

Modelling the dependency of the soil-water retention behavior on the initial void ratio in unsaturated soils

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ABSTRACT

The soil–water retention curves (SWRC) of soil play a key role in unsaturated soil mechanics, which have wide applications particularly in geotechnical and geo-environmental engineering. SWRCs express the ability of unsaturated soils to retain water at different negative pore-water pressure levels. The common approach in approximating the SWRCs is to use empirical mathematical models that cannot distinguish the physical mechanisms responsible for the hydraulic retention behavior. In addition, the SWRCs are strongly dependent on the variation of void ratio during both hydraulic loading such as the infiltration and evaporation or the mechanical volume change or shearing. This paper presents a modified water-retention model that not only distinguishes the physical mechanisms of hydraulic retention, i.e., the capillarity and surface adsorption but also accounts the initial void ratio dependency of this phenomenon. The validity of this model is examined through an array of experimental data obtained at Université de Sherbrooke in addition to datasets collected from literature. The results indicate the soil-water retention model presents a more realistic prediction of hydraulic retention which has important consequences for the interpretation of stress-strain behavior and the numerical modelling of coupled flow-deformation problems in unsaturated state.

Les courbes de rétention sol-eau des sols jouent un rôle clé dans la mécanique des sols non saturés, qui ont de vastes applications notamment en géotechnique et en génie géo-environnemental. Ces courbes expriment la capacité des sols non saturés à retenir l'eau à différents niveaux de pression interstitielle négative. Pour approximer, l'approche courante consiste à utiliser des modèles mathématiques empiriques qui ne permettent pas de distinguer les mécanismes physiques responsables du comportement de rétention hydraulique. De plus, ces courbes sont fortement dépendants de la variation de l'indice des vides pendant le chargement hydraulique comme l'infiltration et l'évaporation ou la déformation volumique due au chargement mécanique et le cisaillement. Dans cet article, on présente un modèle modifié de rétention d'eau qui non seulement distingue les mécanismes physiques de la rétention hydraulique, c'est-à-dire la capillarité et l'adsorption superficielle, mais qui tient également compte la dépendance de ce phénomène à l'indice des vides initiale. La validité de ce modèle est examinée en utilisant les données expérimentales obtenues à l'Université de Sherbrooke ainsi que des ensembles de données provenant de la littérature. Les résultats indiquent que le modèle de rétention sol-eau présente une prédiction plus réaliste de la rétention hydraulique, ce qui a des conséquences importantes pour l'interprétation du comportement mécanique et pour la modélisation numérique des problèmes liés à l'écoulement et à la déformation en état non saturé.

1 INTRODUCTION

The understanding of the hydraulic retention behavior of unsaturated soils is an essential aspect in both numerical studies and theoretical constitutive modellings. It is known that the amount of water retained in pores is related to the state of free energy of in unsaturated state which is commonly depicted using the soil-water retention curve (SWRC). Disregarding the osmotic potential which represents the effect of solutes dissolution in the soil water, the matric potential is considered the main component of free state of energy in unsaturated state. The matric potential shows the deficit in the energy of water with respect to the ambient pore-air pressure. There are two physical mechanisms responsible for the variation of matric potential, i.e., Capillarity and surface adsorption. Water retained by the Capillarity results from the generation of local tensile forces due to the presence of a curved air-water interface in soil pores. On the other hand, the adsorption of water to the soil particles occurs due to the presence of

exchangeable cation hydration, mineral surface, or crystal interlayer surface hydration according to Lu and Khorshidi (2015). Other than particle shape and orientation, the capillarity mechanism depends on the available volume of pores which in turn is represented by the particle granulometry and the state of compaction of the medium.

