Analyses of climatic data to determine the frost penetration depth based on statistical analyses and matching to measured ground temperature profiles

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ABSTRACT

The depth of frost penetration is required for many engineering projects where the ambient air temperature falls below freezing for an extended period of time over which frost susceptible soils could undergo deformation due to the formation of ice lenses. When possible, measured climatic data from a given site should be used to determine the depth of frost penetration, but in many cases, climatic data may only be available nearby a site, which calls into question the validity of the analyses. Furthermore, the calculation of the depth of frost penetration is highly sensitive to the use of empirical design coefficients. The authors propose the use of climatic data that has been assessed statistically allowing for the estimation of return periods for better risk management during design. The authors further propose coupling the calculations for the depth of frost penetration to an actual ground temperature profiles for one or more winter seasons where data is available, which will allow for further refinement of the calculations. The authors provide a worked example based on climatic data for a mine site in Northern Canada, coupled to data from thermistors installed for the proposed project. The authors also provide comments on the assessment of degrees above and below zero degrees Celsius from the available climatic data set for a given site.

RÉSUMÉ

La profondeur de pénétration du gel est requise pour de nombreux projets techniques dans lesquels la température de l'air ambiant est inférieure au gel pendant une période prolongée au cours de laquelle des sols sensibles au gel peuvent se déformer en raison de la formation de lentilles de glace. Lorsque cela est possible, il convient d'utiliser les données climatiques mesurées sur un site donné pour déterminer la profondeur de pénétration du gel, mais dans de nombreux cas, des données climatiques peuvent uniquement être disponibles à proximité d'un site, ce qui remet en question la validité des analyses. De plus, le calcul de la profondeur de pénétration du givre est très sensible à l'utilisation de coefficients de calcul empiriques. Les auteurs proposent d'utiliser des données climatiques évaluées statistiquement permettant d'estimer les périodes de retour pour une meilleure gestion des risques lors de la conception. Les auteurs proposent en outre de coupler les calculs de la profondeur de pénétration du gel aux profils de température réelle du sol pour une ou plusieurs saisons d'hiver où des données sont disponibles, ce qui permettra d'affiner les calculs. Les auteurs fournissent un exemple concret basé sur des données climatiques pour un site minier dans le nord du Canada, couplées à des données provenant de thermistances installées pour le projet proposé. Les auteurs commentent également l'évaluation des degrés au-dessus et au-dessous de zéro degrés Celsius à partir de l'ensemble de données climatiques disponibles pour un site donné.

1 INTRODUCTION

The calculation of the depth of frost penetration is needed in most regions of Canada due to the prevalence of cold weather in the winter months of the year. In southern portions of the country where the average annual temperature is above zero, the depth of frost penetration is used to determine the depth at which to bury sensitive structures so as not to be susceptible to frost heave due to cyclical freeze-thaw cycles. In northern portions of the country, the active zone can be underlain by permafrost, which is ground that has been frozen for longer than two years. The calculation of depth of frost penetration is more about understanding the depth of the active layer, which is the layer that thaws during the warmer months of the year.

The calculation of depth of frost penetration is typically done using climate normals, which is the arithmetic average temperature over a 30-year period. There is no measure of frequency or return period. This leads to a situation where design engineers cannot comment on the return period or annual exceedance probability for a given depth of frost penetration; which is the case when discussing earthquakes and floods. The proposed methodology for the calculation of the depth of frost penetration utilizes traditional statistical analyses in order to equate a given depth of frost penetration to a return period. This will then allow design engineers the opportunity to discuss the level of risk a client wants to take on, which will affect the actual depth used in design.

2 THEORETICAL BACKGROUND

Frost penetration depth calculations were first developed by Stefan in 1889 and have since been revised by others (e.g. Zarling et al 1989, Bianchini and Gonzalez, 2012). The difficulty in modelling frost penetration depth lies in accounting for the effect of latent heat, as its release or absorption introduces non-linearity associated with heat transfer (Bianchini and Gonzalez, 2012). This non-linearity can be observed in the typical bell curve formed by plotting data from thermistors (i.e. the 'trumpet' shape above the uniform temperature with depth that, if negative, indicates the presence of permafrost). The range over which the trumpet shape forms is referred to as the 'active zone', or that depth of soil subjected to seasonal freeze-thaw cycles. For this paper we will be using the modified Stefan's equation (Bianchini and Gonzalez, 2012):

$$X = \sqrt{\frac{2 \times K_u \times n \times I_f}{L}} \quad [1]$$

Where the depth of frost penetration is X, $K_u$is the thermal conductivity of unfrozen soil, n is a surface factor based on empirical coefficients, I_f is the air-freezing index, and L is the volumetric latent heat of fusion. The thermal conductivity of unfrozen soil is defined by Equation 2 for coarse-grained soils and Equation 3 for fine-grained soils. These equations were developed by Kersten (1949) as a mathematical representation of the UFC thermal conductivity charts.

