

Recent Advanced Constitutive Models for Liquefaction and Cyclic Softening Analysis

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ABSTRACT

Five bounding surface constitutive models within the critical state framework are discussed in this study. The selected models are either developed to study the potential of liquefaction or cyclic softening. The studied sand constitutive models are DM04 and PM4SAND while clay constitutive models are SANICLAY-B, SANICLAY-H, and PM4SILT. First, the main features and differences of the models are highlighted, then the performance of each model is compared by a set of undrained cyclic direct shear test simulations. Finally, the merits and limitations of each model are briefly discussed. All selected models have shown to have high abilities in predicting cyclic response of soils in terms of stress path, hysteretic loops, and pore water pressure generation. However, each model has shown limitations that the user should be aware of when using such models.

RÉSUMÉ

La présente étude traite de cinq modèles constitutifs de surfaces limites dans le cadre des états critiques. Les modèles sélectionnés sont développés pour étudier le potentiel de liquéfaction ou d'amollissement cyclique. Les modèles constitutifs de sable étudiés sont DM04 et PM4SAND, tandis que les modèles constitutifs d'argile sont SANICLAY-B, SANICLAY-H et PM4SILT. D'abord, les principales caractéristiques et différences des modèles sont surlignées, puis les performances de chaque modèle sont comparées par un ensemble de simulations d'essais de cisaillement direct cyclique non drainé. Enfin, on discute brièvement des avantages et des inconvénients de chaque modèle. Tous les modèles sélectionnés ont montré de grandes capacités de prédiction de la réponse cyclique des sols en termes de chemin de contraintes, de boucles hystérétiques et de génération de pression interstitielle. Cependant, chaque modèle a montré des limites que l'utilisateur devrait connaître lorsqu'il utilise de tels modèles.

1 INTRODUCTION

Liquefaction is defined as a loss of strength of a loose saturated sand under cyclic loading (Jefferies and Been, 2015). This strength loss can considerably damage manmade and natural earth structure, and cause serious loss of human properties and lives. Liquefaction has been reported in many recent important earthquakes (such as the 1964 Niigata earthquake (Seed and Idriss, 1967) and the 2011 Christchurch earthquake (Bradley and Cubrinovsky (2011) to name a few). Consequently, various guidelines and procedures for evaluation of liquefaction resistance and potential have been published (e.g., Seed and Idriss (1971), Youd and Idriss (2001), Idriss and Boulanger (2006)).

Similar to sands, clays can also exhibit a loss of stiffness leading to severe deformations, and a few case histories have been reported (such as the fourth avenue landslide during the 1964 Alaskan earthquake (Stark and Contreras, 1998), and the Chiufengershan landslide triggered by the 1999 Chi-Chi earthquake in Taiwan (Shou and Wang, 2003)). Boulanger and Idriss (2006) proposed that two different terms be used for sands and clays; they suggested that the significant strength loss or strain development accompanied by a sharp increase in the excess pore water pressure be referred to as "liquefaction" for fine-grained-soils exhibiting sand-like behavior, while the similar phenomenon be referred to as "cyclic softening" for those exhibiting clay-like behavior. Whether a soil is

behaving like a clay- or a sand may be found out in different ways (Ladd, 1991; Ladd and Foot, 1974; Boulanger and Idriss, 2006; Zergoun and Vaid, 1994; Vucetic and Dobry, 1989). Although the cyclic softening of fine-grained soil can be as disastrous as liquefaction, it has been studied to a far lesser extent (Boulanger and Idriss, 2007).

Constitutive modeling is a means to integrate previously known characteristic of a soil under a specific condition (i.e. static or dynamic loading, saturated or dry condition etc.) by means of analytical approaches and mathematical equations. Several constitutive models have been developed for cyclic applications and each model is based on different assumptions and, therefore, lead to precise results only if the model is being used in accordance with said assumptions. Most soil constitutive models are formulated within the critical state framework to predict the stress-strain behavior of soils under monotonic and/or cyclic loading.

