 16.188 and clayey gravels with sand $(\phi') = 0.3556$ SPT + 20.703 based on SPT - N values.

Table 1 Approximate ranges of (ϕ ') and corresponding SPT - N for cohesionless soils (Meyerhoff 1956)

Soil packing	SPT - N values	Internal angle of friction (\u00f6') (°)
Very loose	< 4	< 30
Loose	4 - 10	30 - 35
Compact	10 - 30	35 - 40
Dense	30 - 50	40 - 45

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Verv dense	>50	> 45

Soil	SPT-N	Internal angle of
conditions	values	friction (\phi') (\circ)
Very poor	0 - 4	< 28
Poor	4 - 10	28 - 30
Fair	10 - 30	30 - 36
Good	30 - 50	36 - 41
Very good	> 50	> 41

Table 2 Approximate ranges of (ϕ') and corresponding SPT - N for soils (Terzhagi and Peck 1967)

Table 3 Specific values of (ϕ ') for compacted clays (Carter, M. and Bentley, S.1991)

Soil type	UCS class	Effective friction angle (\phi') (°)
Silty clays, sand – silt mix	SM	34
Clayey sands, sandy – clay mix	SC	31
Silts, and clayey silts	ML	32
Clays of low plasiticity	CL	28
Clayey silts	MH	25
Clays of high plasiticity	СН	19

2.2 The literature review on SPT - N correction

In the literature, most researchers express their concerns in regards to energy correction which was elaborated as follows. The energy delivered to the rods during a SPT expressed as a ratio of the theoretical free fall potential energy, can vary from 30% to 90% (Kovacs and Salomone 1982 and Robertson et al. 1983). Schmertmann and Palacios (1979) have shown that the SPT blow count is inversely proportional to the delivered energy. Kovacs et al. (1984), Seed et al. (1984) and Robertson et al. (1983) have recommended that the SPT - N value has to be corrected to an energy level of 60% (CFEM 2006). The SPT - N values corresponding to 60% efficiency are termed N_{60} . The practice in the United States/Canada the SPT - N value measured to an average energy ratio of 60% (ERR=60%) according to ASTM D1586-11 (2014). In this study energy ratio of 60% (ERR=60%) is adopted.

Currently, there is no such relationship available for cohesive glacial tills in the city of Toronto. This study is performed based on an extensive site investigation and laboratory tests conducted for the Eglinton Crosstown LRT project for the Toronto Transit Commission and Metrolinx.

3.0 ENGINEERING BACKGROUND

The site is situated along Eglinton Avenue from the existing Kennedy subway station in the east to the Mount Dennis station in the west, in Toronto, Ontario, Canada.

The Toronto area acquired at least three glacial and two interglacial periods from the published geological data (Karrow 1967 and Sharpe 1980). The geological history of the Toronto area has included several advances and retreats of glaciers of Illinoian and Wisconsinan ages (Karrow and White 1998). The glacial tills in this area were generally deposited during the early to late Wisconsinan period, the Sunnybrook, represented bv Seminary. Meadowcliffe, Newmarket and Halton tills (Sharpe 1999). The glacial till deposits in Toronto can be divided into low plasticity cohesive glacial tills (silty clay to clayey silt glacial till) and cohesionless glacial tills (sandy silt to silty sand glacial till) (Manzari et al. 2014). This kind of soil is derived due to the wearing away and entrainment of material as a result of the moving ice of a glacier. As shown in Figure 1, this type of soil can be described as high variability materials in both horizontal and vertical axis, and it normally contains complex non-linear stress-strain characteristics (Baker et al. 1998). In addition to that, the tills consist of a heterogeneous mixture of gravel, sand, silt and clay size particles in varying proportions. Cobbles and boulders are common in these deposits (Robert et al. 2011). However, the behaviour of glacial tills in southern Ontario is not fully understood.



Figure 1 Typical glacial till (Source-Mark Clark, (http://www.free-stockillustration.com)

The proposed Eglinton Crosstown LRT is approximately 33 km in length and located approximately 7 km north of Lake Ontario. There are 25 proposed stations along the alignment as shown in Figure 2.



