

# The Piezo-electric Ring Actuator technique (P-RAT) – 16 years of progress

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## ABSTRACT

The shear wave velocity ( $V_s$ ) has been widely recognized as a fundamental design parameter for soils subjected to dynamic loading. The past several decades have seen a significant increase in the use of shear wave velocity in geotechnical applications especially those involving earthquake ground response analysis, liquefaction potential, and soils characterization in terms of geotechnical and mechanical properties. Shear wave velocity can also be used to monitor the setting and hardening of cement-based materials and to characterize rigid materials (rock, concrete, etc.). Unlike conventional geotechnical parameters, the  $V_s$  can be measured either in-situ or in the laboratory on disturbed or undisturbed soil samples. However, only a limited number of laboratory methods are available to evaluate  $V_s$  of soils such as; the resonant column, the ultrasonic pulse, and the piezoelectric bender element. A new technique, Piezoelectric ring-actuator technique (P-RAT), has been developed in the geotechnical laboratory at the University of Sherbrooke. This technique can be incorporated into conventional apparatuses (e.g., oedometer, triaxial, and even resonant column). In contrast to the bender element technique, the P-RAT can be used to measure  $V_s$  of rigid materials since the method obviates penetrating any sensor into the tested specimen. This paper presents a description of the P-RAT developed and improved over the last 16 years at the University of Sherbrooke. Several P-RAT results of various conducted applications during these years are also presented. The exhibited examples demonstrate the superiority, versatility, and the broad applicability of the P-RAT in civil engineering.

## RÉSUMÉ

La vitesse des ondes de cisaillement ( $V_s$ ) a été reconnue comme un paramètre fondamental pour l'analyse et la conception des sols soumis à une charge dynamique. Au cours des dernières décennies, l'utilisation de la vitesse des ondes de cisaillement dans les applications géotechniques a considérablement augmenté, notamment dans les domaines de l'analyse de la réponse des sols au séisme, du potentiel de liquéfaction et de la caractérisation des sols en matière de propriétés géotechniques et mécaniques. La vitesse des ondes de cisaillement peut également être utilisée pour suivre le durcissement et le durcissement des matériaux à base de ciment, ainsi que pour caractériser les matériaux rigides (des roches, des bétons, etc.). Contrairement aux paramètres géotechniques conventionnels, la  $V_s$  peut être mesurée in situ ou en laboratoire sur des échantillons de sol reconstitués ou non intacts. Cependant, seules un nombre limité de méthodes de laboratoire sont disponibles afin de mesurer  $V_s$  des sols tels que ; la colonne résonante, le test des impulsions ultrasonores et le test de l'élément bilames piézoélectriques une nouvelle technique, la technique des anneaux piézoélectriques (P-RAT) a été mise au point dans le laboratoire de géotechnique de l'université de Sherbrooke. Cette technique peut être incorporée dans les appareils conventionnels (œdomètre, triaxiale et même la colonne de résonante). Contrairement à la technique de l'élément bilames piézoélectriques, le P-RAT peut être utilisé pour mesurer les  $V_s$  de matériaux rigides, car il empêche toute pénétration de capteur dans l'échantillon testé. Cet article présente une description de la technique P-RAT développée et améliorée au cours des 16 dernières années à l'université de Sherbrooke. Plusieurs résultats de P-RAT de diverses applications effectuées au cours de ces années sont également présentés. Les exemples exposés démontrent la supériorité, la polyvalence et la grande applicabilité du P-RAT en génie civil.

## 1 INTRODUCTION

In the last few decades, the shear wave velocity ( $V_s$ ) is recognized as a key design parameter, as it is widely employed in several geotechnical applications such as; earthquake ground response analyses (e.g., Kramer 1996), liquefaction potential evaluation (e.g., Youd et al. 2001), and characterization of soil mechanical properties (e.g., Robertson et al. 1995). A prominent advantage of  $V_s$  is the ability to perform in-situ measurements and laboratory measurements as well. Although in-situ measurements may offer an accurate mean to determine  $V_s$  of the examined soil stratum, they only provide information on the specific conditions where the test was performed.

Shear wave velocity can be measured in the laboratory using the resonant column technique (RC) (e.g., Iida, 1937; Hardin & Richart, 1963; Drnevich, 1978) or through piezoelectric elements such as the bender elements (BE) (e.g., Shirley & Hampton, 1978; Dyvik and Madshus, 1985; Brignoli et al. 1996). Despite the popularities of these techniques, they have several flaws and deficiencies. The literature showed examples of these deficiencies as; the near field effect difficulties which associated to BE (e.g., Meyer and Pender, 1995; Wang et al., 2007), the uncertain detection of the first arrival of the shear wave velocity (e.g., Lee and Santamarina, 2005), the boundary effects (e.g., Leong et al., 2005), and the mixed radiation of P- and S-waves, representing a combination between the

compression and the shear waves (e.g., Arulnathan et al., 1998, Lee and Santamarina, 2005).

