Modifying the NorSand soil model to improve static simple shear behaviour predictions

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ABSTRACT

The simple shear laboratory test is nowadays commonly used to investigate soil behaviour under plain strain conditions of deformation. This article focusses on behaviour predictions of such laboratory test using the NorSand soil model. Through the analysis of various test predictions under undrained static simple shear conditions, it is shown that the original isotropic NorSand model is unable to correctly predict contractive behaviour observed in laboratory test results. By implementing the anisotropic critical state theory within the model to develop a new anisotropic NorSand model, the authors were able to satisfactorily predict sand behaviour under simple shear conditions. Prediction results obtained using the anisotropic NorSand model are consistently in better agreement with laboratory data than when using the original isotropic model, especially for anisotropically consolidated simple shear tests.

RÉSUMÉ

L'essais de cisaillement simple est de nos jours couramment utilisé pour étudier le comportement des sols sous conditions de déformations planes. Cet article porte sur la prédiction du comportement d'un sable sous ces conditions à l'aide de la loi de comportement NorSand. Par l'analyse de différentes prédictions de comportement sous conditions de cisaillement simple non drainé, il est montré que le modèle original NorSand isotrope est incapable de correctement prédire le comportement contractant observé dans les essais de laboratoire. Par l'implémentation de la théorie de l'état critique anisotrope dans NorSand pour créer un nouveau modèle NorSand anisotrope, les auteurs sont parvenus à prédire de manière satisfaisante le comportement d'un sable sous cisaillement simple. Les résultats de prédiction de comportement obtenus grâce au modèle NorSand anisotrope sont systématiquement plus près du comportement observé en laboratoire que ceux obtenus grâce au modèle original NorSand isotrope, spécialement pour les essais de cisaillement simple consolidés anisotropiquement.

1 INTRODUCTION

Simple shear conditions of deformation are generally considered close to conditions which might arise in a slope or during earthquake events. These conditions imply zero-displacement in horizontal directions (ensuring plane strain conditions), constant total vertical stress and shear strain increments (γ_{xy}) being applied at the top of the soil sample. Though many simple shear laboratory devices were developed through the years to replicate these conditions, the NGI type apparatus is the most commonly used nowadays. It uses circular Teflon coated confining rings to ensure proper lateral restraint. Simple shear conditions can also be achieved in the hollow cylinder apparatus. Compared to the NGI type apparatus, this device offers the possibility to control and measure all stresses applied to the sample (which is normally impossible due to the use of mechanical lateral restraint).

Simple shear conditions are complex, significantly more so than triaxial compression, in part due to the rotation of principal stress axes during testing. Roscoe *et al.* (1967) were among the firsts to measure the direction of principal stress axes (α) during simple shear. They reported values increasing until stabilization was achieved around a value of $45°$ after extended deformations (see Figure 1).

This article explores undrained static simple shear laboratory results obtained by Yoshimine *et al.* (1998) in the light of numerical modelling. Behaviour predictions using the original isotropic NorSand model and a modified anisotropic NorSand soil model are compared. The importance of anisotropy effects on soil behaviour during simple shear testing are highlighted.

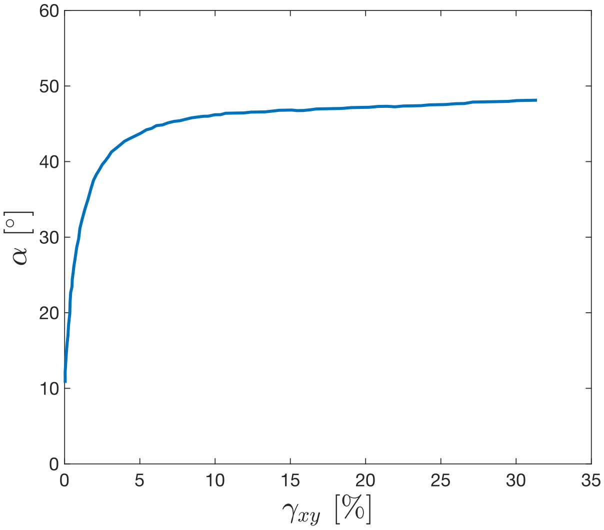

Figure 1. Evolution of principal stress direction (α) during static simple shear (modified from Roscoe *et al.*, 1967)

2 MODELLING PROCEDURE

2.1 NorSand

NorSand is an isotropic critical state based soil model engineered by Jefferies (1993). Initially developed to predict the behaviour of sands under triaxial compression conditions, the model was since then extended to cover additional laboratory test conditions. Recently, the softening effect of principal stress rotation was incorporated into the model as an effort to improve predictions of cyclic simple shear tests (Jefferies *et al.*, 2015). Although these modifications yielded promising results, the full extent of simple shear behaviour was not readily considered, as will be shown in this paper.