$$K_u = \frac{[(0.7 \times \log(w) + 0.4) \times 10^{0.01 \times \gamma_d}]}{12} \qquad [2]$$

$$K_u = \frac{[(0.91 \times \log(w) + 0.2) \times 10^{0.01 \times \gamma_d}]}{12} \qquad [3]$$

The surface factor is chosen based on the values recommended by the 2006 Canadian Foundation Engineering Manual (CFEM 2006). The recommended values of n are summarized in Table 1. The values are not precise and in many cases are defined over a range of possible values due to the inherent heterogeneity and anisotropic nature of natural materials.

Table 1. Surface Factor for given Surface Types

Surface Type	**Surface Factor, n**
Spruce trees, brush, moss over peat – soil surface	0.29 (under snow)
Same as above with trees cleared – soil surface	0.25 (under snow)
Turf	0.5
Snow	1.0
Gravel (Most probable range)	0.6 to 1.0 (0.9 to 0.95)
Asphalt pavement (Most probable range)	0.29 to 1.0 or greater (0.9 to 0.95)
Concrete pavement (Most probable range)	0.25 to 0.95 (0.7 to 0.9)

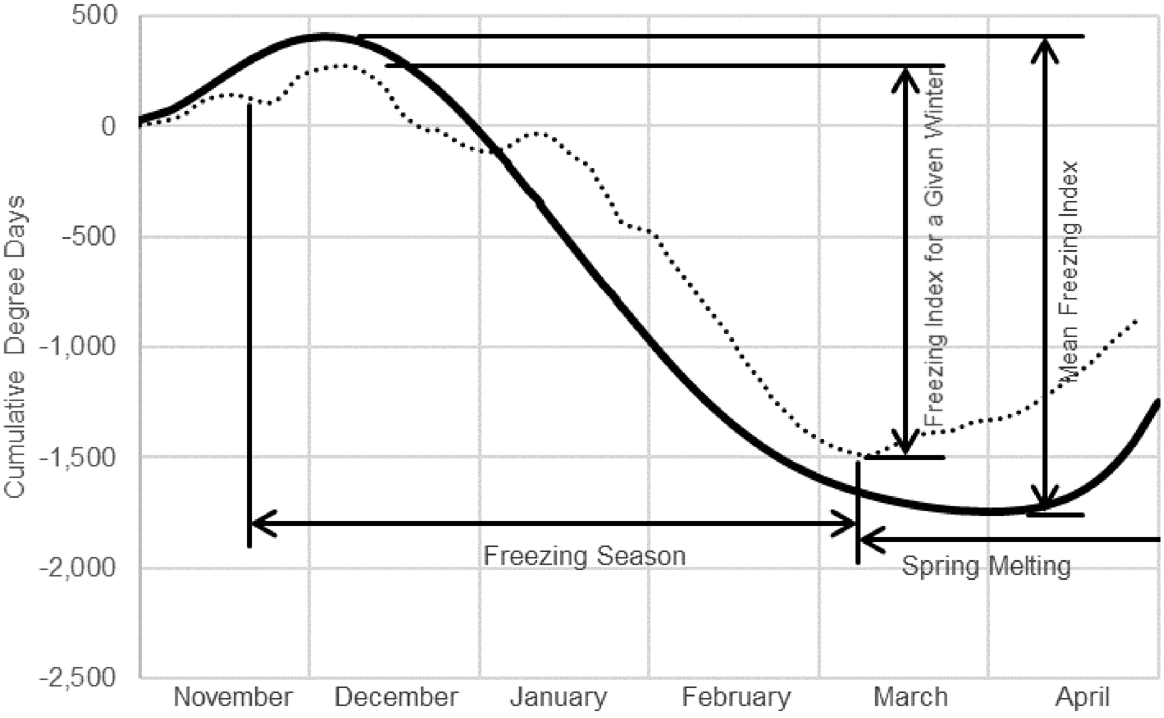

Figure 1. Illustration of Cumulative Degree Days Plotted Over a Freezing Season (after Linell, 1953)