Figure 2 Crosstown route map (<u>http://www.thecrosstown.ca/the-project</u>)

A series of laboratory and in-situ tests were conducted in advance at the stations above. The insitu tests included SPTs, FVSTs, pre-bored TEXAM PMT and seismic tests. The laboratory tests included density and moisture content measurements, grain size and hydrometer analysis, consistency (Atterberg) limit tests, consolidation tests, consolidated undrained and drained triaxial compression tests.

Based on these tests, the soil was classified as a glacial till which further classified as low plasticity cohesive glacial till and cohesionless glacial till according to the current version of TTC Geo-technical Standards (2014). In this area, the low plasticity cohesive glacial till mostly consists of the following soil types such as (i) silty clay till (ii) clayey silt till. The cohesionless glacial till mostly consists of following soil types such as (iii) sandy silt till (iv) silty sand till. The glacial tills are interbedded with silty clay, clayey silt, sandy silt, sand and silt and silty sand.

SPTs conducted near the CU tests at similar depths were selected to develop the relationship between SPT - N values and internal angle of friction (ϕ ') in this paper for the following stations such as Bathurst, Bermondsey, Chaplin, Laird, Victoria Park, West portal, Wynford, Yonge. The pair of readings (SPT - N and (ϕ ') for silty clay, silty clay till and clayey silt till was collected from these tests in this study.

Silty clay from the above stations contains 0 to 6% gravels, 0 to 30% sand, 18 to 82% silt and 13 to 80% clay size particles based on grain size analysis. The water contents are generally between 9 to 37% and unit weight is from $14.9 - 23.2 \text{ kg/m}^3$. Based on the Consistency (Atterberg) limits test the range of LL is 20 to 61%, PL is 9 to 28% and Pl is 4 to 33.

Silty clayey till from the above stations contains 0 to 19% gravels, 9 to 41% sand, 34 to 62% silt and 14 to 36% clay size particles based on grain size analysis. The water contents are generally between 6 to 28% and unit weight is from $14.9 - 23.4 \text{ kg/m}^3$. Based on the Consistency (Atterberg) limits test the range of LL is 17 to 32%, PL is 7 to 19% and PI is 7 to 14.

Clayey silt till from the above stations contains 0 to 13% gravels, 11 to 44% sand, 37 to 69% silt and 11 to 23% clay size particles based on grain size analysis. The water contents are generally between 6 to 28% and unit weight is from $22.1 - 23.1 \text{ kg/m}^3$. Based on the Consistency (Atterberg) limits test the range of LL is 15 to 23%, PL is 9 to 17% and Pl is 4 to 7.

Overall cohesive glacial till from the above stations contains 0 to 19% gravels, 0 to 44% sand, 18 to 82% silt and 11 to 80% clay size particles based on grain size analysis. The water contents are generally between 6 to 37% and unit weight is from 14.9 - 23.4 kg/m³. Based on the Consistency (Atterberg) limits test the range of LL is 15 to 61%, PL is 7 to 28% and Pl is 4 to 33.

4.0 CORRELATION BETWEEN SPT - N AND (ϕ ')

The statistical analysis is carried out in this paper to investigate the relationship between SPT - N value with (ϕ') . The first step is to collect the pairs of internal angle of friction (ϕ ') and SPT - N value at the same depths in the same boreholes. The field measured SPT - N values are corrected according to the CFEM (2006). Because of the variability in equipment and operating conditions, direct use of SPT - N values for geotechnical design is not recommended. As a result, many corrections shall be done on the field SPT - N values. Those corrections are rod length, borehole diameter, sampler, energy and overburden described in CFEM (2006). The practice in the Canada the SPT - N value measured to an average energy ratio of 60% (ERR=60%) according to ASTM D1586-11 (2014). In this study energy ratio of 60% (ERR=60%) is adopted. In case of cohesive glacial tills, overburden correction is not accommodated in this study. In these situations, the SPT - N became SPT - (N) 60.