Although a significant amount of work has been done on the constitutive modeling of liquefaction-susceptible soils, the effort to simulate cyclic softening is only starting. This study reviews some of the most recent constitutive models developed so far to assess liquefaction and cyclic softening potential and provide some details about their abilities and limitations.

2 MODELS FOR LIQUEFACTION ANALYSIS

In this section, the following recent advanced soil constitutive models for liquefiable soils are discussed: 1) Dafalias and Manzari (2004) version of SANISAND and 2) the plasticity model for sands (PM4SAND).

2.1 DAFALIAS AND MANZARI (2004)

The model, developed by Dafalias and Manzari (2004) and hereafter referred to as DM04, is a modified and improved version of the model proposed by Manzari and Dafalias (1997). This model is formulated in $p - q$ and multiaxial spaces for simulation of both monotonic and cyclic responses of sands. The most interesting feature of the model is that it considers the effect of fabric change upon dilation. The model is formulated in terms of stress ratio ($\eta = \frac{q}{p}$) and uses 4 different surfaces which are lines (i.e. stress ratios) in $p - q$ space: yield, critical, dilatancy, and bounding surfaces. The main framework of this model is made such that only a change of stress ratio can cause plastic strains, and a loading condition resulting in no change in stress ratio, such as an isotropic compression, will only develop elastic strains.

The yield surface geometrically represents a wedge in $p - q$ space inside which only elastic strains occur. The center of the yield surface, or slope of the middle line of the elastic wedge, is adjusted by α , and the width of the elastic wedge by m . It is worth mentioning that the generalization of the model makes the yield surface to be a cone in stress space where the back-stress ratio α , the counterpart of α in the multiaxial setting, and elastic parameter, m , defines the center and radius of the cone, respectively.

The model uses a non-associative flow rule. The deviatoric and volumetric plastic strain increments formulations, in $p - q$ space, are postulated as:

$$\varepsilon_q^p = \frac{\dot{\eta}}{H} \quad [1]$$

$$\varepsilon_v^p = d \left| \varepsilon_q^p \right| \quad [2]$$

Therefore, the model requires that the plastic deviatoric strain increment (ε_q^p) depends on the increment of stress ratio ($\dot{\eta}$) and the plastic modulus (H). The increment of plastic volumetric strain (ε_v^p) is associated directly with the plastic deviatoric strain increment (ε_q^p) via the dilatancy parameter (d). H and d are proportional to the distance of the stress ratio (η) with bounding (M^b) and dilatancy (M^d) stress ratios in the contraction and dilation phase of loading, respectively:

$$H = h(M^b - \eta) \quad [3]$$

$$d = A_d(M^d - \eta) \quad [4]$$

The values of M^b and M^d are controlled by the state parameter which is defined as the distance between the void ratio at the current stress point (e) and that at the critical state line (e_c) in the $e - q$ space:

$$\psi = e - e_c \quad [5]$$

2.2 PM4SAND

The first version of the plasticity model for sand (PM4SAND) was published by Boulanger in 2010 (Boulanger (2010, version 1). This model is based on the Dafalias and Manzari model, and similarly, is a stress ratio-based model which is casted within the critical state framework and the bounding surface plasticity. Since the earlier version of the model, it has been modified to improve the model's ability to predict the cyclic response of sands under cyclic loadings. One of the emphasizes in this model is that in its simplest application, the model only requires three input parameters which are soil relative density, the shear modulus coefficient, and the contraction rate parameter. In this study, the last version of the model published by Boulanger and Ziotopoulou (2018, version 3.1) will be briefly discussed.

The PM4SAND model is developed to be only in terms of the in-plane stresses with the aim of simplifying the model and reducing the computations and corresponding time required for solving complex engineering problems Boulanger and Ziotopoulou (2013).

The yield surface of the PM4SAND model is similar to that of the DM04 model and is expressed in terms of the back-stress ratio, α . Also, similar to the DM04 model, generalizing the yield surface makes it to be a cone in stress space where the back-stress ratio and elastic parameter, m , defines the center and radius of the cone, respectively:

Similar to the yield surface, bounding and dilatancy surfaces are also conical-shape surfaces in the stress space which are mathematically shown in terms of back stress ratios as:

$$\alpha^b = \sqrt{1/2} [M^b - m]n \quad [6]$$

$$\alpha^d = \sqrt{1/2} [M^d - m]n \quad [7]$$

Where n is the normal to the yield surface. M^b and M^d are bounding and dilatancy ratios which are related to the critical state, M .