Various laboratory and in-field techniques have been developed to obtain the SWRC. Their main hindrances are that these techniques are often time-consuming and expensive since they need trained technical personnel to perform the tests. Modelling the SWRC is also a challenging research topic since it requires a sound understanding of hydraulic retention. Various conceptual and empirical models have been proposed to predict the SWRC covering full range of matric potential. Nevertheless, the empirical models are in their nature mathematical shape-fitting equations and may not truly represent the soil-water interactions. These

models such as those proposed by Gardner (1958), Brooks and Corey (1964) and Van Genuchten (1980) need the introduction of the fitting parameters to simulate the experimental results. Some of these parameters may be attributed to the physical soil properties such as the air-entry value (AEV) and pore-size distribution whereas others, such as the residual water content and residual potential are still vague. Recently, given the shortcomings of the empirical models, new models such as those by Konrad and Lebeau (2015), Zhou et al. (2016) and Lu (2016) have been proposed that can explicitly account for capillary and adsorptive mechanisms. Yet, these models does not consider the initial state of unsaturate soil as an input. The soil-water interaction depends on the soil type and structure, method of compaciton, stress history and initial void ratio or density. As stated by Sun et al. (2006), regarding the constitutive modeling of a compacted soil, the void ratio and stress history are among the dominant factors affecting the SWRC. Therefore, it is vital to investigate how the initial void ratio variation can affect the hydraulic retention.

The main objective of this study is to investigate the application of a modified soil-water retention model that takes the effect of initial void ratio into consideration. A mathematical basis for the equation is described and a best-fit procedure is outlined to obtain the parameters for the equation. This paper also presents the results of a series of experimental tests obtained at Université de Sherbrooke along with the interpretation of data available in literature that are used to verify the proposed model.

2 PROPOSED WATER RETENTION MODEL

The backbone of the proposed model is given by Lu (2016) where he proposed a generalized hybrid model on the occurrence of physical mechanisms responsible for the pore-water deficiency. Considering the double porosity pore model, the general formulation of this hydraulic retention model is shown in Eq. 1:

$$\theta(\psi_m) = \theta_{cap}(\psi_m) + \theta_{ad}(\psi_m) \quad [1]$$

Where ψ_m is the matric potential, θ_{cap} is the amount of water retained in pores due to the capillarity defined by the volumetric water content. θ_{ad} is the volumetric water content corresponding to the adsorbed water.

The adsorptive component of Eq. 1 is devoted to the solid-liquid phenomenon that can be described using Freundlich model (Jeppu and Clement 2012). The Freundlich model describes the amount of adsorbate on a flat adsorbent at the thermodynamic equilibrium of energy with the ambient adsorbate. Lu (2016) suggested that θ_{ad} can be obtained using Eq. 2 as following:

$$\theta_{ad}(\psi_m) = \theta_{max} \left\{ 1 - \left[\exp\left(\frac{\psi_m - \psi_{max}}{\psi_m}\right) \right]^M \right\} \quad [2]$$

In equation above, θ_{max} is the adsorption capacity that ranges from 0.01 to 0.30 for coarse materials to fine soils

with high level of activation number (Lu, 2016). ψ_{max} is the highest level of matric potential that a soil can undergo which can be viewed as a limiting value of pressure deficiency (Fredlund and Xing, 1994). This value ranges from the 5.8 to 6.0 logkPa which is highly dependent on the temperature and the humidity at near fully dry state (Beal, 2013). M is the adsorption strength which expresses the intensity of intermolecular interaction acted on particle of mineral surfaces and varies from 0.01 to 1.0 as suggested by Lu (2016). In general, the adsorption strength of a given soil is only controlled by mineral type and quantity while it is less affected by pore-size distribution. In other words, the variation of pore volume due to mechanical loading or compaction state might not significantly affect the adsorption capacity of a soil and θ_{ad} is independent of the void ratio changes. This seems logical since the studies of Konrad and Lebeau (2015), Zhou et al. (2016) showed that the inter-particle stress of particles surrounded by liquid film is not dependent on the volume of retained liquid and is solely related to the capillary component of water retention.

