Conventional method to estimate settlement of plate subjected to bi-axial loading – Part I

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ABSTRACT
A plate bi-axially loaded presents general loading case of shallow foundations. Raft columns loads resultant does not lay on its center. Isolated footing subjected to moment and vertical load eccentric or concentric, property-line or interior footings are plate's subjected to bi-axial loadings. Not to mention if wind or seismic is considered. Even though, most conventional methods were developed for settlement of concentric loadings. This research presents novel simplified method for settlement prediction of rigid plate center and corners subjected to bi-axial loadings. The method works for rectangular plate rests on Mohr-Coulomb soil having any B/L ratio, starting from squares up to strips. Also it suits any E_S and μ_S values. This method uses one graph that contains two figures, alternatively two empirical equations. The generalization of this method is proved analytically. The used methodology and method to develop this technique was parametric study, finite element, Plaxis 3D, and first principals'.

RÉSUMÉ
Une plaque soumise à des charges bi-axiales présente le cas général des fondations superficielles dont la résultante des charges ne s'applique forcement au centre de radier. Les semelles isolées soumises à un moment et à une charge verticale sont également des plaques soumises à des actions bi-axiales, en plus de l'effet séismique. La plupart des méthodes conventionnelles de calcul de tassement ont été développées pour le cas de charge concentrique. Cette recherche présente une nouvelle méthode pour la prédiction de tassement au centre et aux coins d'une plaque, sous l'effet des charges bi-axiales. La méthode convient a une fondation rigide reposant sur un sol Mohr-Coulomb, ayant des modules E_S, μ_S et un rapport B/L variable. Le tassement prévu comparé aux solutions analytiques et numériques, montre une bonne concordance. Cette méthode est déduite par une étude paramétrique et numérique (Plaxis 3D). Le résultat est présenté par un graphique et deux équations.

1 INTRODUCTION

This paper consists of five headings. After this introduction, the paper starts with literature review on which the research question, hypothesis gap and theme have been originated. Then, a brief statement about the method and methodology stated. That is to say, numerical study, finite element applied by Plaxis 3D-foundation. A parametric study, well designed then performed to produce sets of qualitative data. Finally, the research results have been presented, discussed and analyzed.

A simplified, novel and general method has been suggested to calculate settlement of the corners and center points of bi-axial loaded plate. It is recommended to be used for settlement prediction as first approximation.

2 LITERATURE REVIEW

This item summarizes selected samples of previously published research oriented to topic. The reviewed references concentrated on three key words, settlement, eccentric and plate. Papers that focused on bearing capacity have been reviewed, kept in mind, and over-looked. For every reference, what was studied? Methodologies, then the main findings, have been stated. The reviewed references present an integration of recently published and pioneer's works. The literature body has been arranged methodologically, and ended-up by research question, hypothesis and theme.

2.1 Numerical Investigations: 2D and 3D

Salimath and Pender (2014) examined the stress underneath footing subjected to vertical load and moment applied at its centroid. A 3D-finite element modeling was carried out using PLAXIS 3D-2001. The footing is square rests on soil surface with contact interface element. The hardening model was used. The results indicated that the existing solutions of effective footing width concept for eccentrically loaded footing are incorrect. Also, a partial width of the footing experienced zero stress even when there is no actual contact between the soil and footing.

Enkhtur et al. (2013) work is an extension of Mayne and Poulos (1999) approach. The study improved the settlement influence and correction factors for foundations subjected to concentric loading of aspect ratios L/B > 2. Finite difference was used based on computer code FLAC3D-2006. Settlement influence factor and correction factor for foundation roughness of circular and rectangular foundations were proposed. A correction chart for rectangular foundation rigidity was offered. The proposed factors were verified by comparing the calculated settlements with the measured from two rafts. Generally, the method of Mayne and Poulos (1999) can be used for estimating settlement of foundations up to L/B = 2. For L/B > 2, the extended approach is a better alternative.

Zedzn and Maulood (2017) presented a study to investigate the behaviour of square shape footing subjected to eccentric inclined loading with respect to the bearing capacity, settlement and horizontal displacement.

A 3D finite element analysis is used. Mohr-Coulomb criterion modeled the soil and rigid foundations were adopted. 15-Node wedge elements were used to model the soil and 5-Node elements with three degree of freedom were used to model the plate. The program PLAXIS 3D TUNNEL has been used. The results of analysis were presented in the form of pressure-settlement and pressure-horizontal displacement. Non-dimensional correlations have been developed for predicting the values of settlement and horizontal displacement of eccentrically loaded foundations.

