

A Numerical Study of the Effects of Contact Friction on CPT Results

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ABSTRACT

The Cone Penetration Test (CPT) is a popular soil characterization tool as it is fast, cost-effective, and provides continuous measurements. Modelling the cone penetration process has been a common goal for many geotechnical researchers due to the difficulties in developing soil-specific CPT correlations through calibration chamber tests. Modeling the penetration problem encompasses various difficulties including large strains in soil around the advancing probe. Mesh distortions resulting in numerical instability have been the main barrier to developing a full model of the CPT. One of the effective ways that researchers adopt to overcome the mesh distortion is to neglect the contact friction between the penetrometer and soil. However, neglecting friction could result in inaccurate results. To investigate the effect of contact friction, a realistic model of the CPT calibration chamber was created using in the Finite Element software package ABAQUS CAE, and the constitutive model NorSand was implemented in it. The effect of contact friction on CPT tip resistance and sleeve friction was studied for loose and dense sand. The results show that contact friction has a significant effect on CPT results. Therefore, contact friction should be considered in modelling the CPT, and correct contact frictions must be assumed.

RÉSUMÉ

L'essai de pénétration au piézocône (CPT) est un outil rapide et rentable qui permet de caractériser de manière continue un dépôt de sol. La communauté géotechnique vise à créer un modèle numérique du CPT parce que, jusqu'à présent, il demeure difficile de développer des corrélations qui soient spécifiques à un matériau à partir des données existantes des chambres de calibration. Plusieurs difficultés se présentent lors de la modélisation du cône tel que les grandes déformations autour de la pointe du cône lors de la pénétration. Le plus grand défi dans le développement d'un modèle du piézocône est de gérer la distorsion du maillage qui cause une instabilité numérique. La manière la plus efficace d'éviter la distorsion du maillage est de négliger le frottement entre le piézocône et le sol. Cependant, cette approche peut mener à des résultats erronés. Dans la présente étude, un modèle numérique de pénétration du piézocône dans une chambre de calibration est utilisé afin d'investiguer l'effet du frottement entre le cône et le sol. Le modèle à éléments finis a été créé à l'aide d'ABAQUS CAE en implémentant NorSand comme modèle constitutif. L'influence du frottement sur la résistance en pointe (q_c) et sur le frottement local du manchon (f_s) est étudié pour des sables lâches et denses. Les résultats démontrent que le frottement a un impact majeur sur les données obtenues du CPT. Par conséquent, le frottement entre le cône et le sol ne peut pas être négligé dans un modèle numérique du CPT. Par ailleurs, l'usure du piézocône doit être surveillée sur le terrain.

1 INTRODUCTION

The Cone Penetration Test, CPT, is the predominant soil characterization test in a variety of geotechnical engineering projects related to mining, infrastructure, or residential as it is a fast and cost effective in-situ testing method. The penetrometer is usually equipped with multiple sensors and provides geotechnical engineers with continuous readings as it is pushed into the soil. Two of the main readings provided by this test are tip resistance q_c and sleeve friction f_s . Engineers use correlations (e.g. Robertson 1998) to calculate various soil properties and design parameters. These correlations are usually empirically based and have limited applicability to soils with properties outside of their database. Developing soil-specific correlations requires comprehensive laboratory testing using calibration chambers. This can be very time consuming and costly. With these difficulties along with the increase in computing power of modern computers many researchers have resorted to modeling the CPT.

Modeling the CPT can be very cost effective and fast; however, modelling the CPT in a useful fashion has taken

decades to come about. Modeling the penetration using typical continuum-based approaches requires both a good constitutive model for representing the soil and a numerical package capable of handling mesh distortions caused by penetrometer as it is pushed into the soil.