An innovative, superior technique, piezoelectric ring-actuator (P-RAT) with a unique interpretation method has been developed in the geotechnical laboratory at the University of Sherbrooke (e.g., Gamal El Dean, 2007; Ethier, 2009; Ben Romdhan et al., 2014; Karray et al., 2015; Mhenni et al., 2015). The primary purpose behind developing the P-RAT is to minimize/eliminate the difficulties associated with other techniques, for example, the unavoidable penetration of specimens by the BE sensors. P-RAT is also an attractive mean to measure  $V_s$  as it can be installed in most conventional geotechnical apparatuses such as triaxial and oedometer cells.

The signal quality is the underlying factor in obtaining more precise results in all  $V_s$  measurement techniques. In other words, to eliminate confusion in the obtained  $V_s$  in term of accuracy, it is imperative to ensure the quality of signals. A defective or bad signal cannot be ameliorated even with the most sophisticated signal processing techniques. Therefore, a substantial improvement in the quality of the received signal is a prerequisite to the development of an excellent  $V_s$  measurement technique, as noted by Jovičić et al. (1996). For this reason, a long history of modification and improvements were applied to the P-RAT to obtain high-quality signals. A brief description of the P-RAT, its unique interpretation method, and the different stages of its evolution are explained below then, the paper presents some applications of employing the technique to measure  $V_s$  of granular, cohesive soils, rock fills, and other cementitious materials as well.

## 2 PIEZOELECTRIC RING-ACTUATOR TECHNIQUE (P-RAT)

### 2.1 Evolution of the P-RAT

The piezoelectric ring-actuator technique essentially consists of two parts: an emitter and a receiver (Fig. 1). These parts are fastened in the bottom and top heads, respectively, of an oedometer cell or any conventional geotechnical apparatus that allows a constrained or partially constrained soil specimen (Fig. 2). Each part (the emitter & the receiver) is an inert piezoelectric ring covered at its outer and inner faces by a thin conductive layer. Both faces are welded to shielded wires that transfer voltage pulses with different durations and shapes into a radial deformation which is transmitted in turn to the specimen via the inner stone. Notably, the component, which comes into contact with the soil specimen, is fastened inside the piezo ring with epoxy. The transversal inner stone deforms creating pure shear waves in the radial direction onto the tested sample. The transferred wave, when strikes the receiver surface, produces a transverse deformation that by inverse piezoelectric effect induces electric voltage to the electrodes of the receiver piezoelectric ring and thus a test cycle is completed.

The sensor's design has many advantages, including a large area of contact with the tested specimen edges that covers more representative contact surface and ensure axisymmetric distribution without favoring specific direction to another. Also, there is no need to penetrate the soil

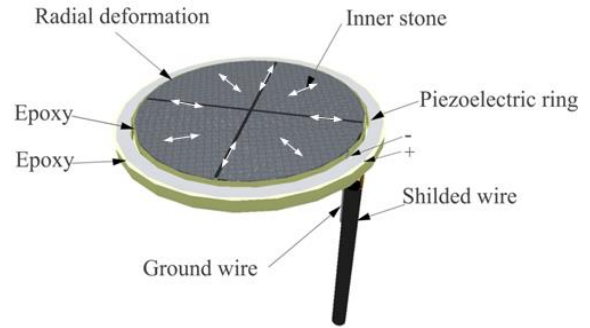


Figure 1. P-RAT emitter or receiver.



(a)



(b)



(c)

Figure 2. Evolution of the P-RAT: a) primary sensor, b) four-pieces sensor, and c) capsulated sensor.

specimen and alter its mechanical/physical properties since the inner stone rests directly on the sample bottom and top surfaces (Fig. 2a).

A tip-to-tip test was performed to validate the concept of the ring actuator technique. The idea was to demonstrate that the radial deformation of the ring induces the radial deformation of the inner stone and then the movement at the surface should be a pure shear movement. The obtained results at this primary stage were promising, although the received signals may be not that accurate as they may be affected by the inducing P-wave due to Poisson's ratio effects.