2.2 Experimental Investigations and Modeling

Patra et. al. (2013) conducted a number of laboratory model tests to study the bearing capacity of strip foundation subjected to vertical and inclined eccentric loads. Based on some of settlement results, an empirical procedure has been developed to estimate the average settlement of the foundation while being subjected to an average allowable eccentric load per unit area, where the applied load is vertical. The empirical relationships presented are for embedment ratio (D_f/B) varying from zero to 1.0 and the eccentricity ratio (e_x/B) varying from zero to 0.15. The settlement calculated by using this approach is an approximate, first estimation.

Braim et. al. (2016) investigate the effect of load eccentricity on settlement of strip footing rested on sandy soil. A series of small-scale 1*g* physical modeling tests was carried out. The foundation soil was medium dry sand. A rigid plate was used to imitate the strip footing. Three different locations of loading were eccentrically located at 0.0 B, 0.05 B and 0.1 B measured from the long axis. Photogrammetry and image velocimetry (PIV) methods were used to examine the failure pattern. It was found that with the increasing of eccentricity, bearing capacity decreased with increasing settlement. A clear soil movement opposite to eccentricity was observed, the largest rotation happened with largest eccentricity ratio.

2.3 Numerical and Experimental Investigation

Kaya and Ornek (2018) performed 48 model tests and numerical investigation concerning the case of T-shaped footings subjected to eccentric load to investigate the effects of footing geometry and soil density. The problem geometry was represented by the load eccentricity (e) and embedded depth (H). The tests were performed on loose and dense sand conditions. The experimental stage was followed by numerical analyses based on 2D computer program. The sand bed was represented by the hardening soil model. At the end of every test and numerical analysis, the load-settlement and bearing capacity-settlement curves were plotted. It is observed that the bearing capacity of the strip footing decreases due to the formation of the failure zone at one side of the footing. With respect to the inserted part of the T-shape, the carrying load increases when the inserted depth increases. The numerical model results underestimated model results, i.e. numerical analyses are conservative.

2.4 Analytical and Theoretical Investigations

Algin (2009) introduced analytical closed-form solutions for estimating the elastic settlement under eccentrically (biaxial) loaded rectangular surface footings resting upon an elastic mass. The presented solutions are determined by evaluating the integration of strain expressions based on the Boussinesq stress equations. The linear contact pressure concept in footing-soil interface is adopted. The solutions are also developed for the location of incompressible soil layer is infinity. The simplified influence factors are presented graphically. The results of the developed equations were compared with the existing solutions and evaluated as valid. Numerical examples are provided for illustration. The paper represents a significant step forward in calculating the elastic settlement, under eccentrically loaded rectangular footings.

Nandakumaran and Senathipathl (1981) suggested a simple and approximate method to obtain the settlement of footings subjected to eccentric loading. Comparison with test data on a plate and one of the existing procedures demonstrate the feasibility of the technique. It is concluded that when footings are subjected to vertical loads and moments, as during a seismic, the proposed method can be employed with advantage.

2.5 State-of-the-Art Report

Lutenegger and DeGroot (1995) prepared a valuable comprehensive state-of-the-art review of the procedures used by the geotechnical engineers to estimate the settlement of shallow foundations resting on granular soil. The review was made of the literature to summarize the existing available methods. Recommendations are given at the end of this report for improvements in settlement analyses. The results of the project indicate that a number of methods are out-dated and should not be used in predicting settlements. The results presented in that report enable researcher to improve the existing methods to allow more reliable settlement estimation.

2.6 Theme, Hypothesis and Questions,

The theme of this research is settlement of a rigid plate has width B and length L subjected to bi-axial load with special interest to the settlements of its central S_{ecc} and corner S_{ecr}. The possibility of relating those two settlements to the settlement of the central point S_{cc} of the same plate when it is subjected to concentric loading is the research hypothesis. The research two questions are; is it possible to predict S_{ecc} and S_{ecr} if S_{cc} is previously calculated by closed form equations? Is it possible to generalize the answer to be a conventional method?

3 METHODOLOGY AND METHODS

Two expressions, methodology and method are closely interlocked. They serve two different roles. Simply, the methodology describes the strategy and outlines the way in which research to be carried out.

Method is the tool which will be used to create the needed data to answer research question. Methodology demands the methods to generate convincing data. With one methodology, several methods may be used to compensate research hypothesis and achieve its goal. In writing-up research methodology comes first, in order.

3.1 Research Methodology

The strategy used to fulfill the goal of this study consists of three steps. The first step is designing a parametric study that covers selected variable with uniform intervals. The second step is assigning values to all variables and constants that will be used throughout the study. The third step is achieving the suggested parametric study. Then, the routine research work will be continue, i.e. presentation, analysis and discussion of the results.

To start with, all variables have been normalized as dimensionless ratios, i.e. B/L, e_X/B, e_Y/L, S_{ecc}/S_{cc} and S_{ecr}/S_{cc}. The considered soil model is Mohr-Coulomb.