One of the applied changes to this model compared to the DM04 is the exclusion of the Lode angle dependency, meaning that the same friction angles are used in compressive and extension loading. Although this exclusion has simplified the model and its implementation, it brings the responsibility of selecting a representative friction angle to the user.

The bounding and dilatancy ratios are associated with the critical state line via the relative state parameter index, ξ_R , which is defined into the model using the empirical Bolton's dilatancy relationship (Bolton, 1986). Based on this relationship $D_{R,cs}$, which is the relative density at the critical state for the current p , is calculated using Bolton parameters, Q and R as:

$$D_{R,cs} = \frac{R}{Q - \ln(100 \frac{p}{p_A})} \quad [8]$$

One of the modifications of the PM4SAND compared to the DM04 is the modification of the shear modulus formulation to be only dependent on one constant, i.e. G_0 , but not both G_0 and void ratio, e , that was the case in DM04 model. In fact, the authors believe that the confining stress has more effect on G than the void ratio does. In addition, removing e eases calibration purposes. Instead, two other factors are added to the shear modulus formulation which will allow better predictions of the soil hysteretic stress-strain response at large shear strains where liquefaction happens.

2.3 SAND MODEL PERFORMANCE AND COMPARISONS

In this section, the performance of sand models compared to each other will be shortly discussed. For this purpose, the models have been implemented to simulate an undrained cyclic shear test. All models have been subjected to a cyclic shear stress ratio ($\frac{\tau}{\sigma'_{vc}}$) of 0.15, with a vertical effective consolidation stress, σ'_{vc} , of 100 kPa, lateral earth pressure coefficient, K_0 , of 0.5, and a OCR value of 1.

Simulation results of DM04 model is presented in Figure 1. a. The results show that the model could predict both the contractive and dilative behavior of sands under a dynamic loading as illustrated by the butterfly-shaped loops that occur in a dynamic event can be simulated in this model. However, the hysteretic loops lock-up happen at large shear strain levels for some values of the critical state slope. This shear strain lock-up does not allow the model to accumulate higher shear strains upon further shearing.

The PM4SAND model simulations result are plotted in Figure 1. b. In general, the model shows reasonable approximations of desired behaviors of sand. In fact, the model performance has remarkably improved compared to the DM04 model in terms of developing significant shear strains. Specifically, the hysteretic loops of the model are improved significantly compared to the DM04 model allowing the model to predict stiffer response at large shear strains. However, the model is built upon an in-plane stresses conditions and is not usable for other stress conditions. Finally, it should be highlighted that the selection of a friction angle should be done cautiously as the Lode angle dependency is removed in this model.

3 MODELS FOR CYCLIC SOFTENING ANALYSIS

The studied constitutive models developed for cyclic behavior of low plasticity soils are: 1) SANICLAY bounding surface model, 2) the SANICLAY bounding surface with a hybrid flow rule, and 3) the plasticity model for low plasticity silts (PM4SILT).

3.1 SANICLAY BOUNDING SURFACE MODEL

The SANICLAY bounding surface model, hereafter referred to as SANICLAY-B, builds upon two previous versions for monotonic applications, namely SANICLAY, developed by Dafalias et al. (2006), and SANICLAY with destructuration, developed by Taiebat et al. (2010) and hereafter referred to as SANICLAY-D. SANICLAY is a clay plasticity model developed for simulations of monotonic softening response by taking into account the effect of anisotropy. SANICLAY-D is a modified version of the SANICLAY model that accounts for destructuration by including undisturbed and remoulded shear strength of clays associated with softening.