As a first trial to cope with the Poisson's effect, it was thought that the inner stone should be cut into four quarter pieces (Fig. 2b). It was interesting to observe that the generation of the P-wave associated with the radial deformation was significantly reduced and the signals have been greatly enhanced. However, the obtained signals were not perfect, especially when testing dry soils due to the accumulation of the magnetic field surrounding the piezoelectric rings. Therefore, another modification was applied at this stage; the four pieces of the stone are not completely separated but were kept radially in contact at their outer edges (i.e., the periphery). This amendment definitively allows grounding the magnetic field. It is also worth to mention that the capsulation of P-RAT sensors into the top and bottom heads of the cells (Fig. 3c) would improve the stress distribution on the surfaces of the sample and contribute to the elimination of the P-wave produced by the ring.

Moreover, a numerical study was conducted by the authors to validate/improve the concept, for more details pertaining to this study, the reader should refer to Mhenni et al. 2015. In brief, Figure 3 shows the general meshing of different components of the piezoelectric sensor. In light of that study, a better understanding of the interaction between the various parts of the sensor as well as the interaction between the sensor and the tested soil specimen were established. A better design, where the first mode is exclusively radial, was selected, then a variety of modifications were applied to the sensor to thrust almost pure shear waves. More findings of this complementary study are:

- 1- The 4-quarters divided inner stone eliminates any longitudinal displacement and restrains the generation of any compression wave.
- 2- The surface state allows better coupling between the sensor and the sample; consequently, better energy transmission.
- 3- The manufacture of a silicone mold provides perfect symmetry between the components, and the regularity of the coating epoxy allows an excellent uniform behavior to generate almost pure shear waves.
- 4- The stainless-steel encapsulation of the sensors absorbs the longitudinal expansion of the piezoelectric ring and offers a better distribution of the stresses onto the sample.

Figure 4 summarizes modifications applied to the P-RAT based on the work of Mhenni et al. (2015).

## 2.2 P-RAT interpretation method: a review

As a solution to the signal analysis problems (mentioned earlier), a new interpretation method was developed in conjunction with P-RAT development. This unique frequency-based analysis method fitted the high accuracy of the P-RAT and facilitated the implementation in any regular geotechnical laboratory. In this method, a phase-shift correction, based on the sensor's dynamic properties, leads to recording the same  $V_s$  value for a given soil, independently of the wave emitted shape and frequency. For more details, the reader should refer to Karray et al. (2015).

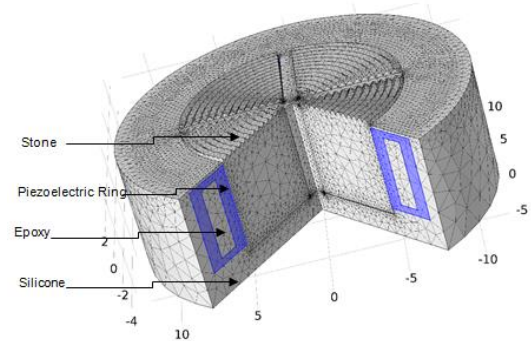


Figure 3. General meshing of different components of the piezoelectric sensor (Mhenni et al. 2015).

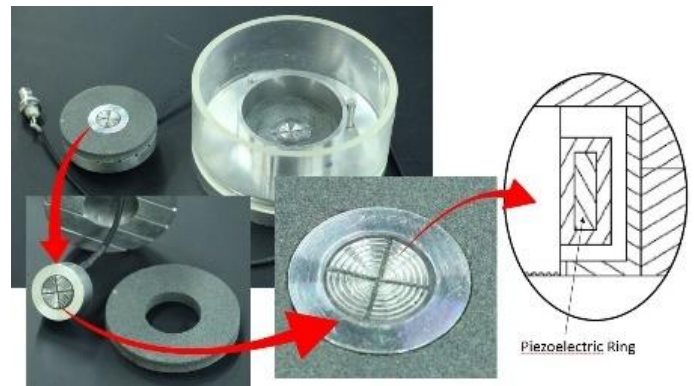


Figure 4. Modification applied to P-RAT after Mhenni et al. (2015).

Figures 5 & 6 illustrate the mathematical methodology of the current interpretation technique, which can be summarized as:

- 1- Projecting the signals from the time domain (Fig. 6a) into the frequency domain (Fig. 6b) to locate the frequency range over which the energy is located.
- 2- Throughout this range, the phase shift between transmitted and received signal should be corrected, by the difference of the experimental phase (data plots in Fig. 6c) onto the theoretical phase (red plotted curve in Fig. 6c) to obtain the corrected phase shift (black plotted curve on the same figure).
- 3- The theoretical phase shift curve (in Fig. 6c) represents the transfer function, which is also the contribution of the sensors. It can be easily determined from a tip-to-tip test, and this spotlights the superior advantage of P-RAT technique. Notably, this curve can be developed

under different confinement pressures (according to the applied testing effective stresses), and different confinement conditions as well (according to the utilized apparatus).