NorSand's greatest quality remains its simplicity. Few input parameters (8) are required to use the model when fixed principal stress directions are considered (such as for triaxial compression). An additional plastic softening parameter (Z) can also be used to take into account principal stress rotation. In depth description of the model is available in Jefferies & Been (2015).

2.2 Anisotropic NorSand

Principal stress direction (α) has a crucial influence on soil behaviour but is often disregarded for simplicity sakes. Such an approach is deemed reasonable when triaxial compression behaviour is studied, as was the case during NorSand's early development. But as soon as principal stress direction is no longer aligned with consolidation direction, as is the case during simple shear, refinement of the model needs to be considered.

As will be seen in Section 3 the current isotropic version of NorSand is incapable of properly predicting static simple shear behaviour of sands. The model wrongly predicts strongly dilatant responses while most corresponding laboratory tests exhibit strong contraction followed by dilation. By focusing on a laboratory test as complex as the cyclic simple shear test to extend the model's capabilities, Jefferies *et al.* (2015) may have overlooked the softening effect of inclined principal stress direction itself, even when no rotation actually takes place.

Castonguay & Konrad (2018) recently proposed an anisotropic version of NorSand, based on the anisotropic critical state theory (Li & Dafalias, 2012). This new anisotropic NorSand model was used to predict the behaviour of sands under various fixed principal stress directions. The anisotropic critical state theory was implemented in NorSand through the use of the dilatancy state parameter (ζ) in lieu of the state parameter (ψ). Per Equation 1, the dilatancy state parameter is the result of the anisotropy parameter (ψ_A) affecting the state parameter to account for anisotropy effects caused by fabric and principal stress direction. Refer to Li & Dafalias (2012) for a thorough description of the anisotropic critical state theory.

$$\zeta = \psi - \psi_A \qquad [1]$$

Full details regarding the implementation of the anisotropic critical state theory within NorSand can be found in Castonguay & Konrad (2018).

2.3 Laboratory test data

Undrained static simple shear tests carried out on Toyoura sand by Yoshimine *et al.* (1998) are used to benchmark accuracy of numerical predictions in this study. These tests were performed using a hollow cylinder apparatus. Several void ratios were tested to investigate the effect of initial state. The first test series considered in this paper was isotropically consolidated ($K_0 = 1$) at an effective mean stress of 100 kPa, while the second test series was anisotropically consolidated ($K_0 = 0.5$) at an effective mean stress of 130 kPa.

NorSand's input parameters used for Toyoura sand are shown in Table 1. These input parameters were calibrated using triaxial compression laboratory test results. Pressure and void ratio dependent shear modulus (G) was calculated throughout the modelling process using Equation 2 (Richart *et al.*, 1970), where G_0 is the initial shear modulus, e is the void ratio, $\bar{\sigma}_m$ is the mean effective stress and p_{atm} is the atmospheric pressure (used for stress normalization).

Table 1. NorSand and anisotropic critical state theory (ACST) parameters for Toyoura sand

NorSand parameters[1]	Toyoura sand
Elasticity	
ν	0.2
G_0	125
Critical state line	
Γ	0.983
λ_e	0.019
Plasticity	
M_{tc}	1.28
χ_{tc}	4.4
N	0.41
H	305
Anisotropy (ACST)	
c	5.7
e_A	0.10
F_0	0.45

[1] Model parameters inspired by Ghafghazi & Shuttle (2008) as well as Gao *et al.* (2014)

$$G = G_0 p_{atm} \frac{\left(2.97 - e\right)^2}{1 + e} \left(\frac{\bar{\sigma}_m}{p_{atm}}\right)^{0.5} \qquad [2]$$

3 ISOTROPIC NORSAND SIMULATION RESULTS

The original isotropic NorSand model was used to simulate undrained static simple shear tests for various void ratios. Comparison with the expected response based on Yoshimine *et al.* (1998) laboratory tests is presented in Figures 2 and 3 for isotropically consolidated tests and Figures 4 and 5 for anisotropically consolidated tests, where $\bar{\sigma}$ is an effective stress, ε is a strain, γ is the shear strain, subscripts "1" and "3" respectively indicate major and minor principal directions, subscripts "q" and "m" respectively indicate deviatoric and mean stresses.

3.1 Isotropically consolidated simple shear tests

Stress-strain responses for various void ratios are shown in Figure 2. The isotropic NorSand model consistently predicts overly dilative behaviour compared to what is observed from laboratory results. This tendency is also shown in Figure 3 where stress paths are presented. Void ratio seems to only have limited effect on the model's capacity to accurately predict sand behaviour in simple shear. Note that, laboratory data exhibits very contractive behaviour for larger void ratios.

