Using a limit states approach for ice road design

Samuel A. Proskin
Thurber Engineering Ltd, Calgary, Alberta, Canada

Al Fitzgerald
NOR-EX Ice Engineering Ltd., Kamloops, British Columbia, Canada

ABSTRACT
Natural resource development in northern Canada has often relied on winter roads and ice roads to provide economical site access and resupply routes. These winter roads provide significant socio-economic benefits to the industrial sites and communities they connect. The state of practice for determining allowable loads has largely been based on empirical ice capacity equations developed in the 1970s that are found in most of the ice bearing capacity guides being used in Canada. Industrial ice roads that have sufficient construction quality assurance and operational controls in place can take advantage of higher allowable flexural stresses without compromising ice road safety. This paper explores the application of limit states design (LSD) to the Tibbitt to Contwoyto Winter Road (TCWR). This road is over 300 km long and about 80% of it is built over ice each winter. A Ground Penetrating Radar (GPR) system collects thousands of ice thickness measurements. By exploiting the ice data generated by GPR profiling and the dispatch system for tracking vehicle loads, we can generate probability distribution curves for ice resistance and vehicle load effects for a typical winter road season. An analysis of ice flexural strength data sets found in the literature provided two ice resistance scenarios: A. 1422 kPa ±396 kPa and B. 1730 kPa ±250 kPa. We determined that in levels II and III reliability analyses, the probability of initiating ice cracks is 1 to 2% for scenario A and much lower for scenario B. Further work is necessary to calibrate the load and resistance factors.

RÉSUMÉ
Le développement des ressources naturelles dans le nord du Canada a souvent reposé sur les routes d'hiver et les routes de glace pour un accès économique aux sites d'exploitation et aux routes d'approvisionnement. Ces routes d'hiver fournissent des avantages socio-économiques aux sites industriels et aux communautés qu'elles relient. L'état de la pratique pour déterminer les charges admissibles a largement été basée sur les équations empiriques de la capacité de la glace développées dans les années 1970, que l'on retrouve dans la majorité des guides sur la capacité portante de la glace en usage au Canada. Les routes industrielles de glace qui bénéficient d'assurance qualité et de contrôles opérationnels suffisants peuvent tirer avantage de contraintes en flexion admissibles plus élevées sans compromettre la sécurité des routes de glace. Le présent article explore l'application de la conception aux états limites (LSD) à la route d'hiver de Tibbitt à Contwoyto (TCWR). Cette route fait plus de 300 km et environ 80% de cette longueur est construite sur la glace chaque hiver. Un système d'acquisition avec Radar à Pénétration de Sol (RPS) recueille des milliers de mesures d'épaisseur de glace. En utilisant les données de profilage générées par le RPS ainsi que le système de répartition pour le suivi des charges sur les véhicules, il est possible de générer des courbes de distribution de probabilité pour la résistance de la glace et les effets de charge des véhicules pour une saison de route hivernale typique. Une analyse des ensembles de données sur la résistance à la flexion de la glace trouvés dans la littérature a permis de générer deux scénarios de résistance de la glace: A. 1422 kPa ±396 kPa et B. 1730 kPa ±250 kPa. Nous avons déterminé que pour les analyses de fiabilité de niveaux II et III, la probabilité de créer des fissures sur la glace est de 1 à 2% pour le scénario A et encore plus faible pour le scénario B. Des études supplémentaires sont nécessaires pour calibrer les facteurs de charge et de résistance.

1 INTRODUCTION

Natural resource development in northern Canada has often relied on temporary roads built over frozen lakes, river and frozen terrain to provide economical site access and resupply routes. These ice or winter roads[1] provide significant socio-economic benefits to the industrial sites (industrial ice roads) and communities they connect (public ice roads). The state of practice for determining allowable loads on ice roads (Gold 1971, 1987) has largely been based on empirical ice capacity equations developed in the 1970s. This approach is found in most of the jurisdictional ice road construction and operations guides being used in Canada (Government of Alberta 2013, Manitoba Infrasture and Transportation 2013, NWT Department of Transportation 2015). Ice road design undertaken with these guides is usually performed without any significant engineering analysis. However these guide are usually adequate for public roads that do not experience extensive vehicle volumes or vehicle loads.