- By correcting the phase difference between transmitted and received signals (canceling the phase shift induced by the sensors), the experimental dispersion curve returns to a constant value referring to the speed of the shear wave. In other words, it is a fact that for a given soil and under the same testing conditions,  $V_s$  is constant and independent of the phase or the frequency content of the transmitted signal as shown in Figure 6d.

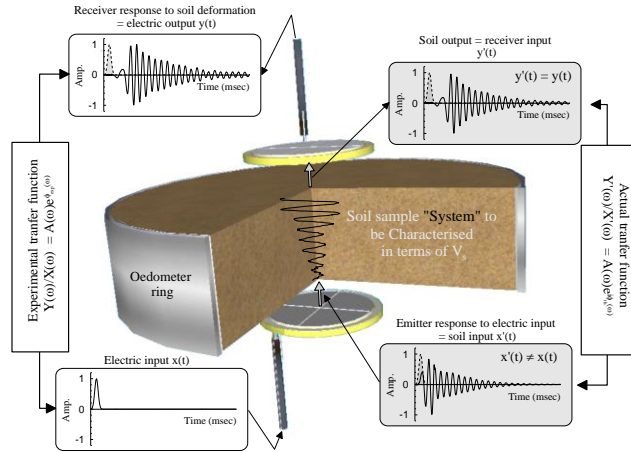


Figure 5. Schematic of experimental and actual transfer functions (Karray et al 2015).

### 3 APPLICATION OF P-RAT IN CONVENTIONAL OEDOMETER

It is widely acknowledged among researchers and practitioners that the sample size should be proportional to the mean particle size of the tested material. Many specifications considered certain conditions that allow more realistic results for the consolidation tests free-to-some extent from the exaggeration of the stress concentration and boundary effects that dominate the overall behavior when these specifications requirements are not applied. In this study, the shear wave velocity of different materials is measured through the P-RAT using three different oedometer cells. Dimensions of the used cells are listed in Table 1. Three various piezoelectric sensors with their corresponding oedometer cells (Fig 7) were employed in this study to measure  $V_s$  of different materials. In particular, the small sensor is used mainly to measure  $V_s$  of clayey soils, while the medium sensor is used to measure shear wave velocity of sands, and the large sensor is used for rock fill. Notably to mention that the P-RAT of Karray et al. (2015) is successfully used in many geotechnical laboratories to assess the shear wave velocity of different geomaterials (e.g., Sherbrooke University; École de Technologie Supérieure).

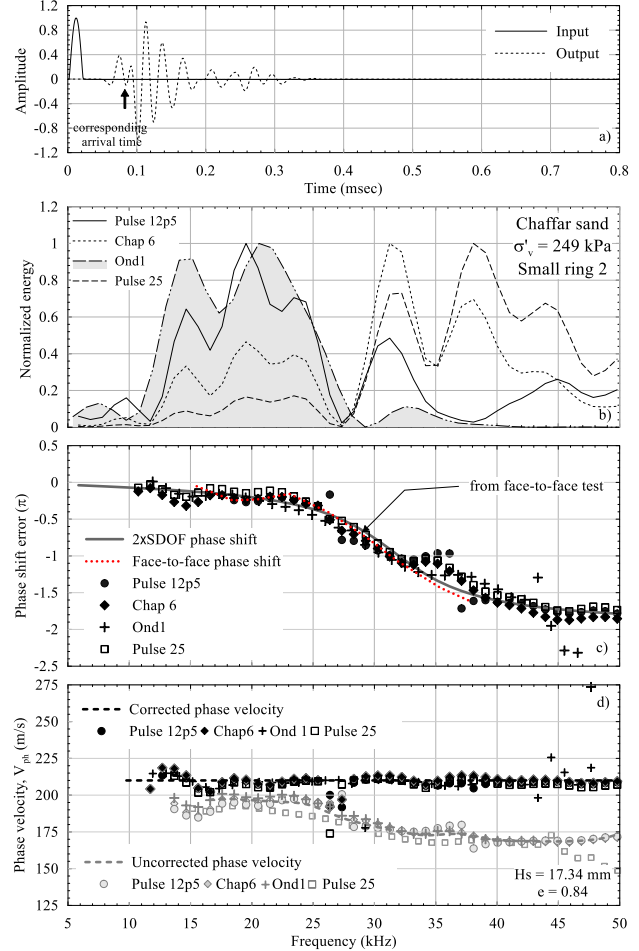


Figure 6. Example of signal processing using P-RAT interpretation technique.