Summary of the parametric study cases is shown in; Table 1, Table 2 and Table 3. They present cases of B/L equal to 1.0, 0.67 and 0.40, respectively. The eccentricity ratio in directions e_X / B and e_Y / L varied from 0.00 to 0.15 in a uniform distributed interval. However, Figure 1 shows a sketch of the plate dimensions and the eccentricity route. Also, this figure has great advantages for the analysis of the results throughout article 4.1, discussed and explained later on.

Table 1. Cases of square plate: B/L = 1.0

Y \ X	0.000	0.025	0.050	0.075	0.100	0.150
0.000	C1	--	C6	--	C11	C14
0.025	--	--	C7	--	--	--
0.050	C2	C5	C8	--	C12	--
0.075	--	--	--	--	--	C15
0.100	C3	--	C9	--	C13	--
0.150	C4	--	--	C10	--	C16

Key: Ci = Case study i, X= ex/B, Y= ey/L

Table 2. Cases of rectangular plate: B/L = 0.67

Y \ X	0.000	0.025	0.050	0.075	0.100	0.150
0.000	R1	--	R6	--	R11	R14
0.025	--	--	R7	--	--	--
0.050	R2	R5	R8	--	R12	--
0.075	--	--	--	--	--	R15
0.100	R3	--	R9	--	R13	--
0.150	R4	--	--	R10	--	R16

Key: Ri = Case study i, X= ex/B, Y= ey/L

Table 3. Cases of rectangular plate: B/L = 0.40

Y \ X	0.000	0.025	0.050	0.075	0.100	0.150
0.000	U1	--	U6	--	U12	U16
0.010	--	--	U7	--	--	--
0.020	--	--	--	--	U13	--
0.025	--	--	U8	--	--	--
0.030	--	--	--	--	--	U17
0.050	U2	U5	U9	--	U14	--
0.075	--	--	--	--	--	U18
0.100	U3	--	U10	--	U15	--
0.150	U4	--	--	U11	--	U19

Key: Ui = Case study i, X= ex/B, Y= ey/L

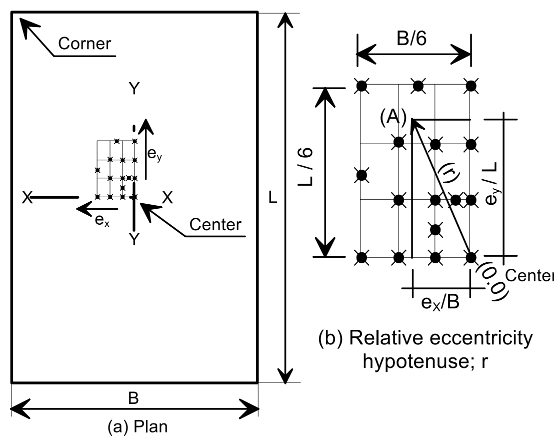

Figure 1. Relative eccentricity "r", diffenition and range.

3.2 Research Method and Tool

The research method and tool are the techniques used to create the research qualitative data. Finite element analysis is an outstanding method to manage settlement calculations. A rigid rectangular plate rests on dense sand is ideal case to be studied by 3D finite element method. Accordingly, Plaxis 3D-Foundation has been selected to be the research tool due to its ranking. Three types of integrated elements have been combined then used to build up a representative finite element model. Namely they are plate, interface and wedge elements.

Triangular plate element is a 2D-structural element has flexural rigidity. It allows deflection due to shear and bending also can change length. It is composed of 6-nodes each having 6-degrees of freedom. Also it has three stress points. The stresses calculated at the stress points then extrapolated to nodes. The quadrilateral plate element is typically triangular element, but is composed of 8-nodes. it is compatible with triangular element and both facilitate the mesh generation. The plate basic parameters are; thickness "t", unit weight "γ", Poisson's ratio "μ", Young's modulus "E" and shear modulus "G".

The interface element is composed of 8-pairs or 6-pairs of nodes to be compatible with the quadrilateral or the triangular side of soil element, respectively. It has zero thickness, i.e. each pair of nodes have identical coordinates. The interface element is used to model the structure-soil surfaces roughness. The main property is the strength reduction factor; R_{inter} relates the plate base friction and the soil shear parameters. Here-in the rough interface option has been considered, i.e. R_{inter} = 1.0, means no slip nor gapping can occur. This study deals with bi-axial loading and the plate tilting is a must during all cases of study. The virtual thickness of the interface element will not be uniform beneath the plate. It is better to avoid this by adopting no slip nor gapping option.

The used soil element is the 15-node wedge element. It is generated from the 6-nodes triangular element during the 2D-stage mesh generation. It is composed of 6-node triangular element in the horizontal direction and 8-nodes quadrilaterals in vertical direction. This element type gives good accuracy based on literature.