While public ice roads typically rely on jurisdictional ice road guides, industrial ice roads may need additional engineering support. Industrial ice road operations are those that service mining, oil and gas or other industrial sites. Industrial ice roads have additional demands in terms of number of loads, size of loads or length of

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[1] ice roads and winter roads will be used interchangeably here to mean temporary roads that make use of floating ice crossings that could be combined with frozen land sections.

season compared to public roads. Vehicles may be hauling large quantities of fuel and construction materials and heavy mining equipment that fall outside the jurisdictional guides experience base (Hayley and Valeriote 1994). Engineering analysis can account for ice properties, load configuration and other operational variables that are not considered explicitly in empirical equations. The most widely adopted engineering analysis models floating ice as a bending plate and calculating an ice thickness that limits the maximum flexural stress to an allowable value. Besides the engineering analysis, industrial ice roads may have construction quality assurance and operational controls in place to maintain overall risk levels despite deploying higher traffic volumes or heavier loads (Hayley and Proskin 2008).

More recently risk management approaches have been developed to account for the quality assurance and operational controls used with ice road design. Proskin and Fitzgerald (2014) discuss the risk management framework developed for the Tibbitt to Contwoyto Winter Road over 7 years. The risk management framework combines developments in GPR ice profiling and vehicle dispatch tracking to better understand and manage the risks associated with ice roads. It also provides more flexibility in handling heavier haul vehicles and higher traffic volumes associated with larger projects or abbreviated hauling seasons.

Limit states design (LSD) offers methods for systematically analyzing the various loads and system resistances (capacity or strength) for geotechnical engineering problems (Canadian Geotechnical Society 2006). The load and resistance factor design approach, based on a factored overall geotechnical resistance (Becker 1996a), offers an opportunity to investigate the limit states of ice bearing capacity. The reliability index and the associated probability of exceeding a limit state may be useful when calibrating the ice resistance factors using field data.

This paper explores the use of the LSD approach in the ice road design by assessing the reliability index for Tibbitt to Contwoyto Winter Road. This winter road typically handles 5000-10000 haul vehicle loads per season bringing in 200,000 to 350,000 tonnes of fuel, supplies and equipment. Over 300 km of road, 80% over ice, is built each winter season and GPR ice profiling collects thousands of ice thickness measurements. By exploiting the ice data generated by GPR profiling and the dispatch system for tracking vehicle loads, we can generate probability distribution curves for ice resistance and vehicle load effects. This enables us to analyze the ice bearing capacity in the context of reliability-based design. We are examining approximate probabilistic methods in generating reliability indices or producing semi-probabilistic load and resistance factors. Our expectation is that we will develop a quantitative basis for assessing risk when considering ice properties and operational controls in the design of ice roads.

2 ICE BEARING CAPACITY AND GOLD'S EQUATION

Floating ice covers carry vehicle loads through the buoyancy of the ice cover and the resulting hydrostatic pressure on the bottom of the ice sheet(U.S. Army Corps of Engineers 2006). For large ice sheets, the ice sheet bends under the load to the extent that the weight of displaced water is equal to the load (Archimedes' principle). The ice sheet is effectively acting as a raft by distributing the load over a large area while floating on the water. If the bending stresses in the deforming ice cover (raft) exceed the strength of the ice, then the ice will crack to relieve the stresses.