Table 1. Dimensions of the used cells.

Cell size	Diameter, D(mm)	Height, H(mm)
1-small	63	19
2-medium	100	33
3-large	282	90

Some of the P-RAT applications with the associated signals and logical correlations with other typical geotechnical parameters are discussed in the following subsections:

#### 3.1 $V_s$ measurement of granular materials

Series of tests were conducted on a vast variety of granular soils. As an example, in this study, the results pertaining to Chaffar sand from northern Africa (portion < 5 mm) is presented. The physical properties of this sand are listed in Table 2, while, the visual description of the tested sand particles indicated subrounded to subangular granular materials.

Figure 6a shows examples of received signals (output) from P-RAT tests conducted on Chaffar sand under effective vertical stress ( $\sigma'_v$ ) of 249 kPa using a pulse



Figure 7. Piezoelectric sensors installed in four different odometer cells to measure shear wave velocity fine grained soils or tailings (a) to sand (b,c) and Rock fills (d).

excitation. The quality of signals presented in Fig. 6a confirms the concept of piezoelectric rings, which doesn't penetrate the soil specimen, therefore, decreases the contribution of the P-wave. In contrast to most of the utilized techniques in literature, the P-RAT produces minimal and negligible noise. Therefore, P-RAT has a quieter environment during signal processing, especially at higher stress levels. In addition, the use of P-RAT technique helps to have easier manipulation and avoid the use of filters. Figure 6a also shows that the emitter-soil-receiver system efficiently reproduces the same wave characteristics that originated from the emitter.

The development of the stress normalized shear wave velocity ( $V_{s1}$ ), where  $V_{s1} = V_s [100 / \sigma'_v]^{0.25}$ , as a function of void ratio ( $e$ ) obtained from eight different P-RAT tests performed on Chaffar sand samples is plotted in Fig. 8.

Table 2. List Characteristics of Chaffar sand.

Specific gravity ( $G_s$ )	2.68
$D_{50}$	0.270 mm
Coefficient of uniformity ( $C_u$ )	1.5
Maximum void ratio ( $e_{max}$ )	0.99
Minimum void ratio ( $e_{min}$ )	0.58

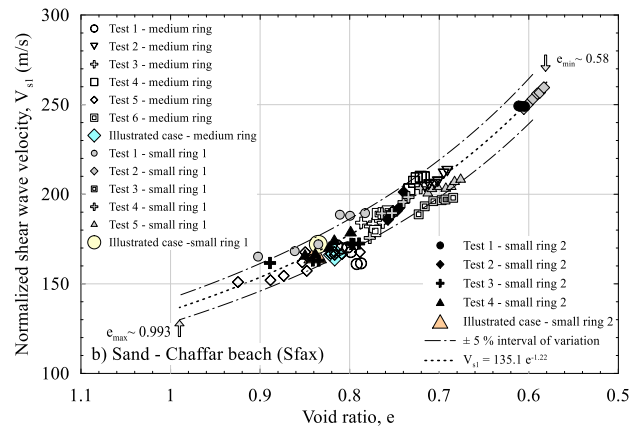


Figure 8. Normalized shear wave velocity of Chaffar sand as a function of initial void ratio using different sensors and different cells (refer to Figs. 7b & 7c).

Each test in Fig. 8 corresponds to a different state of relative density ( $I_d$ ) ranging from 4 to 82%. This variation in  $I_d$  is the result of the change in the corresponding  $\sigma'_v$  in the range 16.5 to 928 kPa. Figure 8 also shows that  $e$  plays a significant role in the variation of  $V_{s1}$ . In details,  $V_{s1}$  increases with the decrease in void ratio. Moreover, it is worth noting that the normalized shear wave velocity values for specimens tested of the same soil (Chaffar sand) under different initial void ratios (different tests) collapse onto almost the same trend. These results confirm the accuracy of the P-RAT and its interpretation method.

### 3.2 $V_s$ measurement of cohesive materials

Series of P-RAT tests were conducted on undisturbed soil samples of Dumont clay extracted from Dumont mining site in Quebec, northern America. Dumont clay is a varved clay of high plasticity (P.I = 45%) with 8% of silt. It has water content, liquid, and plastic limits of 87%, 72%, and 27%, respectively. Figure 9 shows the input signals and typical results of the normalized amplitudes of the received signals (every measured signal is normalized to its maximum value) in time-domain from the P-RAT loading tests on Dumont clay at different vertical stresses. Figure 9 also illustrates the dependency of the arrival time and consequently, the measured  $V_s$  on the applied vertical pressure.