On the other hand, given the premise of double porosity pore model, both capillary and adsorption mechanisms co-exist from fully saturated state to the cavitation potential as stated by Frydman and Baker (2009). Within this range, there is a continuous water exchange between both capillary and adsorbed water since their state of energy may not be equal. At equilibrium and beyond the cavitation potential, the capillarity cease to exit the sole mechanism responsible for water retention is the adsorption mechanism. Lu (2016) expressed the capillary water below the cavitation potential using Eq. 3 as shown below:

$$\theta_{cap}(\psi_m) = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{\psi_m - \psi_c}{\sqrt{2}\sigma_c} \right) \right] \frac{\theta_s - \theta_{ad}(\psi_m)}{[1 + (\alpha\psi_m)^n]^{1-1/n}} \quad [3]$$

Where $\operatorname{erf}()$ is the error function and ψ_c is the the mean cavitation potential. σ_c shows the standard deviation of the pore radius and α, n are fitting parameters of Van Genuchten's model. The mean cavitation potential varies from 0.1 to 25 MPa while $\sigma_c = 0.5\psi_c$ is suggested by Lu (2016) for simplifying the model. As a result, the hydraulic state of an unsaturated soil within the whole range of matric potential can be defined using Lu's model and seven fitting parameters.

However, Lu's model does not account the effect of initial state of soil and its relation on retention capacity of a soil. The effect of initial state of soil on the SWRC is assumed to change in the initial void ratio, that is:

$$\theta_{cap} = f(\psi_m, e_0) \quad [4]$$

Where e_0 is the initial void ratio during the measurement of the SWRC. Based on the experimental results of Warabe, et al. (2000), Kawai et al. (2000), and Sun et al. (2006), it is proven that any change in void ratio results in a shift in the trend of SWRC. This seems reasonable because the void ratio reduction essentially means that the volume of pores is lessen and consequently, a higher level of energy is required to

empty the pores. Many researchers such as Gallipoli et al. (2003), Parent et al. (2007), Nuth and Laloui (2008), Tarantino (2009), Salagar et al. (2010), Salagar et al. (2013) and Pasha et al. (2017) attempted to relate the void ratio-dependency of SWRC to initial void ratio and the deformation induced by matric potential. There are certain shortcomings to these models. For instance, Gallipoli's model homogenously shifts the whole SWRC towards higher matric potential while there are some experimental observations where void-ratio dependency is only related to the capillary domain of hydraulic retention as suggested by Tarantino (2009).

In the proposed model which is on the basis of Lu's model, the void ratio-dependency of SWRC is associated with the changes in the capillary component. In other words, the adsorption component is not affected by the void ratio changes. This assumption seems logical since the adsorbed water to the soil particles is related to the interparticular force interactions, notably van der Waals attraction, electric double-layer interactions that are not dependent on the volume of pores. Such assumption is also supported by the studies of Salagar et al. (2010) and Romero and Vaunat (2000) where the water retention behavior at high suctions is independent of the mechanical parameters such as void ratio. The capillary part of the proposed model is shown in Eq. 5:

$$\theta_{cap}(\psi_m, e_0) = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{\sqrt{2} \psi_m (1 - \varepsilon_v)^{b_1}}{\psi_{c(ref)}} - \frac{\theta_s - \theta_{ad}(\psi_m)}{\sqrt{2}} \right) \right] \frac{1}{\left[1 + (a_{ref}(1 - \varepsilon_v)^{b_2} \psi_m)^n \right]^{1-1/n}} \quad [5]$$

Where $\psi_{c(ref)}$ and a_{ref} are the mean cavitation potential and air-entry value related fitting parameter of the reference test. b_1 and b_2 are the material's constants. ε_v is the volumetric deformation corresponding to the void ratio changes defined by Eq. 6. In this equation, $e_{0(ref)}$ is the initial void ratio of the reference test.

$$\varepsilon_v = \frac{\Delta V}{V} = \frac{\Delta e}{1 + e_{0(ref)}} \quad [6]$$

3 TESTING PROGRAM AND DATASETS

To encompass a large range of retention properties of soils, the present study consists of a granular soil (i.e. cohesionless soil in saturated state), a clayey silt and Ascot Corner silt. The SWRCs of the granular soil and Ascot Corner silt were obtained at Université de Sherbrooke using the modified Tempe cell that is able to measure vertical deformations as low as 0.02% (Maleksaedi et al. 2016; 2019). The SWRCs of clayey silt at different densities were obtained by Gao and Sun (2017) using both pressure plate method and vapor equilibrium technique with saturated salt solutions.