One of the main challenges associated with modeling CPT is the contact friction between the penetrometer and soil. As the penetrometer goes further into the soil the shear force of the friction drags the nodes of the soil along, causing excessive mesh distortions. Many researchers have resorted to ignoring contact friction to avoid this problem (Kioussis et. al. 1988; Huang et. al. 2004; Ahmadi et. al. 2005; Markauskas et. al. 2005; Sheng et. al. 2007; Sheng et. al. 2009, Chai et. al. 2012, Yi et. al. 2012, Ceccato et. al. 2016; Sheng et. al. 2013). Chai et. al. (2012) in their study for determining coefficient of consolidation from piezocone penetration decided to limit contact friction coefficient ($\mu = \frac{\tau}{\sigma}$ where τ and σ are shear and normal forces of contact and μ is the interface friction coefficient) to 0.01 to avoid mesh distortion. Sheng et. al. (2009)

decided to limit their contact friction coefficient to 0.1 as higher friction caused mesh distortion. Moreover, Markauskas et. al. (2005) as well as Sheng et. al. (2013) assumed a contact friction coefficient of zero and justified it based on the steel penetrometer being smooth.

There are many studies on contact friction between metallic surfaces and soils. Uesgi and Kishida (1986) examined influences of steel surface roughness and soil type on the contact friction coefficient μ . As illustrated in Figure 1, they normalized surface roughness by D_{50} of sand particles and showed that the contact friction coefficient, μ , generally increases linearly from $\mu = 0.4$ to $\mu = 0.8$ (which approximately correspond to the surface of new and heavily used steel plate respectively) by increasing normalized surface roughness factor from smooth to rough steel surfaces. Martinez et. al. (2014) looked deeper into the work of Uesgi and Kishida using their own axisymmetric experimental setup and also confirmed that both particles and surface roughness can affect inter contact face friction. Durgunoglu and Mitchell (1973) as part of their report for NASA studied contact friction coefficient between metallic surfaces and Monterey sand. They based their study on surface roughness, sand relative density, and friction angle. The surface roughness was measured for surface of hardened aluminum at three roughness levels: polished, anodized, and sanded which can represent smooth to lower range of heavily used penetrometers. They measured peak friction angle of sand as well as contact friction using direct shear box and found a unique number for friction angle ratio ($\frac{\delta}{\phi}$ where ϕ is the soil internal friction angle and $\delta = \arctan(\mu)$) at each surface roughness levels. The friction angle ratio was reported as $\delta = 0.3\phi$ for polished surface, $\delta = 0.5\phi$ for anodized surface, and $\delta = 0.9\phi$ for sanded surface.

With advancements in software and computation power there has been some success at including contact friction and overcoming mesh distortions in CPT modelling. Moug (2017) modelled the CPT in axisymmetric condition using FLAC3D (Itasca 2016). In order to overcome mesh distortions, she implemented her own distortion control Arbitrary Lagrangian Eulerian (ALE) scheme into FLAC. Based on work of Uesgi and Kishida, she used interface friction angle of $\delta = 0.6\phi$ as it represents a moderately used penetrometer. Susila and Hryciw (2003), Wei et. al. (2005), and Bienen et. al. (2015) used advancements in numerical package ABAQUS (Dassault Systèmes 2017) to overcome distortion problems and include friction in their study of the CPT. Susila and Hryciw (2003) as well as Wei et. al. (2005) based on work of Durgunoglu and Mitchell (1973) used interface friction angle of $\delta = 0.5\phi$ by assuming the penetrometer surface is slightly used. Bienen et. al. (2015) tried to use CPT penetration to predict behaviour of Jack-up rig foundation. They also used work of Durgunoglu and Mitchell (1973) and selected slightly smoother interface friction angle of $\delta = 0.45\phi$.