The air-freezing index is a measure of degree-days between the highest and lowest points on a curve of cumulative degree days vs time for one freezing or thawing season, as shown on Figure 1. The cumulative degree days for a given site are typically derived from a historical climatic record of 20 to 30 years of ambient air temperature readings by taking the mean (the mean freezing index on Figure 1) over the duration of the data set (Linell, 1953). The original Stefan's equation did not consider volumetric heat, L, and as a result tended to overestimate the frost depth. For the modified Stefan's equation, the volumetric latent heat of fusion, the quantity of heat required to freeze water in a unit volume of soil, is used in the denominator of Equation 1 and is defined by Equation 4.

$$L = 144 \times \gamma_d \times \frac{w}{100} \quad [4]$$

Where γ_d is the dry unit weight of subsurface material under consideration and w is the natural water content.

3 PROPOSED METHODOLOGY

3.1 Overview

The proposed risk-based methodology requires the statistical fitting of the air-freezing index to be used as an input in the calculation of frost penetration depth. Once I_f for different return periods has been established, it is possible to calculate a frost penetration depth that corresponds to the specific return period that is to be used for design purposes. In other words, the proposed methodology attempts to mimic the same design methodology employed by hydrologists whereby design flood events are characterized based on return period that corresponds to a given level of risk.

3.2 Calculation of Degree Days

The degree days for the site were calculated using ambient air temperature measurements taken from an Environment Canada weather station. The data was organized by year, then month, and then day. Hourly data was not available for the site. The data in each month of each year was summed as per the example given in Figure 1 (i.e. negative values of ambient air temperature were added to positive values). Months with an overall negative air freezing index were considered as part of the freezing season, as per Figure 1.

A data set of yearly air freezing index values was produced over a period from 1997 to 2015, as summarized in Table 2. This data was then assessed statistically as discussed below.

3.3 Statistical Assessment of Data

3.3.1 Climate Data

Risk is typically assessed using statistical means so that a probability of failure, or probability of exceeding a given threshold, can be determined. This requires a mean and standard deviation. The definition of each depends on the distribution to which the data is ultimately fitted.

The probability density function (PDF) is determined to confirm the fit of the data to a given distribution and then the cumulative density function (CDF) can be determined (Fenton and Griffiths, 2008). The CDF provides a measure of the probability that a given value X will be exceeded, with the probability of exceedance being called the Annual Exceedance Probability (AEP) when the data is related to time, as is the air freezing index. Examples of a PDF and CDF are provided on Figure 2.

Table 2. Summary of Climate Data

Season	**Days with Data**	**Air Freezing Index, I_f (Degree Days Below 0)**	
		Daily Mean Temperature	**Daily Min. Temperature**
1997	50%	1635	2061
1998	100%	3677	4471
1999	100%	4276	5279
2000	100%	4534	5541
2001	100%	4855	5807
2002	100%	4411	5310
2003	100%	4987	5905
2004	100%	4803	5719
2005	100%	3568	4397
2006	100%	4260	5114
2007	78%	3883	4588
2008	40%	1558	1859
2009 [1]	0%	-	-
2010 [1]	0%	-	-
2011	59%	2615	3133
2012	100%	4753	5565
2013	100%	4646	5450
2014	78%	3025	3726
2015	80%	4122	4978

[1] these years not considered in the analysis

3.3.2 Unit weight and water content

As shown in Equations 2 to 4, the dry unit weight and natural water content of the surficial, or subsurface soil if the surface layer thickness is negligible and not considered as an insulating layer, are needed to calculate the depth of frost penetration.

3.4 Calculation of Frost Penetration Depth

The calculation of frost penetration is completed using Equation 1. The equation is used with I_f equated to a given return period or AEP. This then allows for depths to be calculated for a range of return periods. The less frequent the return period, the greater the depth of frost penetration (i.e. value of X in Equation 1 is smaller for the 1 in 100 yr freezing season versus the greater value of X for the 1 in 500 yr freezing season).