The SANICLAY-B model formulation has been developed by updating the two previous models' formulation to improve the model's ability for cyclic applications. However, crucial features of the previous versions such as the effect of anisotropy as well as destructuration (or softening) have been preserved. The modifications on SANICLAY-B have been mainly done by substituting the yield surface of the previous versions with a bounding surface. The benefit of using a bounding surface is to develop elastic and plastic strains simultaneously anywhere inside the bounding surface allowing to develop nonlinearities at very low strain levels, which is consistent with the true soil behavior.

It should be reminded that this is as opposed to classical plasticity that limits the plastic strain developments to conditions where the stress state goes beyond the yield surface, and that any stress state inside the yield surface only develop elastic strains.

In this model, the distance between the current stress state and its projection on the bounding surface is used to compute the plastic strains. The projection of the stress state on the bounding surface is called the image stress. For this purpose, the bounding surface concept requires using a radial mapping rule to project the stress point inside the bounding surface (q, p) to the corresponding image point on it (\bar{p}, \bar{q}) from the projection center (p_c, q_c).

$$\bar{p} = p_c + b(p - p_c) \quad [9]$$

$$\bar{q} = q_c + b(q - q_c) \quad [10]$$

Where b is a similarity ratio between the surface of the current stress, or loading surface, and the bounding surface.

The corresponding expressions for the volumetric and deviatoric plastic strain rates are obtained as follow using the flow rule:

$$\dot{\varepsilon}_v^p = \langle L \rangle \frac{\partial g}{\partial p} = \langle L \rangle \bar{p} (M^2 - \bar{\eta}^2) \quad [11]$$

$$\dot{\varepsilon}_q^p = \langle L \rangle \frac{\partial g}{\partial q} = \langle L \rangle 2\bar{p}(\bar{\eta} - \alpha) \quad [12]$$

Where $\bar{\eta} = \bar{q}/\bar{p}$ is the image stress ratio, and L is the loading index. The model is only taking advantage of the isotropic destructuration mechanism. For simplicity, the frictional destructuration mechanism introduced in the previous version of the model (Taiebat et al., 2010) is excluded.

3.2 SANICLAY BOUNDING SURFACE WITH A HYBRID FLOW RULE

The SANICLAY bounding surface with a hybrid flow rule is another plasticity model developed for cyclic softening. This model, SANICLAY-H, was developed by Shi (2016) upon the framework of SANICLAY-B (Seidalinov and Taiebat, 2014). The radial mapping rule that is used to find the image stress by projecting the current stress point on the bounding surface from a projection surface is referred to as image stress flow rule by Shi (2016) and Shi et al. (2018).

The main modification in the SANICLAY-H compared to the SANICLAY is that an image stress flow rule is substituted by a hybrid flow rule to improve the model's ability to simulate the excess pore water pressure build-up. The new hybrid flow rule is using a plastic strain operator (flow rule) that depends not only on flow characteristics of the image stress, but also on those of the current stress meaning that not only the current stress state but also the image stress state are used to calculate the volumetric plastic strains:

$$R = R_i + R_c \quad [13]$$

Where R is the direction of plastic strains which is decomposed into R_i and R_c that are plastic directions at the image (\bar{p}, \bar{q}) and current stress (p, q) point, respectively.

The new hybrid flow rule proposes that the image and the current stress point gradients of the volumetric plastic flow direction (R_v^i and R_v^c) as well as the image stress point gradient of the deviatoric plastic flow direction (R_d^i) be used to find the plastic strains as follow:

$$R_v = R_v^i g_i + R_v^c (1 - g_i) \quad [14]$$

$$R_d = R_d^i \quad [15]$$

Where g_i is a distribution variable which is given by:

$$g_i = \left(\frac{1}{b}\right)^w \quad [16]$$

Where w is a material constant coupling the relative contribution of R_v^i and R_v^c . In fact, this constant adjusts the intensity of the butterfly shape of the stress path. The special case of the model, which is the image stress flow rule of Seidalinov and Taiebat (2014), is obtained when $w = 0$.