After corrected the SPT - N, the pair of data were collected for both SPT - (N) $_{60}$ values and (ϕ ') for cohesive glacial tills. In order to analyze more accurately, the compiled data were filtered by using the following methodology:

- The data situated far from the trend line was discarded by visual inspection compared to other data.
- (2) The SPT's often reached refusal, i.e. blow count (N) values were greater than 50 for 300 mm or less increment when the SPT sampler hits a cobble or boulder within the glacial till. As a result, the SPT - N values were assigned values of more than 50. The SPT - N values greater than 50 were disregarded.

4.1 General Range of SPT - (*N*) $_{60}$ and (ϕ ') for cohesive glacial tills

The ranges of SPT - (N) $_{\rm 60}$ and ($\varphi^{\prime})$ values are determined for cohesive glacial tills of the data are

collected from in-situ tests. The ranges of (*N*) ₆₀ and (ϕ ') values of cohesive glacial tills are shown in Figure 3 and 4 and Table 4 respectively. The percentages (%) marked in Figure 3 and 4 represents most of the range values that belong to the thick portion of the range diagrams.



Figure 3 Range of SPT - (N) 60 values for cohesive glacial tills



Figure 4.Range of (ϕ') values for cohesive glacial tills

Table 4 Approximate range of SPT - (N) $_{60}$ and ($\varphi^{\prime})$ for cohesive glacial tills

Soil type	SPT - (<i>N</i>) 60	(ϕ') (°)
Silty clay	10 - 50	26 - 37
Silty clay till	8 - 33	31 - 37
Clayey silt till	11 – 15	33 - 34
All soil	8 - 50	26 - 37

4.2 Correlation between SPT- (N) $_{60}$ values and (ϕ ')

The correlation between SPT - (*N*) ₆₀ values and (ϕ ') has been plotted for a cohesive glacial till is shown in Figure 5. In this analysis, origin linear best fit line method used. The correlation functions and correlation coefficients are given in Table 5.



Figure 5 Correlation between (ϕ') vs SPT - (N) ₆₀ for cohesive glacial tills (Linear relationship)

Table 5.Summary of correlation between SPT - (N) $_{60}$ values and (ϕ ') for cohesive glacial tills

Soil type	Correlation equation (R ²)	
	(ϕ') (°)	
Silty clay	0.97 <i>(N)</i> 60 (1)	
Silty clay till	1.56 <i>(N)</i> 60 (1)	
Clayey silt till	2.42 (N) 60 (1)	
All soil	1.11 <i>(N)</i> 60 (1)	

5.0 DISCUSSIONS

There is limited information available about the correlation between SPT - (*N*) ₆₀ values and (ϕ ') for clayey soil, sparse for cohesive glacial tills. This paper presents a study on the correlation between SPT - (*N*) ₆₀ values and (ϕ ') for cohesive glacial tills in the city of Toronto.

5.1 Comparison between SPT - (N) $_{60}$ values and ($\varphi^{\prime})$ for cohesive glacial tills

The approximate correlation between SPT - (*N*) $_{60}$ and (ϕ') was plotted with the studied data was shown in Figure 6. In this plot, linear best fit regression line was plotted for the studied corrected and filtered data. For the preliminary estimation of the (ϕ') for the cohesive glacial tills, the (ϕ') can be estimated from the SPT - (*N*) $_{60}$ values using the following relationship:

$$(\phi')(\circ) = 1.11(N)_{60}$$
 R² = 1 [1]



Figure 6 Correlation between SPT - (*N*) $_{60}$ and (ϕ ') for cohesive glacial tills (Linear relationship)

The predicted (ϕ') values were calculated by using "Equation 1" and the measured (ϕ') and predicted (ϕ') graphs were presented in Figure 6.

In this comparison, there is good agreement with measured and predicted graphs ($R^2 = 1$). In addition the studied specific values of effective friction angles also have a good agreement with literature values.

6.0 CONCLUSION

In conclusion, the study was performed based on an intensive site investigation program conducted for the Eglinton Crosstown LRT Project in the city of Toronto. The data were collected from in-situ tests (SPT) and laboratory tests (CU triaxial test) analysed statically. In this study, the linear correlation equations between SPT - (*N*) ₆₀ values and (ϕ ') is established for cohesive glacial till. Further the ranges of SPT - (*N*) ₆₀ and (ϕ ') were suggested for cohesive glacial till in the city of Toronto

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