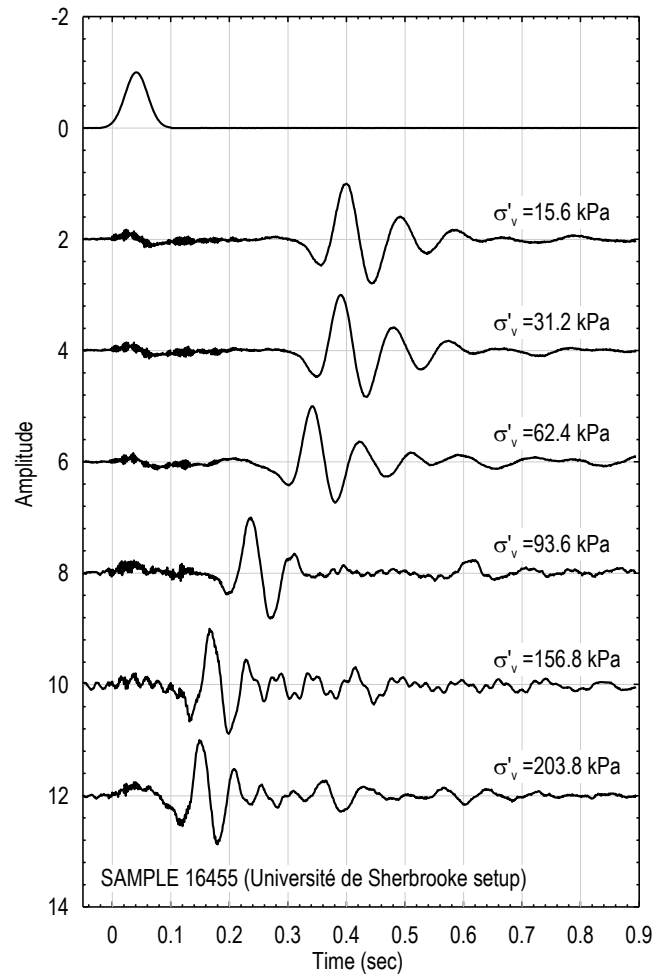


Figure 9. Example of signals obtained during consolidation test on Dumont Clay (sample 16455).

### 3.3 $V_s$ measurement of rock fills

The P-RAT is also employed to measure  $V_s$  of rock fills using a large cell with the large sensor (shown Fig. 7c) in a special setup (Fig.10). The large sensors are used in this series of tests on rock fills to produce enough vibration energy thus guarantee the propagation of the shear wave throughout the whole of the sample height. Table 3 summarize the particle characteristics of the 4 different rock fill samples tested in the study.



Figure 10. Experimental setup for testing Rock fills samples.

Table 3. Rock fills particle characteristics.

Materials	$D_{50}$ (mm)	$C_u$
3M	$10 < D_{50} < 13$	$35 < C_u < 46,25$
3N	$8 < D_{50} < 9$	$40 < C_u < 46,6$
3O	$7,5 < D_{50} < 10$	$55 < C_u < 75$
3P	Ind.	Ind.

For the 4 tested materials (3M, 3N, 3O & 3P), Fig 11 shows that the  $V_s$  increases with the increasing of the vertical stress [ $V_s = f(\sigma'_v)^{0.25}$ ] for both dense (Fig 11a) and loose state (Fig 11b) is also in good agreement with the literature (e.g., Hardin and Richart, 1963, Robertson, 1995).