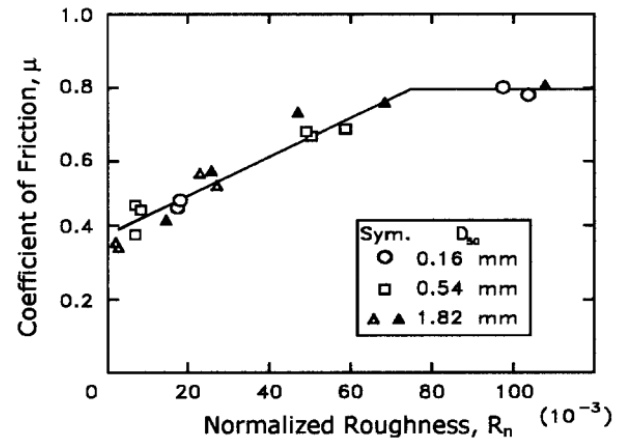


Figure 1 - Normalized surface roughness vs. friction coefficient for sand-steel interface (Uesgi and Kishida 1986)

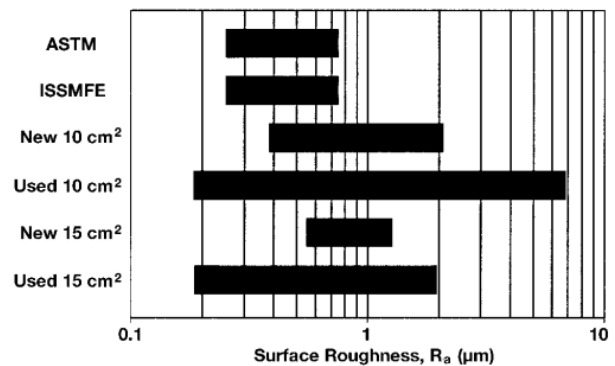


Figure 2 - Surface roughness of CPT sleeves (DeJong et al. 2000)

This paper provides an investigation into the importance of contact friction on CPT modeling results. The CPT ASTM standard (D5778-12) assumes the penetrometer is newly manufactured. DeJong et al. (2000) in Figure 2 summarizes common surface roughness of used and new CPT sleeves and compares them with ASTM D3441 guideline. Based on the wide range of variation this study will try to assess how important it is to define the soil-CPT contact friction more precisely.

2 FINITE ELEMENT MODEL

2.1 Geometry, Loading, and Boundary Conditions

Figure 3 represents the mesh and boundary conditions used in an axisymmetric CPT model in ABAQUS. The mesh and element (four nodes full integration) size was optimised for accuracy and efficiency. The dimensions of the model are based on the University of Toronto calibration chamber (Been et al., 1987). The chamber has a diameter of 1.4 m and height of 1.0 m. The horizontal and vertical confining stresses can be controlled independently

to simulate different K_0 conditions. The penetrometer has standard area of 10 cm^2 and a 60-degree apex angle. The penetrometer is pushed into the soil by applying 2 cm/s velocity boundary condition to the top of the penetrometer. The contact between soil and penetrometer was defined using surface to surface contact elements with penalty formulation. The penetrometer was pre-embedded into the soil down to slightly past the apex to avoid additional problems associated with corner nodes. The shear behaviour of the contact is defined with a constant friction coefficient. As discussed earlier, mesh distortion is a major issue with CPT modeling. The ALE scheme was activated for this analysis with all the default parameters to overcome this problem.

2.2 Soil Constitutive Model

Modeling CPT in sand requires a constitutive model capable of representing sand behaviour, including its compressibility, dilation and contraction tendencies, and peak and critical state shear strengths. This study uses NorSand (Jefferies 1993) as the constitutive model due to its simplicity and performance in its predictions. NorSand achieves a better representation of sands by relating yield surface to the state parameter, ψ . The state parameter is defined as $\psi = e - e_c$ where e_c is the critical state void ratio at current mean stress (Been and Jefferies 1985). A positive state represents loose sands and a negative state represents dense sands. NorSand was implemented in ABAQUS by user material interface VUMAT. A Cutting Plane return mapping (Simo and Taylor 1986; Yapage and Liyanapathirana 2013) technique was used to integrate plasticity equations. The implementation was verified (Mozaffari 2017) against closed form solutions of triaxial compression and simple shear tests provided by Jefferies (2015).