The plastic potential formulation of SANICLAY-B is used. The volumetric (R_v^i) and deviatoric (R_d^i) components of the plastic flow direction at image stress as well as the volumetric component of the plastic flow direction at the current stress (R_v^c) are given as:

$$R_v^i = \frac{\partial g}{\partial \bar{p}} = \bar{p} (M^2 - \bar{\eta}^2) \quad [17]$$

$$R_d^i = \frac{\partial g}{\partial \bar{q}} = 2\bar{p}(\bar{\eta} - \alpha) \quad [18]$$

$$R_v^c = p (M^2 - s_l \eta^2) \quad [19]$$

where s_l is a variable that alternates between 1 and -1 depending on the loading direction and the stress ratio.

3.3 PM4SILT

The PM4Silt model has been developed by Boulanger and Ziotopoulou (2018) for geotechnical cyclic applications of low plasticity silts and clays. The model is casted in the same framework of the PM4Sand model (version 3), and therefore, shares several features with that model. Like the PM4SAND, the PM4SILT is a stress-ratio controlled model that has been framed in a bounding surface plasticity and critical state concept. Improvements of the model have been made such that the model be able to predict undrained cyclic and monotonic response of low to non-plastic silts with a high precision. One of the main emphasis in the model, that also holds true for the PM4SAND, is that the model be pre-calibrated so that the practitioners may use the model by only providing three input parameters that are: the undrained shear strength ratio (or undrained shear strength), the shear modulus coefficient, and the contraction rate parameter. However, assigning different values other than default values to the internal or secondary parameters of the model is possible if desired by the user.

The critical state is considered in this model using the state parameter, ξ . The state parameter, which calculates the difference between the current void ratio (e) and the void ratio at the critical state line (e_{cs}) at the same mean effective stress (p), is adopted in this model from the work

of Been and Jefferies (1985) by which (e_{cs}) is calculated by the following expression:

$$e_{cs} = \Gamma - \lambda \cdot \ln\left(\frac{p}{1kPa}\right) \quad [20]$$

This expression implies a linear critical state line in void ratio versus natural logarithm of mean effective stress space with a slope of λ and an intercept of Γ when $p' = 1 kPa$.

The model uses the same bounding, M^b , dilatancy, M^d , and critical stress ratio, M , surfaces that are incorporated in the PM4SAND model. Similar to the PM4SAND model, reaching the critical state causes the bounding and dilatancy surfaces to collapse onto the critical stress ratio surface.

3.4 CLAY MODEL PERFORMANCE AND COMPARISONS

The performance of the discussed clay models will be illustrated and shortly compared to each here. For this purpose, like for sand models, the clay models have been implemented to simulate an undrained cyclic shear test with a cyclic shear stress ratio ($\frac{\tau}{\sigma'_{vc}}$) of 0.15, a vertical effective consolidation stress, σ'_{vc} , of 100 kPa, a lateral earth pressure coefficient, K_0 , of 0.5, and a OCR value of 1. The SANICLAY-B model implementation results are shown in Figure 2. a. The model renders a realistic cyclic stress-strain response. The broad hysteretic response of the model which can be changed with a damage parameter is one of the great features of the model; however, the model shows an unrealistic effective stress lock-up that occurs after a few cycles of loading. This hinders the model's ability to realistically predict the pore water pressure build-up of a cyclically loaded soil.

The performance of the SANICLAY-H mode is shown in Figure 2. b. As can be seen, the applied hybrid flow rule has improved the performance of the earlier version of the model, i.e., SANICLAY-B. This improvement allows the SANICLAY-H model to show a fair response of excess pore water pressure build-up. Also, the butterfly shape of the stress path, which was missing in the earlier version, can be simulated and adjusted by the new constant, w , that is added to the model. However, the problem of the stress path lock-up still exists in this model in a way that the model does not generate any further pore water pressure as soon as it reaches the critical state line.

The PM4SILT model has been implementation results are presented in Figure 5 and 10. The simulations show a broad hysteretic loop which are very similar to observed behavior of clay under cyclic loading reported by Zergoun and Vaid (1994). Also, the model shows that it can predict a significant strength loss and pore water pressure generation without reaching a pore water pressure ratio of 1. It should be highlighted that the model is formulated for low plasticity silts and its performance for high plasticity soils has yet to be meticulously investigated. Also, the lack of a volumetric cap makes the model an imperfect choice when it comes to study of over consolidated clay response.