Short term moving loads on ice covers are typically modeled as an elastic plate floating on an elastic foundation. A number of solutions have been developed depending on the assumptions made about the ice properties or the geometry (Wyman 1950, Kerr 1976). The solution used in this paper is one proposed by U.S. Army Corps of Engineers (U.S. Army Corps of Engineers 2006) with the equation:

$$\sigma_{max} = \frac{CP}{h^2} \qquad [1a]$$

$$P = \frac{1}{C}\sigma_{max}h^2 \qquad [1b]$$

where P is the load over a uniformly loaded area of radius a, h is the ice plate thickness, $\sigma_{max}$is the maximum flexural stress

$b = \sqrt{(1.6a^2 + h^2)} - 0.675h;$ when a <1.724 h or b=a ; when a>1.724h γ_w is the unit weight of water, E is the elastic modulus of ice and v is the Poisson's ratio.

Using this equation, the maximum ice flexural stress can be calculated for a given vehicle load, contact area and the ice thickness and then this stress is compared with the allowable flexural strength for the ice. Alternatively, an allowable ice stress and vehicle load is used to estimate the required ice thickness for the associated vehicle.

Although this equation is well established, most ice road users rely on simplified equation developed by Lorne Gold of the National Research Council through his influential study of ice bearing capacity (Gold 1971).

$$P = Ah^2 \qquad [2]$$

P is the load in kg, h is the ice thickness in cm A is a semi-empirical parameter related to C and σ_max but is interpreted from field data.

Although the survey showed an appreciable overlap between the successful and failed ice observations, some of the failures occurred when the vehicles left a prepared area and travelled over untested ice cover. Based on the survey data, three lines representing slopes of A=3.5, 7.0 and 17.6 have been proposed for providing limits on ice bearing capacity depending on the user's risk tolerance:

- A=3.5 is interpreted as low risk as it gives the lowest load and almost all failures occur at higher loads.
- A=7 is what Gold recommended as an upper limit.
- A=17.5 is very high risk but there are a few successful ice covers at loads above this.

The jurisdictional guides prepared in 1970s to the 2000's recommended A values between 3.5 and 7 with varying levels of information on risk management practices such as ice thickness measurements and vehicle speed limits. However, the increasing demands of industrial roads have raised questions about quantifying the relative risk of failure using this approach.