Norsand was calibrated to Ticino4 sand (Baldi et. al. 1986) for the CPT modeling. Ticino4 sand is mostly comprised of Quartz minerals with sub-rounded particles and $D_{50} = 530 \mu\text{m}$. The e_{min} and e_{max} of this material are reported as 0.6 and 0.9 (Ghafghazi 2011). The NorSand parameters of this material is summarized in table 1. In table 1, Γ and λ_e are two parameters defining critical state line in $e-\log(p)$ space and are usually obtained from triaxial compression tests on loose materials. M_{tc} is the critical state stress ratio ($\eta = \frac{q}{p}$ at critical state). χ_{tc} quantifies the dependency of maximum dilatancy on the state parameter (ψ) and N is the volumetric coupling coefficient. Both χ_{tc} and N are obtained from triaxial compression tests on dense materials. H_0 is the base hardening parameter of the yield surface and H_ψ is the dependency of this hardening parameter to the state parameter. Both H_0 and H_ψ are calibrated using few triaxial compression tests on soils at different states. Finally, ν is the Poisson's ratio.

Table 1- NorSand parameters for Ticino4 sand (after Ghafghazi 2011)

Γ	λ_e	M_{tc} (φ°)	χ_{tc}	N	H_0	H_ψ	ν
0.986	0.0243	1.27 (31.6)	3.0	0.4	60	350	0.2

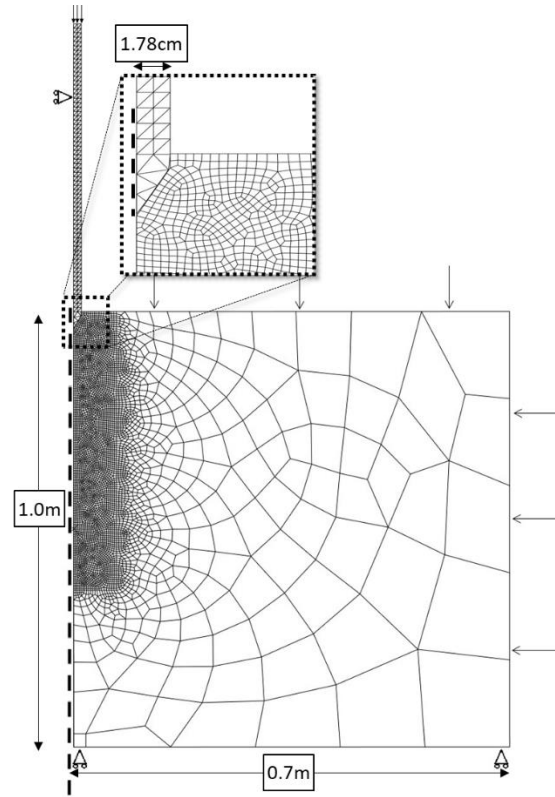


Figure 3 - Axisymmetric mesh and boundary conditions, modelled in ABAQUS.

3 CPT MODELLING

All the CPT models studied here are under drained conditions which corresponds to dry tests in chamber or drained penetration in the field. All the models are performed at the same overburden stress of 74 kPa . The models are simulated at least to a depth where a steady CPT response is obtained. Based on works of Uesgi and Kishida (1986) as well as Durgunoglu and Mitchell (1973), it is apparent that both surface roughness and soil properties are important for contact friction. As mentioned before, this study uses friction angle ratio, $\frac{\delta}{\varphi}$, approach by Durgunoglu and Mitchell. The ratios to study here were selected to cover ideally smooth to slightly rough surface of steel which is interface friction angles of $\delta = 0.0\varphi$, $\delta = 0.35\varphi$ and $\delta = 0.7\varphi$ ($\mu = 0.00, 0.20, \text{ and } 0.40$).