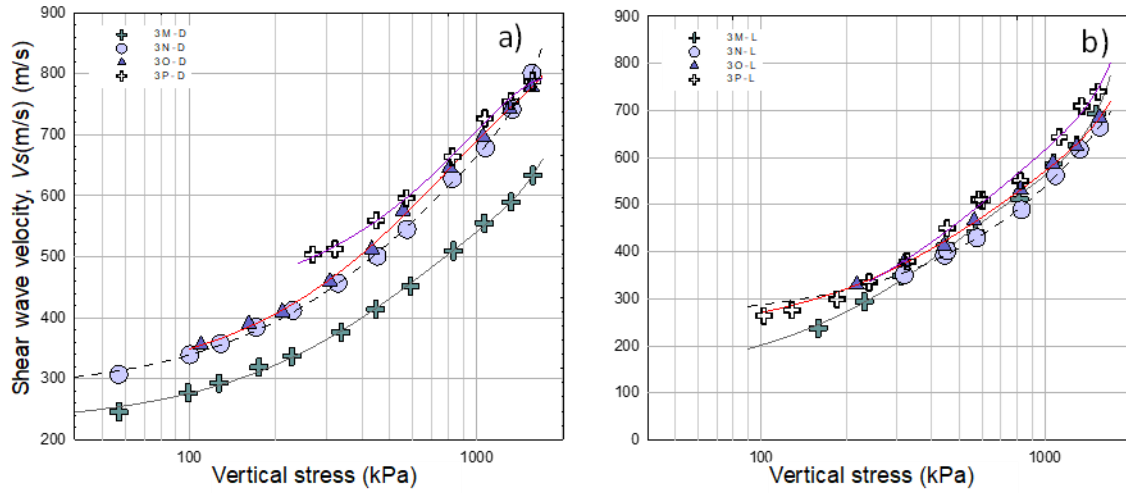
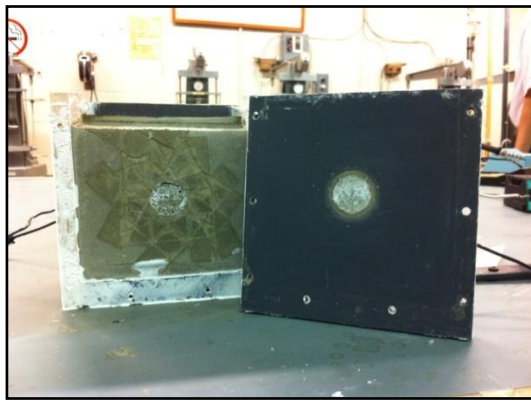


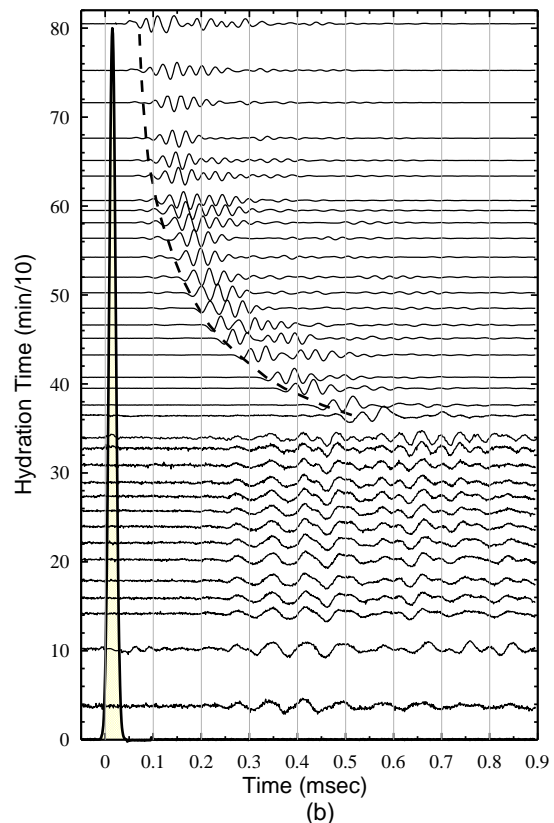
Figure 11 Variation of Shear wave velocity as a function of vertical stress a) Dense soil, b) loose soil

### 3.4 Cementitious materials (Test and Control)

P-RAT is also used to measure  $V_s$  during setting and hardening of some cementitious materials (e.g., concrete and grout) using medium sensors (Fig. 12a). In other words, this technique was successfully adopted to examine several grouts and concrete mixtures with various sand-aggregate contents as well as several water-cement ratios, during its hydration time, in order to correlating the variation of  $V_s$  to the development of the elastic properties of fresh poured concrete (particularly  $f_{cu}$ ) with time.



(a)



(b)

Figure 12. Experimental setup and sample results of measuring shear wave velocity of concrete (transit state) using P-RAT.

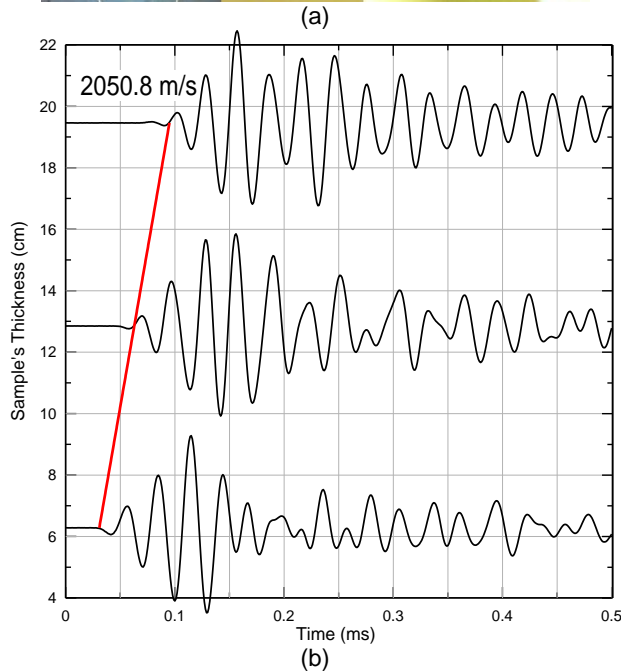


Figure 13. Experimental setup and sample results of measuring shear wave velocity of concrete (hard state) using P-RAT.