Table 1 shows the physical characteristics of studied materials. For the glacial till and Ascot Corner

silt, the specific gravities, G_s , were obtained according to ASTM D854-14, Standard Test Methods for Specific Gravity of Soil Solids by Water Pycnometer. The particle size distributions were measured by following ASTM D6913/ D6913M-17, Standard Test Methods for Particle-Size Distribution (Gradation) of Soils Using Sieve Analysis, and ASTM D7928-17, Standard Test Method for Particle-Size Distribution (Gradation) of Fine-Grained Soils Using the Sedimentation (Hydrometer) Analysis. The minimum and maximum void ratios, i.e., e_{min} and e_{max} , were measured as reported by Muszynski (2006). In the case of Ascot Corner silt, the obtained e_{min} and e_{max} are merely given as a suggested range of the possible deformation and must not be accounted as extreme values of void ratio. The clayey silt known as Pearl clay, is composed of 26% clay fraction ($< 2\mu m$) and about 74% silt fraction. The clay mineralogy compositions were determined by the X-ray diffraction test showed that the soil includes quartz, pyrophyllite, and kaolinite in the dominant order.

Table 1 Physical properties of studied soils

Soil ID	Glacial till	Ascot Corner silt	Clayey silt
G_s	2.73	2.66	2.71
e_{max}	0.686	1.179	-
e_{min}	0.298	0.592	-
c_u	8.33	7.15	-
c_c	0.69	1.79	-
LL(%)	-	-	0.43
PI(%)	-	-	17.5

4 RESULTS AND DISCUSSIONS

4.1 Calibration of proposed model

Using Eq. 2 and Eq. 5, the SWRCs of three materials were examined. The model is only capable of modeling the main drying or wetting paths and cannot take the hydraulic hysteresis into consideration. Shifting the SWRC due to the initial void ratio changes is attributed to α which is commonly considered as the inverse of the air-entry value. In addition, the cavitation potential, ψ_c depends on the pore size distribution of soil and as soil structure is densified, the pore size distribution moves towards smaller radii and consequently increasing the cavitation potential.

For glacial till, as shown in Figure 1, the initial void ratios were 0.66, 0.62 and 0.60 which respectively resulted in the air-entry values of 1.30 kPa, 1.78 kPa and 4.96 kPa. Figure 1 also depicts the fitting parameter and the corresponding prediction of SWRC by the proposed hybrid model. To fit the experimental data, the first test with the void ratio of 0.66 was set as the reference SWRC. Afterwards, drawing the changes in the cavitation pressure and air-entry value with volumetric deformation suggested that both parameters can be fitted by a power function. As a result, by assuming θ_{max} , M , ψ_{max} and n constant, the SWRCs can be predicted in comparison to reference test result. As it can be seen,

the predictions are in good agreement with test data. It must be reminded that the model requires further adjustment to take the suction-induced volume changes into consideration. Yet, the scope of this study, such assessment was ignored.

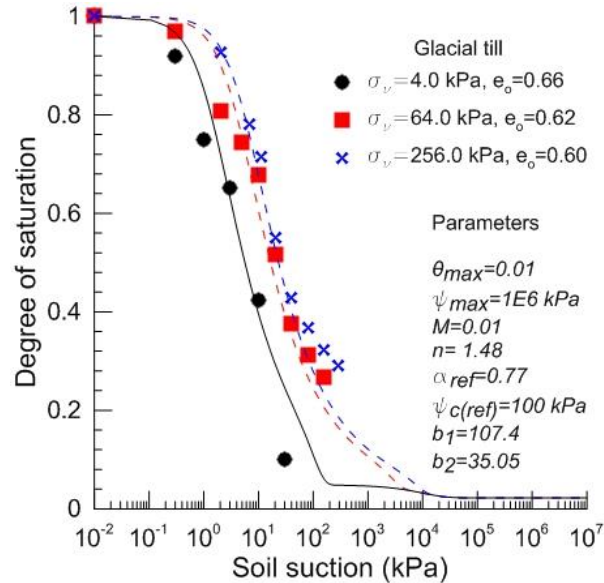


Figure 1 Comparison between the predicted SWRCs with the proposed model and the experimental results for the glacial till.