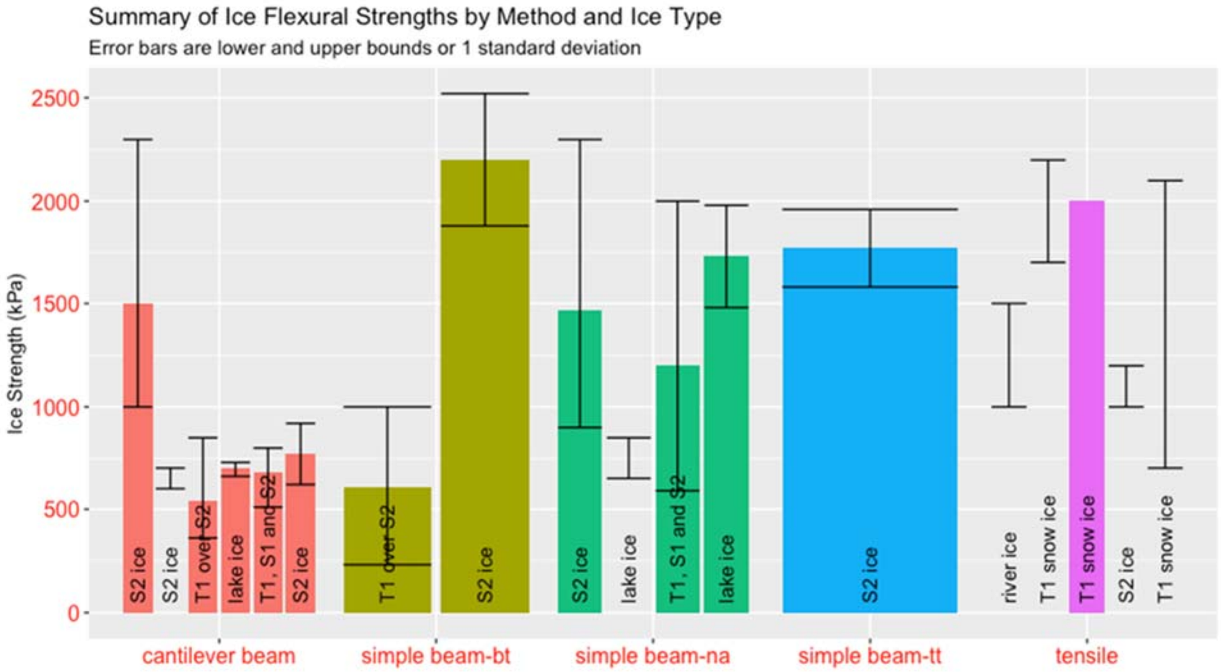

Figure 1. Summary of Ice Flexural Strengths

3 FAILURE CRITERION AND ICE STRENGTH

The most widely accepted failure criterion for ice cover bearing capacity under short term loads is taken as the maximum bending stress that would exceed the ice tensile strength and initiate a radial crack in the ice (Michel 1978). This criterion is considered conservative because the breakthrough failure occurs at loads about twice this value and breakthrough failure is preceded by radial and circumferential cracks.(Sodhi 1995). Since ice behavior at typical vehicle -induced strain rates is brittle, radial and circumferential cracking can occur rapidly or be hard to detect. (Peters et al. 1982) also relied on initial cracking because there was no reliable method for predicting the ultimate bearing capacity for an ice sheet. Under the conditions of a bending ice sheet, the stresses within the entire ice sheet at crack initiation in the sheet are related to the tensile or flexural strength of the ice (Timco and O'Brien 1994).

The strength of freshwater ice is required for a number of ice engineering problems including ice breakup, ice forces on structures and ice bearing capacity (Gold 1987). Because ice is a solid that is within a few degrees of its melting point, the strength of ice varies with its temperature along with grain size, ice type, specimen orientation and size, loading rate and strain rate (Timco and Frederking 1982). Temperature, grain size, and ice type reflect environmental conditions while loading rate and strain rate reflect loading conditions. In general ice strength increases with reduced (colder) temperatures and increased strain rate. Ice is ductile under low strain rates and becomes brittle at higher strain rates. Although these variables should be considered when reviewing ice strength data, many datasets do not document them making it difficult to assess their influence on the results.

Due to the technical difficulty in conducting reliable tensile tests on ice, flexural strength is viewed as alternative method to assess ice strength under bending. Ice flexural strength measurements are conducted as either cantilever beam tests or as 3 or 4-point simple beam tests. All require making assumptions about the mechanical behavior to interpret the load results. Cantilever beam tests are commonly used in the field to measure ice strength as a beam can be cut into the ice cover to test a large section of ice in situ conditions. Three -point simple beam tests are usually done on smaller laboratory specimens, as they require careful alignment in the loading frame to reduce torsional stresses. Four-point beam tests are preferred over simple beam tests as failure takes place over a larger volume of material. However, they require more complex loading arrangements to ensure uniform loading conditions. The cantilever beam measurements of fresh water ice strength is less than simple beam results due to the stress concentrations at the root of the beam.

Flexural strength data for freshwater ice has been compiled by Ashton (1986) and Timco and Frederking (1982). Figure 1 provides a summary of results compiled by (Ashton 1986) for cantilever beam tests and three categories of simple beam tests: tb: tensile bottom; tt: tensile at top; na: not available. The freshwater ice type (if identified) is also noted at the bottom of each bar. The coloured bars provide the mean strength (when available) and the error bars provide the upper and lower bounds (or 1 standard deviation if available). The variation in the flexural strength results makes it difficult to choose an allowable ice strength. For example, both simple beam and cantilever beam mean flexural strengths can vary as much as a factor of 3 to 4: 590 to 2400 kPa for simple beam and 500 to 1500 kPa for cantilever beam. In comparison the ice tensile strength varies between 500 and 2200 kPa. Part of the variation is due to ice temperature variation, ice type and strain rate. In an compilation of 1556 measurements of flexural strength, (Timco and O'Brien 1994) estimated the simple beam mean flexural strength for freshwater ice as 1730 kPa ±250 kPa (temperature less than -4.5 °C). This is comparable to 1770 ±190 kPa for simple beam (bottom tension) flexural strength reported by (Timco and Frederking 1982).