The soil behaves as a non-linear material, but in our cases settlement and loads lie within the early beginning of the stress-strain curve, i.e. within the allowable bearing capacity. It is still not elastic but also it is not highly non-linear. Accordingly, Mohr-Coulomb model is suitable one to analyze case of dry dense sand deformations. Five parameters are required for the model. They are; Young's modulus "E_s", Poisson's ratio "μ_s", frictional angle "φ", dilatancy angle "ψ" and cohesion "c".

The used modulus of elasticity is the secant modulus at 50% "E_{50}" Also, it has been considered constant with depth. The manuals recommended the use of higher values for "μ", i.e. between 0.3 and 0.4 . Later on, this choice had been evaluated by matching the value obtained from the "K_o" procedures, i.e. initial conditions. The value of sand cohesion "c" should be zero, but for the numerical stability it was advised to be less than 0.2 kPa. The manuals advised to use dilatancy angle "$\psi \subseteq \varphi$ - 30^o".

The model geometry has been divided into elements to form finite element mesh. "K_0" procedure is used within initial phase, while plastic calculation is used to perform elastic-plastic analysis. No construction stages specified because the plate rests on soil surface, i.e. no excavation and the loads applied at once with its full value.

3.3 Model Parameters and Verification

The soil and plate parameters have been abstracted based on Patra et al. (2012 a). His paper is experimental investigation and modeling, sharing with Behera R.N and Das B.D. The names of authors give the results credibility. However, the used here-in soil propertie present the case of dense sand as shown in Table 4.

Table 4. Dense sand Properties

γ_d (kN/m^3)	E_s (kN/m^3)	μ_s (ratio)	Φ (deg.)	Ψ (deg.)
14.37	42000	0.33	40.8	10.8
γ_d dry unit weight, E_s Young modulus, μ_s poisson's ratio, Φ :friction angel and Ψ dilatancy angle.				

As a first step, several models with different dimensions have been suggested then tested with respect to closed form solutions. Based on the evaluation of their results the following dimensions have been adopted throughout the rest of the study. For a plate has width B and length L, the dimensions of the finite element model are; [7B by (L+6B)] in plan and 5B deep and standard conditions for boundary condition are used.

The vertical stresses σ_{vi} beneath the center of a square flexible loaded area has been calculated vs. depth. This has been done based on Fadum (1948) integration of Boussinesq equation as well as the aid of his charts, Eq. 1:

$$\sigma_{vi} = q.I_R \qquad [1]$$

Where σ_{vi} is the vertical stress with depth, q is the applied uniform distributed load and I_R is Fadum influence factor either calculated or obtained from the charts. Then, the vertical stresses vs. depth based on the finite element model results, linear elastic, have been reported. Fadum vs. finite element results have been drafted in two curves for comparison, Figure 2 . The dashed line presents Fadum results, while the solid one presents the finite element results. The two curves agree perfectly. Accordingly two conclusions have been abstracted. First, the model dimensions avoided completely the boundary effects, moreover the modelling process, is error free.

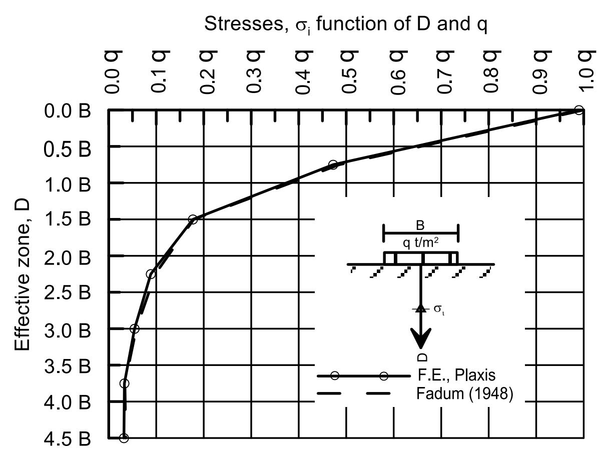

Figure 2. Fadum (1948) vs. F.E.A, normalized stress under central point of flexible square loaded area.

Table 5, gives the reinforced concrete properties of the plate based on average of typical values. The plate thickness (t) needs to be suggested as follows:

Table 5. Concrete plate element properties

γ_c (kN/m^3)	E_c (kN/m^3)	μ_s (ratio)
24.00	2 E+07	0.20
γ_c unit weight, E_c Young modulus, μ_c poisson's ratio		

Here-in, it has been defined the rigid plate as a plate its thickness (t) gives settlement of three of its points nearly the same. They are center, corner and mid side point. Different plate thickness has been examined, the finite element model run and the settlements recorded and plotted. The Y-Axis of Figure 3 is the normalized settlement of the three points. Its X- Axis is the rigidity coefficient, K_R as given by Eq. 2, Borowicka (1936).