3.5 Calculating Site Specific Frost Penetration Depth

In many cases the available climatic data is collected some distance from the site in question and the weather station could be set at a different elevation and slope aspect. This could result in the use of non-representative data for the calculation of the depth of frost penetration.

The authors propose back-calculating the n value for use in Equation 1 by comparing the seasonal data from a thermistor installed at a site for the equivalent year's value of I_f. This would then allow the design engineers to develop site-specific n-values for use in design. The site-specific n-value can be back-calculated by rearranging Equation 1, as depicted by Equation 5.

$$n = \frac{X^2 \times L}{2 \times K_u \times I_f} \qquad [5]$$

4 RESULTS

4.1 Case Study Overview

The case study considers a site in northern Canada where the depth of frost penetration is needed to understand the potential influence of frost jacking at the base of an embankment dam. The embankment dam will be founded on soil that will be submerged below a lake before the embankment begins to retain water. Once the foundations soils are exposed to the ambient air temperatures, the authors expect freezing to be generated in the soils and rock underlying the embankment.

The climatic data summarized in Table 2 is from a weather station some 20 km from the site. As the embankment dam will only be in service for 10 to 25 years, the designers have considered cost reduction measures by quantifying the risk associated with elements of the embankment. The depth of frost penetration is one aspect of the project that is needing to be assessed. The reliability of the data is also in question due to the distance between the site and the weather station.

4.2 Statistical Assessment of Climate Data

The air freezing indices over the valuable range of time are summarized in Table 2. The data was fitted to a statistical distribution as described in Section 3.3.1 using the commercially available software program @Risk (Palisade Corporation, 2016). The results of the analyses are summarized in Table 3.

4.3 Matching Calculated Frost Penetration Depth to Site Thermistor Data

Data from a thermistor installed through waste rock, fill, overburden and bedrock was used to assess a site-specific n-value. The n-value would be used for assessing the depth of frost penetration into a rockfill embankment dam constructed of a material similar to waste rock. The thermistor data is summarized on Figure 3. All 16 of the thermistor data series considered in the matching analysis indicate the presence of an active zone, with the depth of frost penetration varying depending on the time of year.

Given that the ambient air temperature measurements were made approximately 20 km from the site, a check was made against thermistors installed at the site to confirm that the estimated depth of the frost penetration was representative of the site ground conditions. This provides a greater sense of reliability in the statistical data than would otherwise be the case. Degree-days were calculated for the specific time periods for the thermistor data illustrated on Figure 3. Equation 5 was then used to estimate the site-specific n-Factor for estimating the depth of frost penetration using Stefan's equation (Bianchini and Gonzalez, 2012).

The results of the data matching analysis are presented in Table 6. The mean of all the n-Factors is 0.92, but with a coefficient of variation of 2.1 due to two periods in the thermistor data where the depth of frost penetration reached the depth of permafrost. When ignoring the two outliers, the mean is 0.24 with a coefficient of variation of 0.78.

When considering the mean, minimum, and maximum daily temperature data independently, the average n-Factors are 0.76, 0.89, and 1.09, respectively. The coefficients of variation are 1.5, 1.8, and 2.3, respectively. The minimum daily temperature data correlates well with an n-Factor of 0.9 suggested in literature for the 'most probable range' for 'gravel' (Bianchini and Gonzalez, 2012), which is the closest material type listed in literature (Bianchini and Gonzalez, 2012) that compares to a rockfill or waste rock.

Table 3. Summary of Statistical Parameters for the Air Freezing Index

Parameter	***I_f***	
	Daily Mean Temperature	**Daily Min. Temperature**
N	17	17
Min	4,987	5,905
Max	1,558	1,859
µ	3,859	4,641
σ	1,039	1,218
median	3,727	5,114
mode	3,859	4,500
ν	0.27	0.26

4.4 Quantification of Risk

The quantification of risk, defined as failure probability times consequence, needs to correspond to the measurement of said risk. For instance, when considering dam safety, risk due to a given flood event or a specific earthquake with a given magnitude and PGA is quantified using return periods or AEPs. . The risk-based proposed herein is based on the same concept. Basing the estimation of depth of frost penetration on the mean freezing index over a 20 to 30 year period does not provide any measure of risk relating to frost penetration. By relating frost penetration to a return period or AEP, a quantification of risk can be achieved.