The allowable stress in the ice for ice bearing capacity has been estimated from field experience. Hayley and Valeriote) 1994 back-calculated the maximum ice tensile stresses from the operations of the Lupin Winter Road and found them to range between 600 and 700 kPa. (Masterson 2009) recommended 550 kPa as a flexural strength for design. Assuming an ice flexural strength of 1730 kPa and an allowable ice strength of 600 kPa, the ratio of ultimate to allowable provides a putative factor of safety of 2.9

4 LIMIT STATES APPROACH TO ICE BEARING CAPACITY

Limit state design was first proposed by Peters et. al. (1982) when their design procedure was for analyzing sea ice bearing capacity. Their procedure adopted a serviceability criterion by reducing the likelihood for initiating tensile cracking of the ice sheet due to vehicle loads. Their paper was a significant advancement in showing how limit states design can be used to individually factor the loads and resistances. However, it did not discuss the reliability or the probability of initiating ice cracks.

For this paper the authors adopted the load and resistance factor design (LFRD) approach used by Becker (Becker 1996a, 1996b) in his work on the foundations of the National Building Code. In LFRD partial factors allows one to separately assign factors to resistances (e.g., materials strengths) or loads (e.g., static and dynamic) that may have individual variations instead of relying one global value. This also allows one to individually calibrate factors to engineering experience using probability and reliability concepts. The LRFD equation is:

$$\Phi R_n \geq \sum \alpha S_n \qquad [3]$$

where $\Phi R_n$is the factored resistance with:
Φ is the resistance factor;
$R_n$is the nominal (characteristic) resistance= $\bar{R}/k_R$
$\bar{R}$ is the mean resistance
$k_R$is the ratio of mean value to the characteristic value for resistance;
and $\sum \alpha S_n$is the sum of the factored loads with:
α is the load factor
S_n is the nominal (specified) load = $\bar{S}/k_S$
$\bar{S}$ is the mean load(s)
$k_S$is the ratio of mean value to the specified value for the load(s).

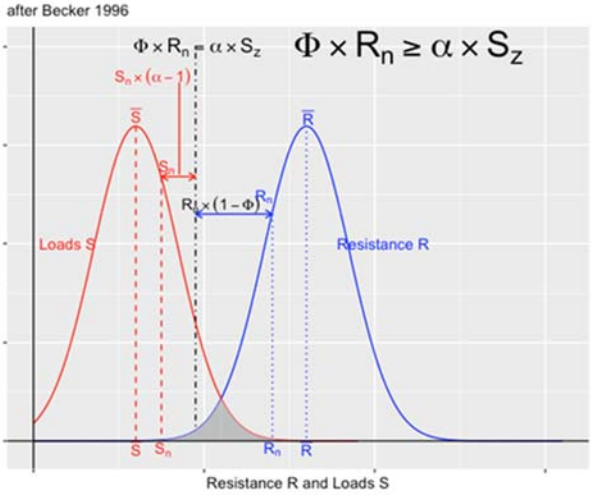
after Becker 1996

Figure 2. LSD nomenclature in probability density plots

Reliability based design can extend the LFRD analysis by treating the resistances and loads as random variables instead of deterministic ones through probabilistic design. In this case you require the actual probability density functions (PDF) or the frequency distributions (e.g., histograms) are measured or known. Provided these PDFs are known or can be estimated, the probability of failure (i.e., exceeding the resistance) is related to the shared area representing the overlap between the load and resistance curves in Figure 2.