In details, Figure 12b illustrates the variation of velocity and time required to transmit shear wave with the hydration time for a grout mixture of zero sand content during the first 14 hours from casting. It was noticeable that, at the beginning of the test, the grout was liquid and the wave could not be transmitted. While, during the initial setting time (about 5 hrs) the speed of transmission was less than the speed during the final setting time. The speed starts to increase gradually with time and tends to be constant after full consolidation which attended after about 12 hours from casting the sample.

In addition, experiments have been repeated for samples of different heights in order to include the effect of the sample height on the transmitted shear wave, as illustrated in Figure 13. To expand the use of sensors on the cementitious materials and concretes at early ages, it is worth to mention that, the P-RAT technique can be also used during the curing time to study and represent the development of the concrete characteristic strength ( $f_{cu}$ ) of

the same tested sample without using any compressive destructive tests for several samples. In addition, continuous unique trend for the development of  $f_{cu}$  with time can be developed under the fact of testing the same sample (concrete block) during all the hydration and curing times (P-RAT is a non-destructive test).

#### 4 APPLICATION OF P-RAT IN CONVENTIONAL TRIAXIAL

The P-RAT was also successfully installed in the conventional triaxial cell to measure shear wave velocity of soils during triaxial tests. An example of such a setup of the piezoelectric sensors in the triaxial cell is shown in figure 14. This figure also displays the very high quality of the resulted signals. A full package of P-RAT results incorporated in triaxial cells with their interpretation, discussion, and analyses will be presented in a separate paper.

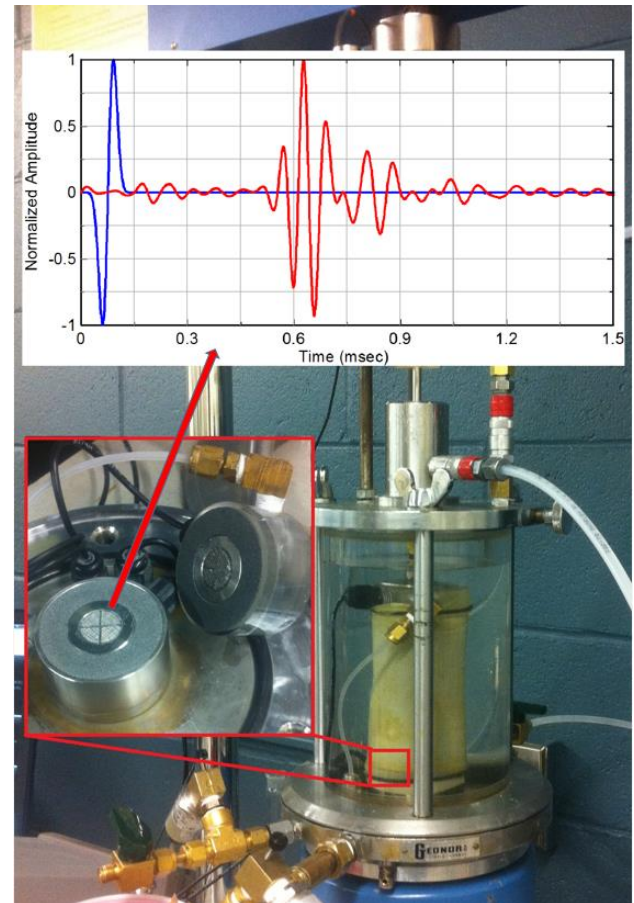


Figure 16. Piezoelectric sensors implemented in triaxial cell.

#### 5 CONCLUSION

In this paper, a description of the innovative piezo-electric ring actuator technique (P-RAT), which developed and improved over the last 16 years at the University of Sherbrooke to measure the shear wave velocity of different

soils and granular construction materials is presented. Stages of evolution, as well as the unique wave interpretation method, were also discussed. Examples of the applicability of the technique to measure  $V_s$  of granular, cohesive soils, rock fills, and other cementitious materials at different hardening stages are detailed. Eventually, typical results and high-quality signals obtained from the P-RAT tests have been displayed and analyzed in the paper proofing the reputation of the proposed P-RAT.

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