The results of the statistical analysis considering the CDF described in Section 4.2 are used to determine a range of return periods and AEPs. The frost penetration depth using the Daily Mean Temperature and Daily Minimum Temperature of the data set are summarized in Tables 4 and 5. The calculations for the depth of frost penetration are performed assuming a unit weight of 17 kN/m^3 and a natural water content of 31%.

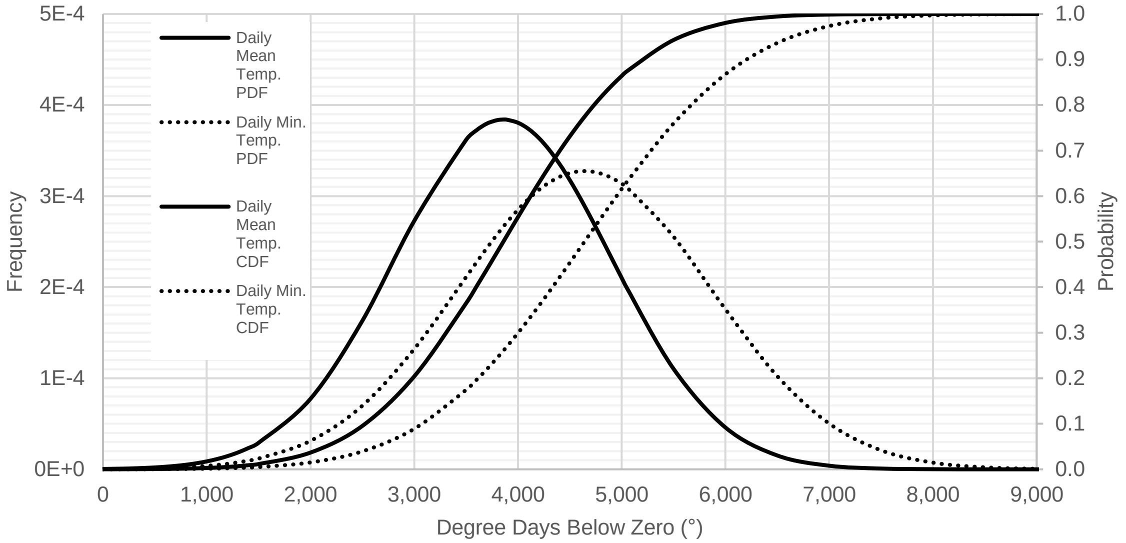

Figure 2. PDF and CDF plots for Daily Mean Temperature and Daily Minimum Temperature data

Table 4. Summary of Statistical Parameters using the Average Mean Temperature

Return Period	Probability of Exceedance per Annum		Probability of Exceedance in X Years		Air Freezing Index	Depth of Frost Penetration
	Probability	Δ	Probability	Years	Degrees Days Below 0°C	m
1 in 10	0.100	---	0.50	5	5,029	3.4
1 in 50	0.020	0.80	0.20	10	5,387	3.6
1 in 100	0.010	0.50	0.10	10	5,514	3.6
1 in 200	0.005	0.50	0.125	25	5,629	3.6
1 in 500	0.002	0.60	0.20	100	5,770	3.7

5 DISCUSSION

5.1 Consideration of Acceptable Risk

The acceptance of risk is not a simple topic and typically requires a significant amount of discussion between responsible parties to determine the acceptable level of risk for a given dam project.

Dam design engineers need to have the tools necessary to quantify risk in order to communicate, preferably through illustration (i.e. pictures are worth a thousand words), the risk associated with a specific design option. This requires design engineers to consider multiple options concurrently so that decisions on the preferred option can be made in a timely manner by parties with which the dam design team may never meet.

With many dam projects, an independent review board (IRB or Independent Tailings Review Boards (ITRB) for mine embankment tailings dams) is utilized to review the overall design of the dam as a level of quality assurance of the design process. Commonly the same board is used during construction. The IRB typically has the final ‘technical voice’ regarding acceptable risk. This is a factor of the IRB members being selected based on their many years of dam design and construction experience, with members typically having a broad range of experience.

However, at the end of the day, dam safety guidelines (CDA 2013, CDA 2014) will dictate the overall acceptable risk for dams. Professional members of the Canadian Dam Association (CDA) have spent many volunteer hours developing dam safety guidelines that provide the design criteria for water retention and mining dams. Although dam safety guidelines in many cases do not address the frost penetration, the level of risk still needs to be considered no differently than earthquakes or floods, which are discussed in dam safety guidelines.