In this paper the authors chose to evaluate the probability of exceeding the resistance associated with initial ice cracking through two methods:

1. level III probabilistic design where the probability density plot (PDP) of the ice resistance is estimated from published flexural strength data for freshwater ice and the PDP of the applied load ice stresses are estimated from dispatched vehicle traffic and their associated ice stresses.
2. level II (approximate) probabilistic design where we use the mean and standard deviations of the ice resistance (ice flexural strength) and the vehicle ice stresses, assume a normal distribution, and then calculate the reliability index and the probability of exceeding the resistance.

In both cases vehicle traffic and design load data are extracted from the 2014 Tibbitt to Contwoyto Winter Road season Exdocs database (TCWR JV 2014). The data analysis was done with the R programming language(R Core Team 2015) and several of its packages.

5 LEVEL III PROBABILISTIC DESIGN FOR RELIABILITY

5.1 Determining Load PDP from Vehicle Loads

The Tibbitt to Contwoyto Winter Road (TCWR) Joint Venture dispatches and tracks vehicles transporting fuel, equipment, and supplies to the Joint Venture mines along its route. The database records of all of the haul vehicles dispatched, along with their destinations, payloads, operators, carriers, payload weights, dispatch times, and Gross Vehicle Weights (GVWs). In 2014 there were 7055 vehicle that moved over 240,000 tonnes. By combining the dispatch times and GVWs from Exdocs with the ice thickness monitoring, the stresses in the ice cover can be estimated for each vehicle using the bending plate equation [1]. Through this exercise an ice load (ice stress) PDF can be prepared for the 2014 season. To carry this out efficiently the following assumptions were made:

- only Super B fuel haul vehicles were selected as they account for most of the traffic and tonnage.
- a standard Super B vehicle configuration was used
- the ice thickness used in the calculation was the minimum recorded for the ice road at the time the vehicle was dispatched
- ice stresses were estimated using an algorithm based on linear regressions between vehicle load and ice stress for various ice thicknesses.

Figure 3a below shows a histogram of the vehicle counts for each bin of ice stress while the Figure 3b provides the probability density plot of the same data. It is clear this is not a normal distribution as the data is skewed towards a region around the nominal allowable ice stress of 500 to 600 kPa. The stresses below 500 kPa represents lightly loaded vehicles or vehicles that travelled over the road near the end of season when the ice thickness reached a maximum. Higher stresses are permitted for special loads that have additional risk measures in place. The PDP in Figure 3b will be compared with the ice stress PDP. The mean vehicle stress is 564 kPa with standard deviation of 45 kPa.

Figure 3 Vehicle ice stress plots (a) histogram and (b) density

5.2 Determining Ice Resistance PDP from Lab and Field Measurements

In this section we prepare a PDP going back to the original ice flexural strength datasets prepared by Gow and Frankenstein (Frankenstein 1963, Gow and Langston 1975). The database contains 1017 laboratory and field measurements but we have taken a subset by filtering by test method and ice temperature:

- simple beam flexural strengths with tension at the bottom of the beam to simulate the ice loading
- ice temperatures colder than -1.9 C to simulate ice temperatures that represent an average (air temperature at surface and 0 C at the bottom).

Figure 4a below shows a histogram of the ice flexural strengths for each bin of ice stress while the Figure 4b provides the probability density plot of the same data. This more closely resembles a normal distribution compared to the vehicle load PDP. The mean ice flexural strength is 1422 kPa with a standard deviation of 396 kPa.