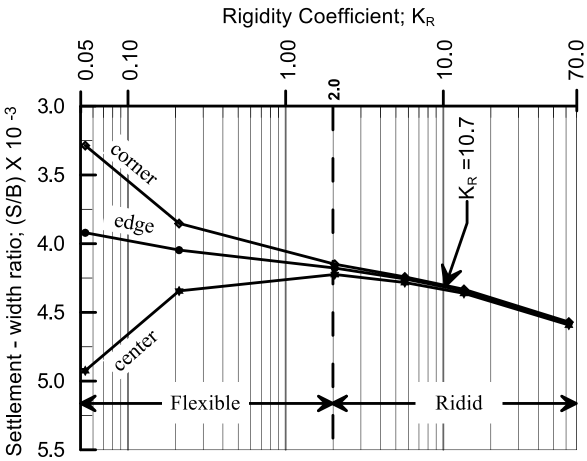

Figure 3. Plate rigidity vs. settlement for different “d”

$$K_R = \frac{1}{6}\left(\frac{1-\mu_f^2}{1-\mu_s^2}\right)\left(\frac{E_f}{E_s}\right)\left[\frac{t}{B}\right]^3 \qquad [2]$$

As the rigidity coefficient, K_R increases the settlement of the plate center decreases while the settlement of the corner and the mid side edge increases. At K_R = 2.0 the three curves became nearly one. Accordingly if K_R o 2.0 the plate is rigid. Based on Tables 4, 5, and Eq. 2 plate of t = 1.0 m and B =2.0 m, K_R is 10.7, i.e. rigid with respect to the foundation soil.

4 PRESENTATION AND ANALYSIS OF RESULTS

To recap, Table 1, Table 2 and Table 3, presents the parametric cases of study for B/L equal to 1.0, 0.67 and 0.40 respectively. Figure 1 (a-b), shows a sketch with the plate dimensions and the eccentricity route in both directions where e_X/B and e_Y/L varied from 0.00 to 0.150 in a uniform distributed intervals.

4.1 Axes Titles and Their Normalization

During the rest of this study all figures axes have been normalized to be dimensionless ratios. Also, they have been given significant headings with consistent notation.

Figure 1 (a-b), the arrow that join plate center, i.e the origin (0,0) with any point (A) within the middel third in both directions has been called “relative eccentricity hypotenuse”, r . It presents the X-axis for all figures, from now and on. The values of the bi-axial eccentricities e_x and e_y are absolute values. When they are used to calculate r, they will be divided by the parallel side-length of plate, i.e. (e_x/B) and (e_y/L). For that reason, r has been called relative eccenteiciy and the term hypotenuse came from its way of its calculation, Eq.3:

$$r = \sqrt[2]{\left(\frac{e_x}{B}\right)^2 + \left(\frac{e_y}{L}\right)^2} \qquad [3]$$

This strategy not only normalized the X-axis values, but also generalized the solution for any plate dimensions B and L, as it will be shown later-on throughout item 4.4.

Y-axis normalized by dividing the corner settlement, S_{ecr} and center settlement S_{ecc} of eccentrically loaded plate by the settlement of its center when concentrically loaded, S_{cc}. This introduced a ratio denoted by R_S and called “Settlement correction factor”, Eq. 4 and Eq. 5.

$$R_S = \frac{S_{ecr}}{S_{cc}}\ ; for\ plate\ corner \qquad [4]$$

$$R_S = \frac{S_{ecc}}{S_{cc}}\ ; for\ plate\ center \qquad [5]$$

This strategy normalized the X-axis and Y-axis co-ordinate and generalized the solution for any values of the deformation parameter, i.e. E_S and μ_S, as it will be discussed in item 4.4, layer on.

4.2 Presentation of Table 1 Results

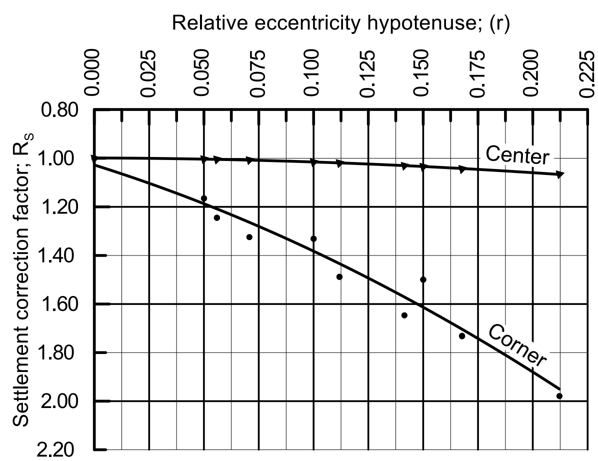

Figure 4. Relative eccentricity vs. correction factor, cases of Table 1, B/L = 1.0, center and corner.