As mentioned before sand behaviour is state-dependent. In order to investigate how CPT response is affected by the state parameter, the CPT models here were performed for a wide range of $\psi = 0.1$ to $\psi = -0.3$.

4 RESULTS AND DISCUSSION

The modeling results in Figures 4 to 6 provide a representation of the difference in CPT results caused by contact friction. The difference in the results is more pronounced in denser materials. The modelled tip

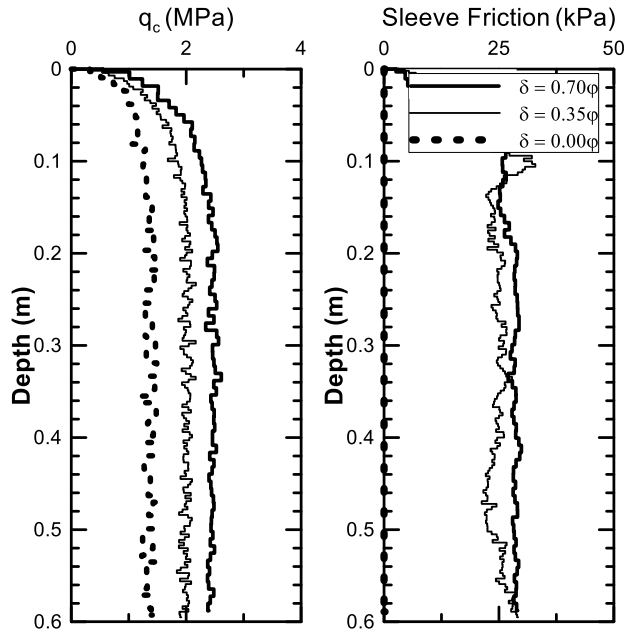


Figure 4 - Modeled CPT tip resistance q_c and sleeve friction vs. depth for loose Ticino4 sand ($\psi = 0.076$)

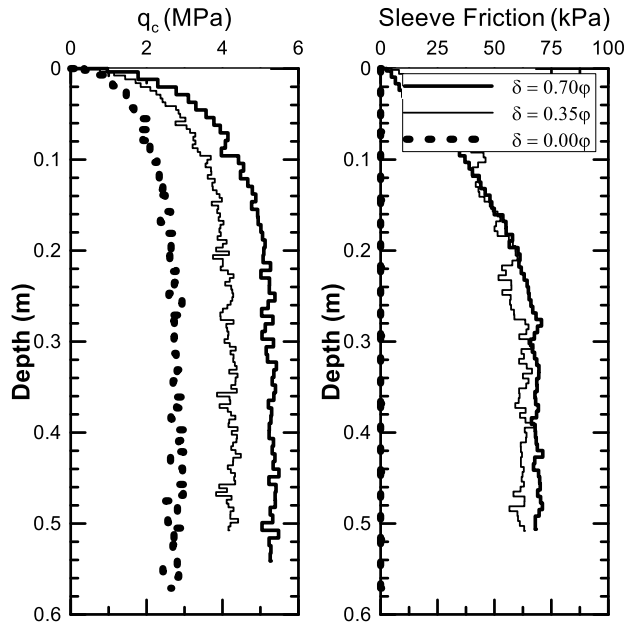


Figure 5 - Modeled CPT tip resistance q_c and sleeve friction vs. depth for medium dense Ticino4 sand ($\psi = -0.044$)

resistance at highest friction compared to lowest friction is 45% higher for loose materials (Figure 4), 85% higher for medium dense material (Figure 5), and 110% more for dense Ticino sand (Figure 6). The sleeve friction is not affected by friction for looser materials (Figures 4 and 5); however, similar to the tip resistance the sleeve friction

shows a stronger dependency on contact friction for denser materials (Figure 6). Overall the sleeve friction is less influenced by the contact friction than the tip resistance, which seems somewhat counterintuitive at first. DeJong et. al. (2001) explained that sleeve friction measurements are less reliable than tip resistance because the friction sleeve is exposed to varying levels of horizontal stress and the soil surrounding the sleeve is disturbed by the advancing tip.