The advantage of using the proposed model is that it removes the confusion over the definition of residual state and the required treatment to correctly predict its void ratio-dependency. The results herein confirmed the finding of Gao and Sun (2017) and Gao et al. (2018) the residual suction value and the residual water content merge into a unique value even when the specimen densities changes.

Figure 2 shows the prediction and experimental results of Ascot corner silt using Eq. 2 and Eq. 5. The initial void ratios of tested specimens were 1.22, 1.20 and 1.15 respectively. These values were corresponded to the 14 kPa, 17 kPa and 28 kPa of air-entry value. The slight difference between the predicted and measured values are attributed to the suction-induced volume changes that were occurred during hydraulic loading. In fact, emptying the voids as soil suction increases may lead to the reduction of pore size which in return can influence the degree of saturation. However, taking into account this volume change during water retention test is out of the scope of this study.

It should be noted that in the capillary part of the model uses the same equation as Van Genuchten (1980) by introducing a simplified fitting parameter, n which reflect the overall geometry of SWRC. Fredlund and Xing (1994) suggested that such using only n reduces the flexibility of the Van Genuchten's model. Yet, in Lu's model, in order to reduce the number of fitting

parameters, the suggestion of Fredlund and Xing (1994) was ignored.

Regarding the Clayey silt results obtained by Gao and Sun (2017), as demonstrated in Figure 3, the air-entry values of 37kPa, 52 kPa and 73 kPa were corresponded to the initial void ratios of 1.32, 1.21 and 1.09 respectively.

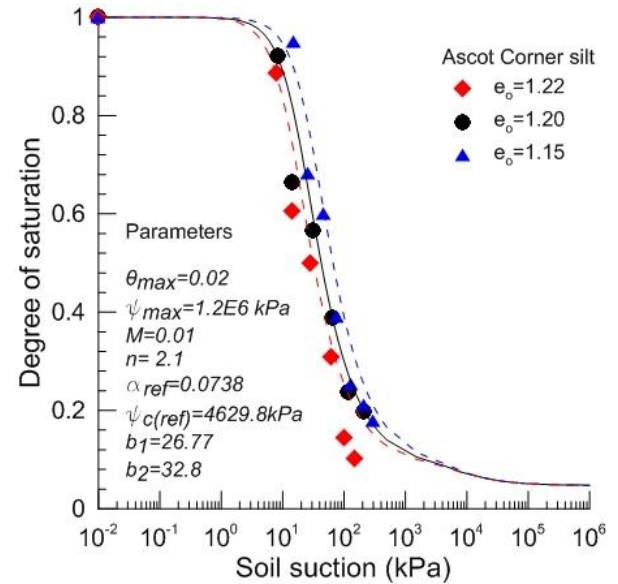


Figure 2 Comparison between the predicted SWRCs with the proposed model and the experimental results for Ascot corner silt.

Similar to their observations, using the proposed model, it was revealed that near residual state, all predicted SWRCs converge to a unique value. Yet, unlike their interpretation, using the proposed model, one can distinguish between the mechanisms responsible for retaining water in pores. In fact, as soil potential, or soil suction passes the cavitation potential, the sole responsible for water retention is the adsorption mechanism which is not void ratio-dependent.

The proposed model unlike other models such as Gallipoli's model (2003), resolves the unrealistic prediction of SWRC over a wide range of suction. This is due to the fact that the implemented Van Genuchten's model in Eq. 5 is forced to reach zero using the adsorption component. Thus, the model is suitable for capturing the void ratio-dependency of SWRCs over the entire range of soil potential. This leads the model to have the ability to explicitly describe capillary water at potentials much higher than 100 kPa while providing a physically based transition from the capillary to the adsorbed water retention regimes.

Based on the evaluations performed in this study, it can be deduced that the proposed SWRC model not only can provide an accurate prediction of water retention behavior, it can adequately consider the void ratio-dependency of the hydraulic retention.