Table 1, presents plate having the ratio B/L = 1.0. Ten cases were performed then the settlements of its corner and center have been recorded. Also, the settlement of its center when there is no eccentricity has been recorded too. Based on Eq. 3, Eq. 4 and Eq. 5, the values of, r and

R_S have been calculated, plotted to relate r with R_S for the corner and center of the plate. The two curves have been drafted based on curve fitting technique; polynomials of the second order, Figure 4. The regression factor for center and corner is 99.9% and 99.8%, respectively.

4.3 Presentation of Table 1,2 and 3 Results

The same procedures that explained in full details just above have been repeated for the cases of Table 2 and Table 3. Then the results of the three tables have been drafted throughout two graphs each has three curves. Figure 5, intreduces the cases of the plate center while Figure 6, presents the cases of the plate corner.

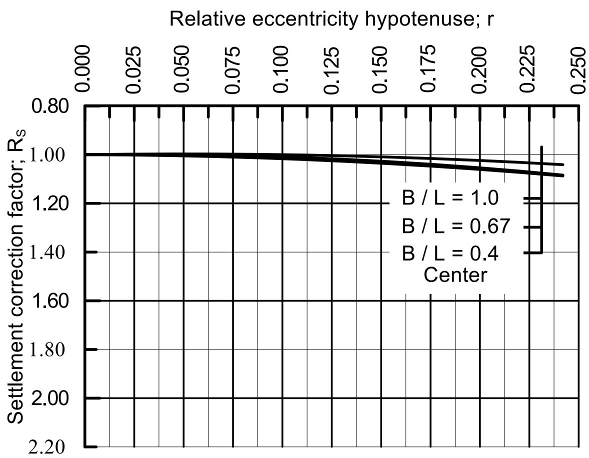

Figure 5. Relative eccentricity vs. correction factor, cases of Table 1,2 and 3, B/L = 1.0, 0.67,0.4 center.

The drafted curves have been done based on the curve fitting technique as polynomials of the second order. It is obvious from the two figures that the curves within each group are very close to each other and could be approximately the same.

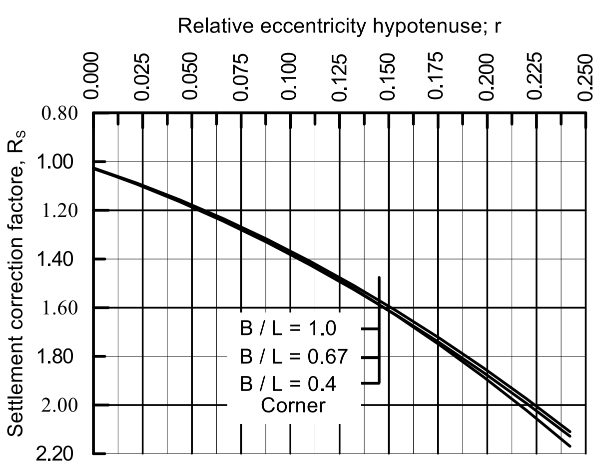

Figure 6. Relative eccentricity vs. correction factor, cases of Table 1,2 and 3, B/L = 1.0, 0.67,0.4 corner.

4.4 Results Analysis and Discussion

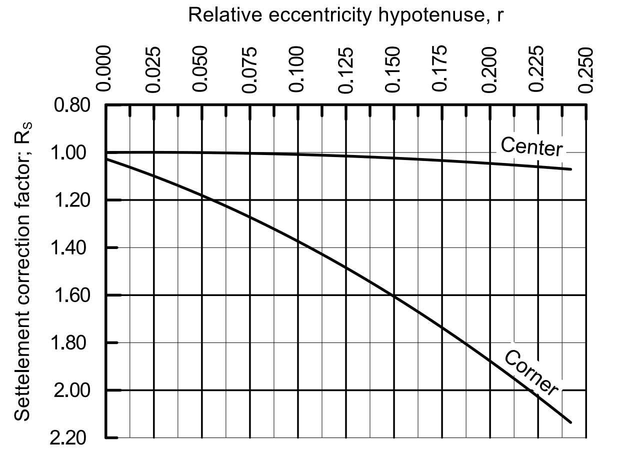

Figure 7. Relative eccentricity vs. correction factor, valid for any value of B/L, center-corner; "Khater curves".

The last note mentioned at the end of item 4.3 is the best start to this subtitle. It says " … *It is obvious from the two figures that the curves within each group are very close to each other and could be approximately the same*". This note encourage one to plot the six curves as two curves only, one for the corner and the other for the center of the plate. Of course the curve fitting technique has been adopted and performed on 48-points, 24 per each curve.

Figure 7, "Khater curves" introduces the goal of this research work. Simply it presents a conventional method to calculate the settlement of the plate center and corner which is bi-axially loaded. The method procedures will be explained in details later on throughout section 5, "conclusions" as well as a numerical example.