Table 5. Summary of Statistical Parameters using the Average Minimum Temperature

Return Period	Probability of Exceedance per Annum		Probability of Exceedance in X Years		Air Freezing Index	Depth of Frost Penetration
	Probability	Δ	Probability	m	Degrees Days Below 0°C	m
1 in 10	0.100	---	0.50	5	5,962	3.7
1 in 50	0.020	0.80	0.20	10	6,347	3.9
1 in 100	0.010	0.50	0.10	10	6,482	3.9
1 in 200	0.005	0.50	0.125	25	6,607	3.9
1 in 500	0.002	0.60	0.20	100	6,757	4.0

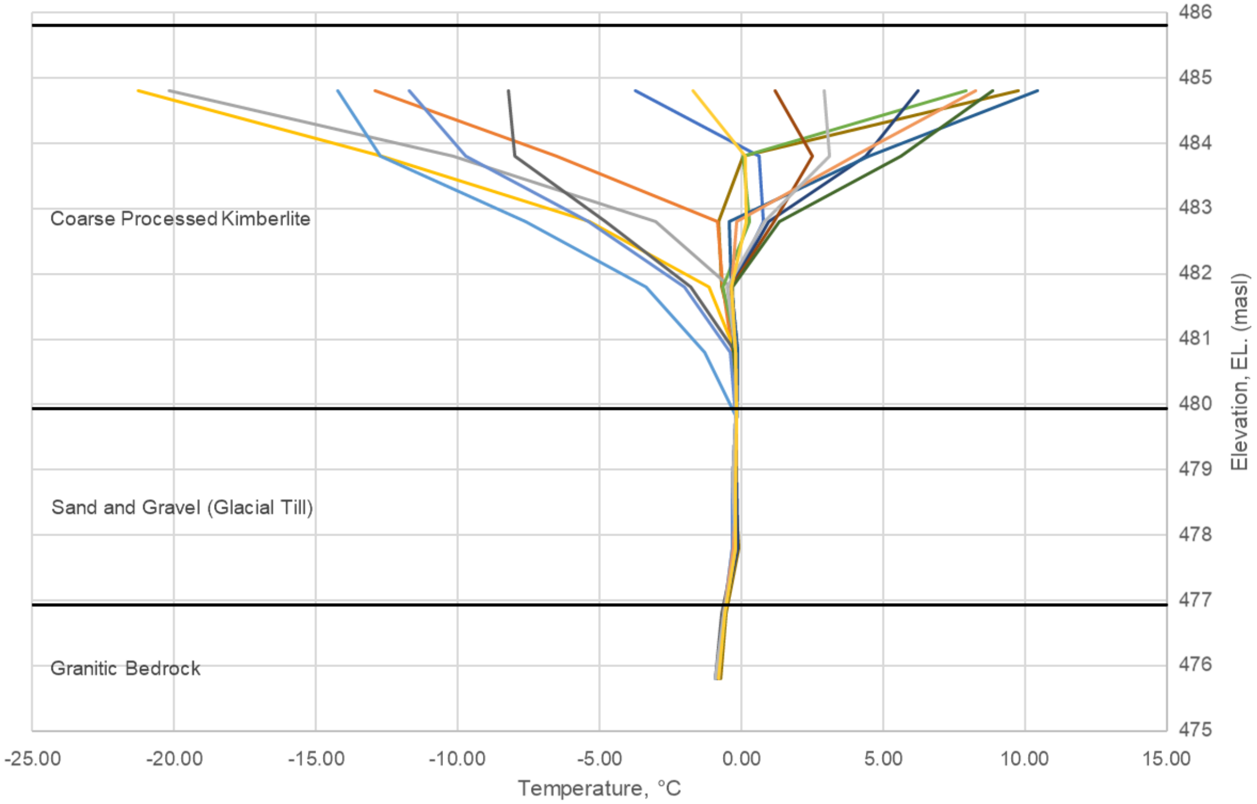

Figure 3. Summary of Thermistor Data

5.2 Use of a Scaling Factor for n

The thermistor data is infrequent such that in a given year there are only two to three data series available. Thermistor data collected on a more frequent basis would provide a more realistic assessment of the frost penetration depth throughout the year.