More comparisons between models can be found in other publications (Kamalzare and Olgun, 2014; Carey and Kutter, 2017; Yang et al., 2018; Eslami et al., 2019).

4 SUMMARIES AND CONCLUSIONS

Five recent advanced constitutive models have been discussed in this study. All the presented models are formulated using the bounding surface plasticity and framed in the critical state concept. The performance of the models is compared based on their ability to predict liquefaction potential in soils having sand-like behavior and cyclic softening potential in those having clay-like behavior. For liquefaction potential study, DM04 and PM4SAND, and for cyclic softening potential, SANICLAY-B, SANICLAY-H, and PM4SILT models have been investigated.

The DM04 model could predict contractive and dilative behavior of sands as well as the butterfly loops of the stress path when reaching a zero mean effective stress. However, the model may show shear strain lock-up at higher number of cycles. The PM4SAND model is improving DM04 model ability by simulating up to very sharp hysteretic loops and avoiding shear strain lock-up at higher shear strain levels. Also, the equation level calibration of the model that has made it to work only with three input parameters is of great interest for practitioners. However, the fact that the model has been simplified to the Lode angle independency and limited in plane strain conditions and cannot be used for other conditions.

The SANICLAY-B model show that it can simulate a broad hysteretic loop the shape of which can be adjusted by an internal input parameter. The main limitation of the model is the stress path lock-up that occur after a few numbers of cycles. However, this is a suitable model for studying different features of clays under cyclic loading such as those concerning, shear modulus reduction curves, damping curves, and cyclic stress ratio versus the shear strain. SANICLAY-H has shown to improve the SANICLAY-B model by adding a butterfly shape to the stress path, modifying the stress path lock-up, and consequently, modifying the pore water pressure build-up.

The PM4Silt model shows that it can generate high level of pore water pressure ratio without necessarily reaching zero mean effective stress. This ability along with broad hysteretic loops of the model can make it a proper choice for problems associated with low plasticity soils under cyclic loading. In addition, like PM4SAND, the dependence of the model to only three input parameters will ease the calibration process of the model. However, the model is limited to in-plane stress conditions and simplified by the removal of the Lode angle dependency, which means it cannot be used for other conditions. Also, the model is not developed for problems associated with consolidation as it is missing a cap. The aforementioned limitations for the clay models will open up a space for a precise comprehensive clay model capable of predicting cyclic softening under dynamic events and complex soil conditions.