Here-in, its validity and range of application will discuss and examined as follows:

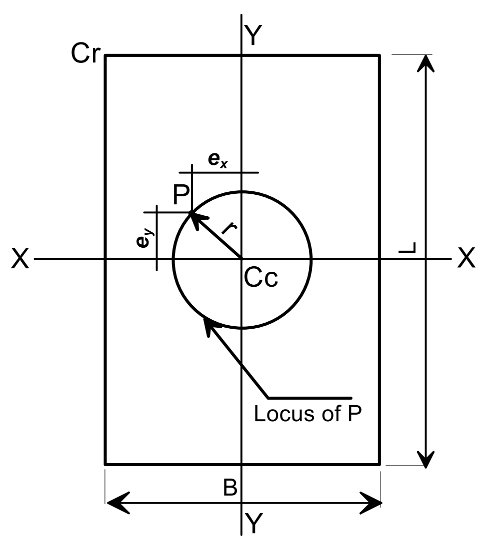

Figure 8. The validity of Figure 7 for any value of B/L.

Figure 7 capture stated; “it is valid for any value of B/L”. No doubt it is valid for B/L equal 1.0, 0.67 and 0.4 and in between, based on the performed cases of study. To ensure its validity for any other values of B/L, the following argument proves the validity for any B/L.

The strategy used to investigate validity of Figure 7 for any value of B/L ratio depends on the normal stress equation Eq. 6 and Figure 8.

$$\sigma_{Cr} = \frac{N}{A} \mp \frac{M_x\, y}{I_x} \mp \frac{M_y\, x}{I_y} \qquad [6]$$

Where;

$M_x = P.\, e_y$, $M_y = P.\, e_x$, $A = BL$, $N = P$, $y = 0.5\, L$, $x = 0.5\, B$, $I_x = B\, L^3/12$ and $I_y = L\, B^3/12$.

Substituting in Eq. 6 and re-arrange the terms;

$$\sigma_{Cr} = \frac{P}{BL}\left(1 \mp 6\,\frac{e_y}{L} \mp 6\frac{e_x}{B}\right) \qquad [7]$$

$$\sigma_{Cc} = \frac{P}{BL} \qquad [8]$$

$$\frac{\sigma_{Cr}}{\sigma_{Cc}} = \left(1 \mp 6\,\frac{e_y}{L} \mp 6\frac{e_x}{B}\right) \qquad [9]$$

σ_{Cr} is the stresses at the plate corner when it is eccentrically loaded, while σ_{Cc} is the stresses at the plate center when it is concentrically loaded. The elastic settlement, S_e equation in its simplest form is a function of the increases of stresses σ, the soil layer thickness h, and the soil elastic modulus E_s.

$$S_e = \frac{\sigma\, h}{E_S} \qquad [10]$$

As the values of h and E_s are constant beneath the same plate, if one multiply the numerator and denominator of the left hand side of Eq. 9 by (h/E_S) and then combined Eq. 9 and Eq. 10 we get:

$$\frac{\sigma_{Cr}\, h\,/\mathrm{Es}}{\sigma_{Cc}\, h\,/\mathrm{Es}} = \frac{S_{Cr}}{S_{Cc}} = \left(1 \mp 6\,\frac{e_y}{L} \mp 6\frac{e_x}{B}\right) \qquad [11]$$

Referring to Eq. 4 and Eq. 5 in addition to Eq. 11, the equation of settlement correction factor, R_S is:

$$R_S = \left(1 \mp 6\,\frac{e_y}{L} \mp 6\frac{e_x}{B}\right) \qquad [12]$$

Relative eccentricity hypotenuse, r equation is Eq. 3, to recap here $r = \{(e_y/L)^2 + (e_x/B)^2\}^{0.5}$, it presents equation of a circle it center is the plate center and its circumference is the locus of the bi-axial load. Eq. 3 and Eq. 12 are the X-axis and Y- axis, respectively. On which Figure 7 has been constructed, all the variables are in dimensionless forms and repeated along both axes with the same power. This presents a general form for any B/L, e_y/L and e_x/B. Also, Figure 7 could be used to all elastic soils regardless its properties. Based on Eq. 11, the modulus E_s exists in numerator and denominator.

A hint for logical verification, write-up Eq. 3 and Eq. 12 for case of strip footing where $L = \infty$, infinity and $e_y = 0.0$

$$r = \sqrt[2]{\left(\frac{e_x}{B}\right)^2} = \frac{e_x}{B} \qquad [13]$$

$$R_S = \left(1 \mp 6\frac{e_x}{B}\right) = (1 \mp 6r) \qquad [14]$$

It is obvious that r and R_S as well as Figure 7 are independent on the absolute values of B and/or e_x, for the case of strip footing which is a special case of rectangular plate. This conclusion is extended to the case of $B = \infty$ and $e_x = 0.0$. Those two special cases presented the case of two ends of the quarter of the circle shown in Figure 8, logically any conclusion is valued in between, i.e. Figure 7 is valid to any B/L and any E_S, i.e. general.