3.2 Anisotropically consolidated simple shear tests

Similar to results presented in Section 3.1, the isotropic NorSand model fails to accurately predict soil behaviour for anisotropically consolidated simple shear tests. As shown in Figures 4 and 5, laboratory test results display strongly contractive behaviour (even static liquefaction in the case of the loosest sample $e = 0.880$). However, the isotropic NorSand model again predicts very dilative behaviour and fails to capture very important behaviour features displayed by laboratory test results. Predicted stress paths shown in Figure 5 are generally inconsistent with stress paths observed in laboratory tests for equivalent void ratios.

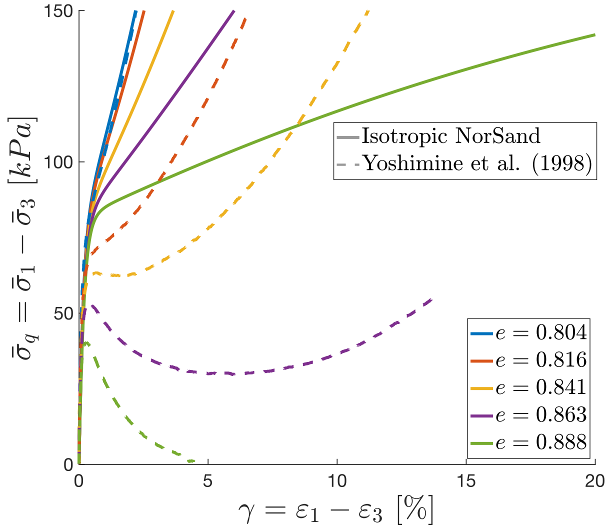

Figure 2. Isotropically consolidated – Isotropic NorSand predictions – Stress-strain behaviour

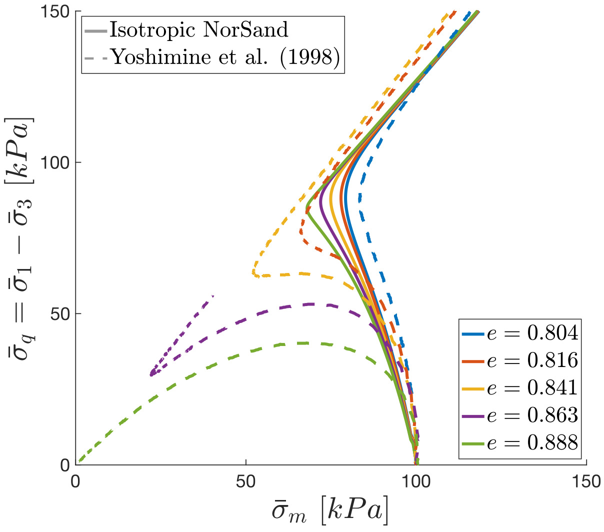

Figure 3. Isotropically consolidated – Isotropic NorSand predictions – Stress path

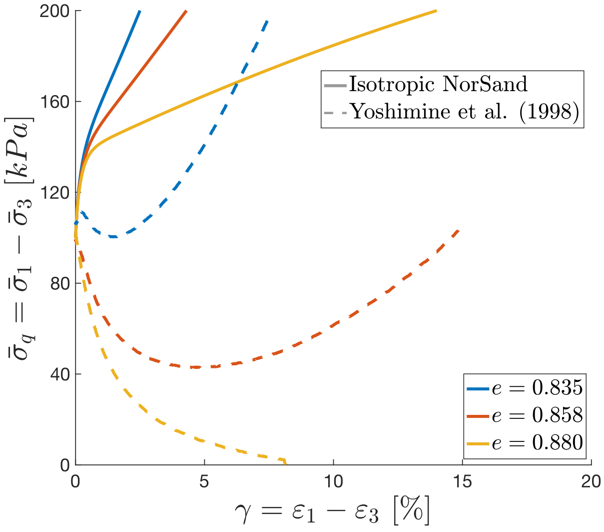

Figure 4. Anisotropically consolidated – Isotropic NorSand predictions – Stress-strain behaviour

As previously mentioned, Jefferies *et al.* (2015) recently proposed to modify NorSand to account for the softening effect of principal stress rotation. As principal stress direction (α) rotates during simple shear, NorSand's yield surface would soften, allowing for additional plastic strains. However, as shown in Figure 6, principal stress rotation happens very quickly from 0° to $45°$ in simple shear modelling (within $\gamma = 0.1\%$ for isotropically consolidated tests and within $\gamma = 1\%$ for anisotropically consolidated tests). The softening effect of this rotation is thus minimal and doesn't affect results past small strain territory. Including NorSand's plastic softening parameter Z within these predictions essentially yields no additional benefits.