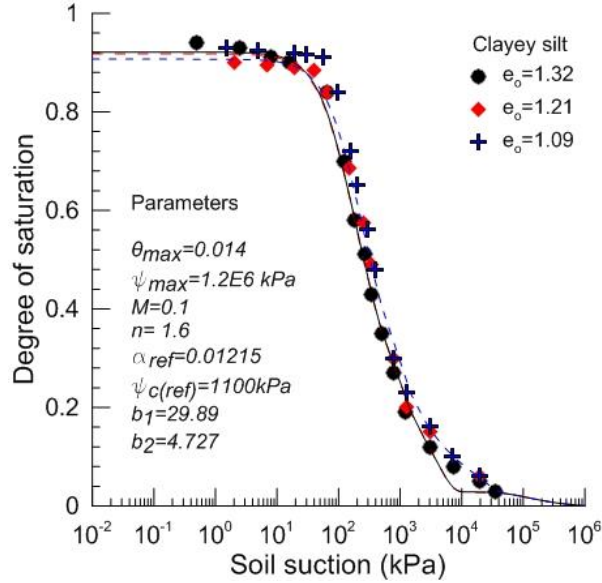


Figure 3 Comparison between the predicted SWRCs with the proposed model and the experimental results for Clayey silt.

4.2 Sensitivity analysis of proposed model

Figure 4 and Figure 5 show the effect of varying four fitting parameters, θ_{max} , ψ_c , α and n on the shape of the SWRC. From Figure 4.a, when other parameters are constant, i.e. $\psi_{max} = 1E6 \text{ kPa}$, $\psi_c = 1000 \text{ kPa}$, $\alpha = 0.01 \text{ kPa}^{-1}$ and $n = 1.5$, the changes in the θ_{max} shift the SWRC upwards. Figure 4.a indicates that the adsorption capacity controls the shape of the ending part of SWRC. The concave shape in the middle of SWRC appears as θ_{max} increases from 0.01 to 0.05.

By keeping all parameters constant, Figure 4.b suggests that ψ_c tends to push the ending part of SWRC slope to the right as its value increases gradually. The smoothness of SWRC in this case is more evident as ψ_c reaches 5000 kPa.

Figure 5.a illustrates the impact of α on the overall shape of SWRC while other parameters are set as constant. As expected, this parameter is highly related to the air-entry value. Based on the studies of Fredlund and Xing (1994), the value of parameter is in general higher than air-entry value but for small values of n , α can be considered as the air-entry value. Thus, the assumption made in this study to assign the variation of SWRC with void ratio to the parameter α is subsequently justified.

Figure 5.b shows that n controls the slope and the overall shape of SWRC. As mentioned earlier, this parameter relates to the pore size distribution of soil. Thus, as the size of particles increases as in granular soils, the value of n becomes higher which in return makes the slope of SWRC deeper. Although, it is out the scope of this study, it is suggested to evaluate how this parameter is affected by the suction-induced volume changes as observed for the tested materials.

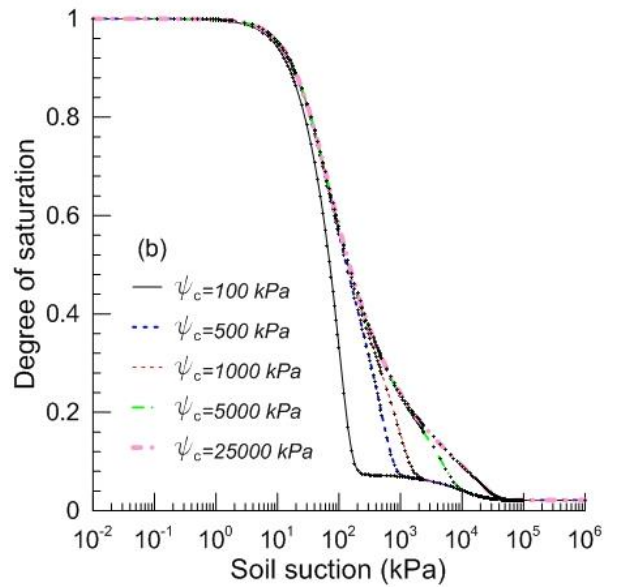
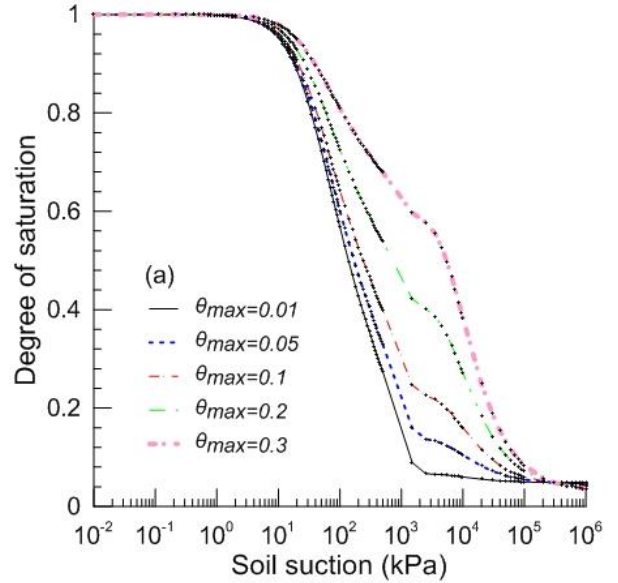