Based on Figure 7, the following two empirical equations, Khater's equations, have been created by using the curve fitting technique, polynomials of the second order with regression factors R equal to 98%:

$$R_{S-Corner} = 1.03 + 2.68\, r + 7.67\, r^2 \qquad [15]$$

$$R_{S-Center} = 1.06 - 0.06\, r + 1.47\, r^2 \qquad [16]$$

4.5 Numerical Example

Khater's method procedures consists of four steps, first calculate the settlement of the plate center which is loaded by concentric load P, based on any conventional method. Then use Eq. 3 to calculate, r. Third step use Figure 7 twice for the same value, r to calculate two values, R_S. Last step, use Eq. 4 and Eq. 5 to calculate the settlements of eccentrically loaded plate, e_x and e_{ye}.

Problem: Square rigid plate 3.0-m side-length subjected to eccentric vertical concentrated load of 1800 kN and eccentricities, $e_x = e_y = 0.30$-m. Rests on the top of dense sand its $E_S = 50000$ kN/m^2, Poisson's ratio $\mu_S = 0.35$ and thickness of 8.0-m. Calculate the settlement of its center and corner, as a first approximation.

Solution :(No equation numbering within this item)

Based on Fadum (1948), z = 0.5 h = 4.0-m.

m.z = B/2 and n.z = B/2, n = m = 0.375 and $I_r = 0.055$

$\sigma_{center} = 4.0\ \{ q\ I_r \}_{quarter}$

= 4.0 % 1800/9 % 0.055 = 44.0 kN/m^2
S_{center} (flexible) = 44 % 8.0 % 1/50000 = 0.704 cm.
S_{center} (rigid) = 0.85 % 0.704 = 0.598 cm = 0.60 cm
e_x / B = e_y / L = 0.3/3.0 , use Eq. 3, Eq. 15 and Eq. 16;
r = { $(0.1)^2 + (0.1)^2\}^{0.5}$ = 0.141
$R_{S\text{- Corner}}$ = 1.03 + 2.68 (0.141) + 7.67 $(0.141)^2$ = 1.56
$R_{S\text{- Center}}$ = 1.06 - 0.06 (0.141) + 1.47 $(0.141)^2$ = 1.08
$S_{e\text{-corner}}$ = 1.56 % 0.60 = 0.936 cm
$S_{e\text{-center}}$ = 1.08 % 0.60 = 0.648 cm

5 SUMMING-UP AND CONCLUSIONS

The result of this research work suggested a conventional method, Khater method, to predict the immediate elastic settlement of a rigid plate corner and center. The plate could be square, rectangular or strip having any breadth to length dimensions, B/L and for any type of sand density and modulus of elasticity. The plate is subjected to bi-axial vertical load. The method may be applied by the help of Figure 7 or by using the two empirical equations mentioned below.

The method procedures consists of four steps, first step calculates the settlement of the plate center when it is loaded by co-centric load P, based on any available conventional method. Second step uses Eq. 3 to calculate the relative eccentricity hypotenuse, r. third step uses Figure 7 twice each for the same value of r to calculate two corresponding values of settlement correlation factor, R_S. Fourth step uses Eq. 4 and Eq. 5 to calculate the settlements of the plate corner and center due to the eccentrically concentrated load P, i.e. having e_x and e_y.

Alternatively in step three, Figure 7, the two empirical equations Eq. 15 and Eq. 16, “Khater’s” equations, could be used.

With reference to Eq.’s 15 and 16, if r = 0.0, i.e. concentric case, the accuracy of their results ranges from +3% to +6%, on the conservative side. This degree of accuracy as a first approximation is quiet enough. Also, from plate geometry point of view the factors influence the settlements of a plate subjected to bi-axial load are B and L and not only its B as for the case of concentric loaded plate. For constant values of e_y / L or e_x / B as B or L increases, r dramatically decreases and consequently its settlement decreases.

The importance of this work depends on its generalization and practicality. It is suitable for footings subjected to bi-axial load, concentric load with bi-axial moment, raft foundation its columns loads induced eccentric resultant with or without bi-axial moments due to seismic. The method deals with any B / L values for rectangular plate rests on cohesion-less soli having any elastic deformation parameters. It is a first approximation method, simple and fast.

For further research work it is advised to perform laboratory physical model to verify the accuracy and validity of the suggested method. Also, it is recommended to perform series of in-Situ plate load tests to add more credibility of the suggested method results. Also, other foundation shapes need to be investigated, such as trapezoidal, L-Shape, U-Shape and etc.

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