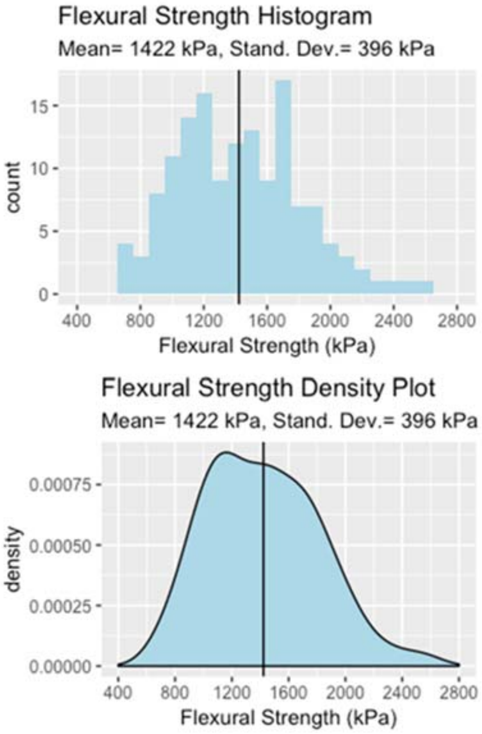

Figure 4 Ice flexural strength plots (a) histogram (b) density.

5.3 Probability of Exceeding the Flexural Stress

The probability of the vehicle stresses exceeding the flexural stress of the ice is estimated from the overlapping zones of the vehicle stress PDP and the ice strength PDP. Figure 5a shows the PDP for both of them and 5b is a zoom of the overlap area. Using the R package overlapping (Pastore 2018), the estimated overlap area is 1.43%. Consequently there is a 1.43% chance of that the vehicle induced ice stress would exceed the ice flexural strength and cause initial cracking of the underside of the ice sheet.

6 LEVEL II RELIABILITY ANALYSIS

The level II reliability analysis was conducted for two ice strength scenarios each with a mean strength and standard deviation:

- scenario A: 1422 kPa ±396 kPa; temperatures less than -1.9 C, simple beam with bottom in tension (noted above)

- scenario B: 1730 kPa ±250 kPa; temperatures less than -4.5 °C; simple beam for freshwater ice (Timco and O'Brien 1994)

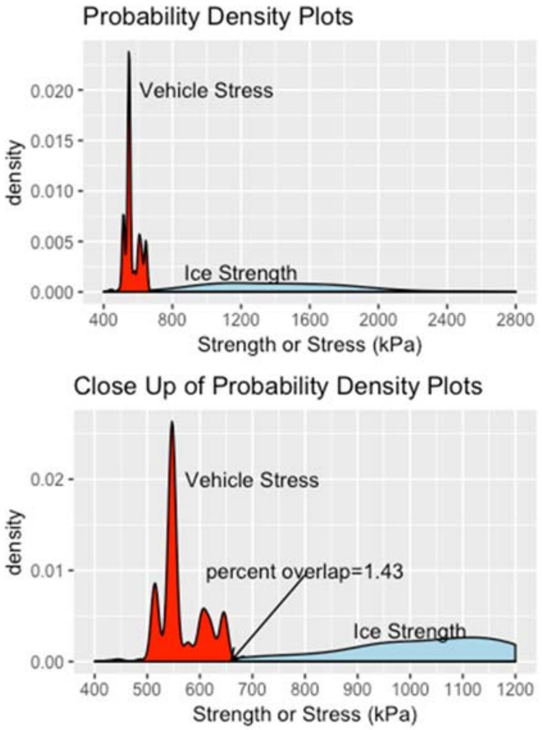

Figure 5 Probability density plots of load and ice resistance (a) full scale (b) close-up.

The simplified reliability index β is defined for $V_R \, and \, V_s < 0.5$:

$$\beta = \frac{ln\frac{R_n}{S_n}}{\sqrt{V_R^2+V_S^2}} \quad [4]$$

$V_R = \frac{\bar{R}}{sd_R}$ with $\bar{R}$ the mean resistance and sd_R being its standard deviation and $V_S = \frac{\bar{S}}{sd_S}$ with $\bar{S}$ the mean load and sd_S being its standard deviation. The probability of the load exceeding the flexural strength of the ice p_es estimated from the reliability index using:

$$p_e = 460 \times e^{-4.3\beta} for \; 2 < \beta < 6 \quad [5]$$

Table 1 summarizes the parameters and the results of the level II reliability analysis calculation for these two ice flexural strength scenarios.