Figure 4 Sensitivity of proposed model to (a) adsorption capacity (b) to cavitation potential

5 CONCLUSIONS

A modified hybrid water retention model was proposed that not only can distinguish the mechanisms responsible for the water retention in the pores but also accounts the effect of initial void ratio on the interpretation of the experimental results. Using the proposed water-retention equation, the experimental data obtained at Université de Sherbrooke and datasets from the literature were investigated. Based on the results, the proposed model, is able to capture the effect of initial void ratio on the main drying over the complete range of matric potential. This proposed equation

provides an accurate model to simulate hydraulic retention data along with a better insight to physical phenomena responsible for soil-water interaction.

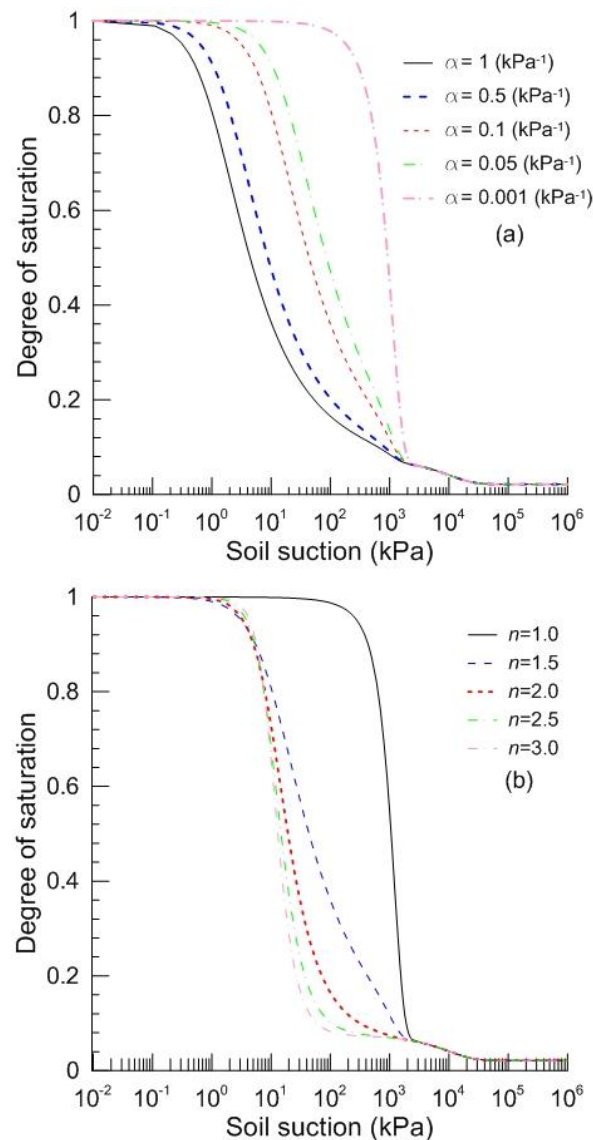


Figure 5 Sensitivity of proposed model to (a) inverse of air-entry value (b) pore size distribution parameter

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