Figure 7 provides a summary of all the models and compares them with calibration chamber data. Figure 8 tests the CPT response against Robertson's (1998) soil identification chart. In both Figures 7 and 8 normalized tip resistance is defined as:

$$Q = \frac{q_c - \sigma_{vo}}{\sigma_{vo}} \quad [1]$$

And normalized friction ratio is defined as:

$$F = \frac{f_s}{q_c - \sigma_{vo}} \times 100 \quad [2]$$

where q_c is the CPT tip resistance, f_s , is the CPT sleeve friction, and σ_{vo} is the overburden stress.

Figure 7 validates the ability of the developed numerical model to estimate CPT tip resistance. The numerical results are in close agreement with the experimental data with regards to both value and their trend vs. the state parameter ψ . Figure 8 examines both tip resistance and sleeve frictions obtained from the model. The model is also in agreement with the soil identification chart as all the obtained results fall within zone 5 (sand and sand mixtures). Additionally, denser specimens (lower state parameter) move upwards in the figure and approach zone 8 (stiff sands) as expected. The soil-CPT contact friction does not appear to significantly affect the soil behaviour type inferred from the data.

A Closer look at Figure 7 reveals more insight into the effect of friction on the CPT tip resistance. Models with rougher interface ($\delta = 0.7\phi$) result in a tip resistance that is more than double that of a completely smooth interface ($\delta = 0.0$). This confirms that ignoring friction in modeling CPT can greatly underestimate the tip resistance. However, the actual effect of contact friction on CPT results is likely bound to a smaller range due to a tighter range of friction ratios within ASTM (D3441) tolerance for wear and tear on penetrometer surfaces. For Ticino4 sand, $D_{50} = 530 \mu m$ and R_n range is between 0.4×10^{-3} and 13×10^{-3} for a used 10 cm^2 cone (Figure 2). This range can result in a μ range of 0.35 to 0.45 (Figure 1) or δ range of 0.34ϕ to 0.42ϕ suggesting that the results presented for $\delta = 0.35\phi$ likely represent the most realistic set of analyses presented here.

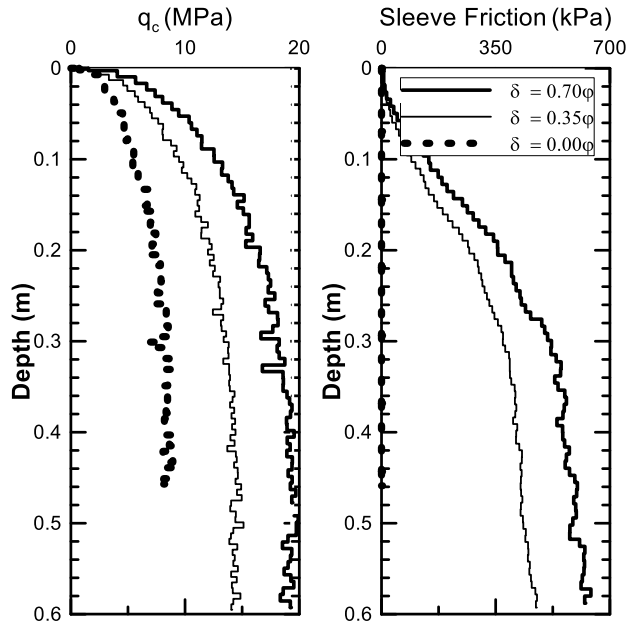


Figure 6 - Modeled CPT tip resistance q_c and sleeve friction vs. depth for dense Ticino4 sand ($\psi = -0.284$)