Table 1. Parameters and results of level II reliability analysis.

Parameter	Scenario A	Scenario B
$\bar{R}$ (kPa)	1422	1730
k_R	1.26	1.12
R_n (kPa)	1125	1543
V_R	0.278	0.145
$\bar{S}$ (kPa)	564	564
k_S	0.98	0.98
S_n (kPa)	575	575
V_S	0.080	0.080
β	2.32	5.98
p_e	2.2%	3.2E-07%

The reliability index for scenario A converts to a probability of exceeding the flexural strength of 2.2% that is consistent with the Level III calculation. In comparison the scenario B reliability index is essentially zero. The increase in mean strength from 1422 to 1730 kPa and the reduction in standard deviation from 396 kPa to 250 kPa both have dramatic effects on reducing the probability of the loading exceeding the ice flexural strength.

The reliability analysis suggests that there is 2% chance of exceeding the ice flexural strength and initiating a tensile crack in an ice sheet with strength characterized by scenario A. However, this does not mean there's a 2% chance of vehicle breaking through the ice for the following reasons:

- the failure criterion is initiating the first crack by exceeding the flexural strength of the ice sheet;
- ice sheets have the capacity to support loads after the initial cracking with breakthrough loads being 3 to 5 times higher in field tests.
- dynamic loading of the ice has been excluded assuming vehicles are travelling below the critical speed
- ice resistance is not only related to flexural strength but also pre-existing cracks, ice type, temperature, and loading rate.
- stresses developing in the ice sheet are more complex than the bending stresses used in the current bearing capacity model (Peters et al. 1982)

The reliability analyses will require additional refinements and calibration to empirical data. One option is to link the crack initiation stresses to field observations of cracking under vehicles or ice road operational conditions. Another is to find a relationship between the frequency of ice cracking to the frequency of breakthrough failure. For example, Hayley and Proskin (2008) observed that the between 2000 and 2007, 58000 vehicles traveled over Tibbitt to Contwoyto Winter Road with only one potentially fatal ice incident. This converts to a probability of ice failure below 0.002% compared to 2% for ice cracking found in this study.

7 CONCLUSIONS

This initial exploration of the applying LSD concepts to ice bearing capacity analysis has tried Level II (approximate) probabilistic and Level III probabilistic design methods. The ice resistance and ice road vehicle load probability density datasets. The serviceability criterion for ice cover bearing capacity under short term loads is taken as the maximum bending stress that would exceed the ice flexural strength and initiate a crack. This criterion allows us to characterize ice resistance in terms of flexural strength. About 1500 ice flexural strength measurements from the literature were grouped according to ice type, temperature and test method and analyzed to derive probability density plots and characteristic strengths for two scenarios: A with a mean of 1422 kPa ±396 kPa and B: mean of 1730 kPa ±250 kPa. Using the TCWR's vehicle dispatch records for 2014, the serviceability criterion bending stress analysis was used to estimate the applied vehicle loads (stresses) to provide a probability density plot and mean value of 564 kPa ±45 kPa. From the reliability analyses we estimated a probability of exceeding the flexural stresses in scenario A as 2.2% and less than 3.2-07 % in scenario B.

Calibrating the load and resistance factors is the next step in applying LSD to ice engineering bearing capacity. Additional work is also needed to explore the relationship of probability of initiating cracks to the probability of vehicle breakthrough failure. The practicing engineer continues to exercise judgment in selecting the appropriate ice flexural strength similar to selecting the appropriate A value in Gold's equation and the associated risk management controls.

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