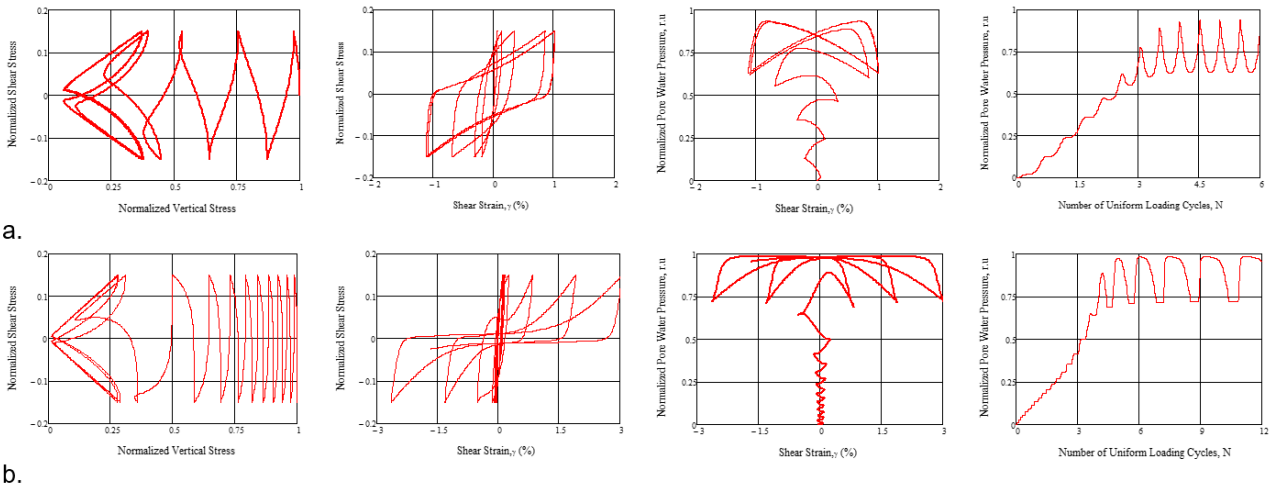


Figure 1. Stress path and pore water pressure response of sand models: a) DM04, b) PM4SAND

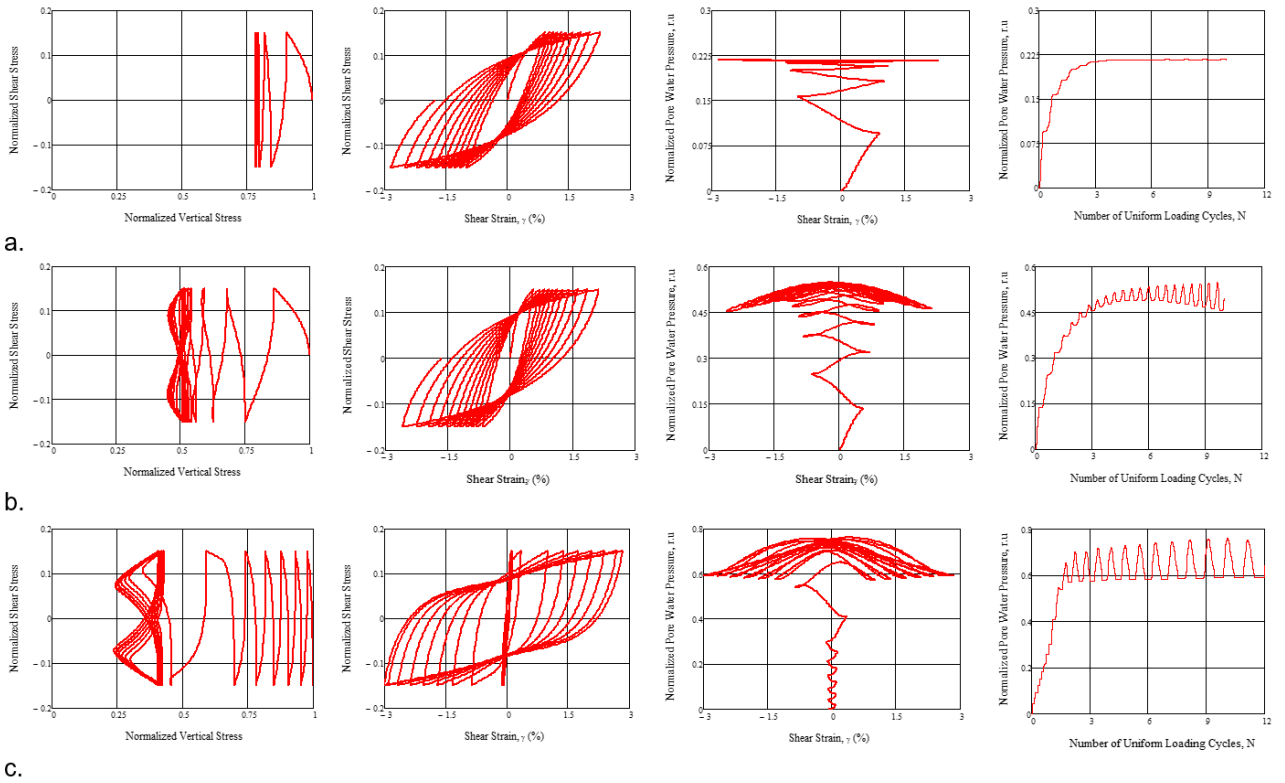


Figure 2. Stress path and pore water pressure response of clay models: a) SANICLAY-B, b) SANICLAY-H, c) PM4SILT

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