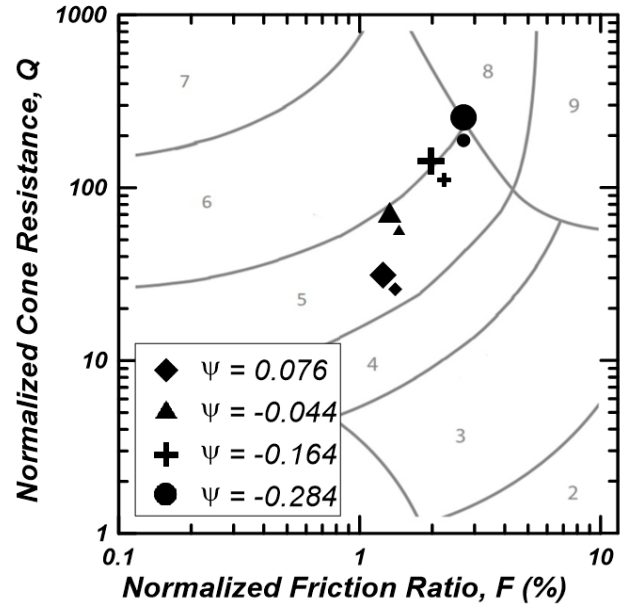


Figure 8 - Modeling results compared with the soil identification chart (Robertson 1998) at $\delta = 0.7\phi$ (large symbol) and $\delta = 0.35\phi$ (small symbol). Regions 5 and 6 identify sand and sand mixtures. Region 8 identifies Stiff sand.

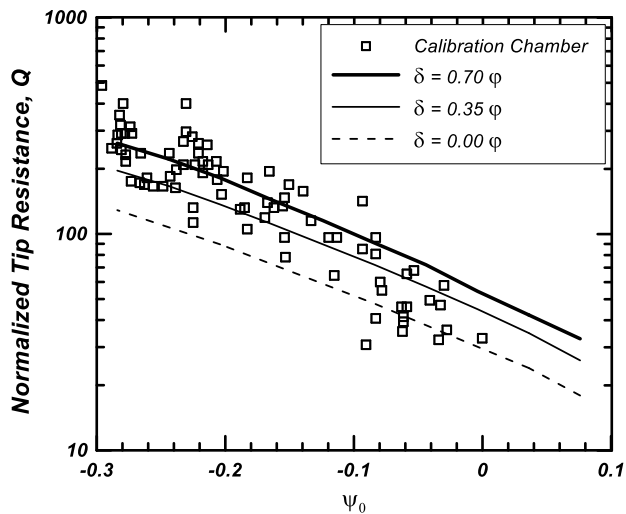


Figure 7 - Comparing modeling results of normalized tip resistance vs. the state parameter, ψ , at different interface frictions, δ , against calibration chamber test data of Baldi et. al. (1986).

5 SUMMARY AND CONCLUSION

This paper investigated the effects of contact friction on two main measurements of the Cone Penetration Test: tip resistance and sleeve friction. Many past studies have assumed zero soil-CPT contact friction and no in-depth analysis on the importance of contact friction has been performed on results of CPT models.

A real-geometry axisymmetric CPT model was created by implementing Norsand (Jefferies 1993; Shuttle and Jefferies 2005) in ABAQUS. A series of drained penetration models in a calibration chamber were created to study the effects of the contact friction angle, δ , and the state parameter, ψ , on CPT response. The model was validated against calibration chamber data.

Both CPT tip resistance and sleeve friction are functions of the contact friction, with the tip resistance being much more dependent on the interface friction. The effect of contact friction on CPT sleeve friction was observed to be more pronounced for dense materials. While the soil-CPT contact friction was demonstrated to have as much as a doubling effect on the tip resistance, in practice, cones appear to have a relatively tight range of contact frictions with soil. The purpose of this paper is demonstrating the importance of including the correct contact friction in CPT modelling.

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