The case study supports the detailed design of an embankment dam in northern Canada. If constructed, the dam will have over two dozen thermistors strings installed in the dam, with each string connected to an automated data acquisition system. The frequency of data collection can then be set to collect data at a frequency that will allow a more thorough assessment of the seasonal fluctuation in the depth of frost penetration.

5.3 Limitations of the Case Study

The case study only has climate data for a period of 17 years, as described in Table 2. The available climate data was based on daily readings and not the preferred hourly increments that would provide a more reliable estimate of I_f. The natural water content of the soil is equivalent to 100% saturation since the foundation soils were submerged below a lake at the time of the site investigations. The actual water content of the soil once the foundation soils are exposed to the ambient air temperatures is expected to decrease.

Table 6. Summary of Analysis Matching Site-Specific Measurements of Depth of Frost Penetration and the n-Factor

Probability of Exceedance per Annum	Depth of Frost Penetration	$I_{f\text{-mean}}$[1] Degree-Hrs	$I_{f\text{-min}}$[2] Degree-Hrs	$I_{f\text{-max}}$[3] Degree-Hrs	n-Factor from Site Specific Matching to Thermistor Data		
	m	°F-hrs	°F-hrs	°F-hrs	Mean	Min	Max
2001-07-24	2.1	81,132	66,478	52,452	0.05	0.06	0.08
2001-11-18	3.7	99,571	80,714	62,928	0.13	0.16	0.20
2002-01-15	7.4	8,760	7,222	5,669	5.85	7.09	9.04
2002-02-15	11.7	37,346	32,892	28,397	3.45	3.92	4.54
2002-03-15	5.2	59,138	52,339	45,473	0.43	0.49	0.56
2002-05-02	6.1	86,894	75,204	63,382	0.41	0.47	0.56
2002-08-26	3.3	93,744	79,169	65,177	0.11	0.13	0.16
2002-09-22	3.7	94,008	79,298	65,213	0.14	0.17	0.20
2002-10-17	3.8	99,216	83,098	67,637	0.14	0.16	0.20
2003-04-30	5.1	79,932	69,521	59,004	0.31	0.36	0.42
2003-08-11	2.9	85,620	72,612	60,470	0.09	0.11	0.13
2003-09-16	3.8	85,848	72,684	60,470	0.16	0.19	0.23
2004-04-30	7.0	79,932	69,521	59,004	0.58	0.67	0.79
2004-08-21	3.0	85,620	72,612	60,470	0.10	0.11	0.14
2004-09-26	3.6	86,244	72,794	60,494	0.14	0.17	0.20
2004-11-19	1.9	99,840	83,122	67,752	0.03	0.04	0.05

[1] Based on Mean Daily Degree Days Below Zero
[2] Based on Minimum Daily Degree Days Below Zero
[3] Based on the Maximum Daily Degree Days Below Zero

6 CONCLUSIONS

The results using the proposed methodology provide a method of quantifying the risk associated with frost development in a similar fashion as is done for earthquakes and floods by equating depth of frost penetration to the climatic conditions with a given return period.

Matching the depth of frost penetration to site-specific measurements of the active zone provides a means of calibrating the n-Factor for a given soil condition to site-specific data.

The results provide a tool for design engineers to report the risk associated with their designs and assumptions. It is not the role of a design engineer to decide the level of risk a client should take on, but only to inform a client as the level of risk associated with a given design. The proposed methodology also provides an efficient process for providing a client with a sensitivity analysis.

7 FUTURE WORK

As shown in Equations 2 to 4, the dry unit weight and natural water content of the surficial, or subsurface soil if the surface layer thickness is negligible and not considered as an insulating layer, are needed to calculate the depth of frost penetration. The same statistical analysis, as discussed in Section 3.3.1, could be used on the material properties assuming a sufficient number of data points are available. This would allow for a probabilistic assessment of the depth of frost penetration to be carried out using a method such as the First Order Reliability Method (FORM) or a Monte Carlo assessment (Fenton and Griffiths, 2008).

Acknowledgements

The authors would like to acknowledge their employer for encouraging technical excellence in dam and cold regions engineering, and providing the time and funding for travel to, and involvement in, GeoStJohns 2019. The authors also wish to acknowledge their client who has worked in tandem to develop a collaborative design process so that the design process is effective and cost effective.

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