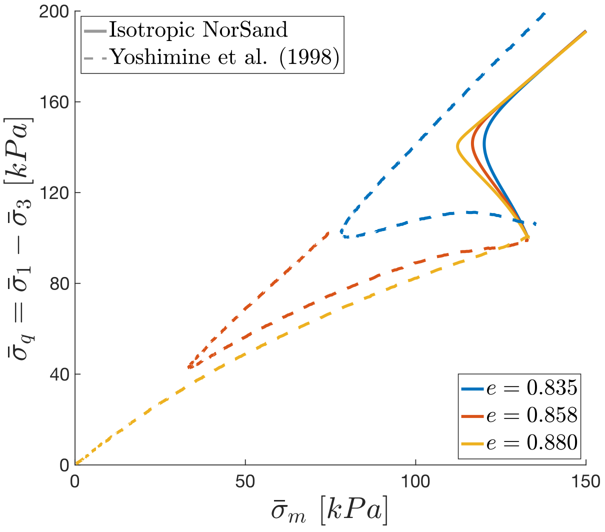

Figure 5. Anisotropically consolidated – Isotropic NorSand predictions – Stress path

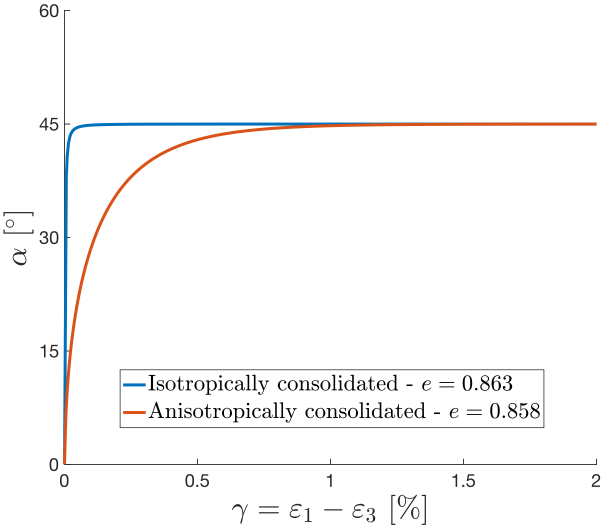

Figure 6. Evolution of principal stress direction (α) during simple shear modelling

4 ANISOTROPIC NORSAND SIMULATION RESULTS

Improved undrained simple shear behaviour predictions can be achieved by accounting for anisotropy effects. Initially, the soil's fabric is globally oriented perpendicular to the consolidation direction ($\alpha = 0°$), leading to a stiffer reaction if soil is loaded following that consolidation direction (as is the case during triaxial compression, for example). As principal stress direction rotates during simple shear testing, the soil is no longer loaded following its strong consolidation direction, leading to a softer response.

The anisotropic critical state theory implemented within anisotropic NorSand uses the angular difference between the current loading direction and the current fabric orientation to modify the soil's response through the anisotropy parameter (ψ_A of Equation 1). As shown in Figure 7, while the state parameter (ψ) remains negative throughout a modelled test, the dilatancy state parameter (ζ) is initially positive because of anisotropy effects brought by the anisotropy parameter. Anisotropic NorSand would hence predict contractive behaviour as long as the dilatancy state parameter would remain positive.

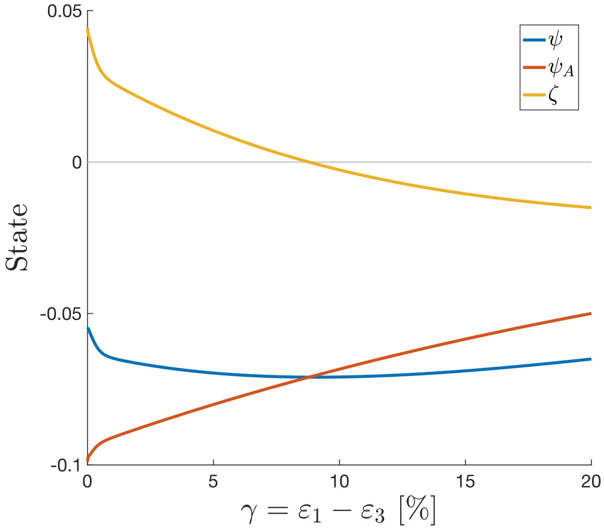

Figure 7. Evolution of state – Isotropically consolidated – Anisotropic NorSand ($e = 0.841$)

4.1 Isotropically consolidated simple shear tests

The softening effect of anisotropy brought by the inclusion of the anisotropic critical state theory within NorSand is evident in Figures 8 and 9 where stress-strain behaviour and stress paths are presented for isotropically consolidated simple shear tests. Contrary to what was observed in Figures 2 and 3 for the original isotropic NorSand model, the anisotropic NorSand model displays a wide range of behaviour: strong contraction for larger void ratios ($e = 0.888$) and dilation for smaller void ratios ($e = 0.804$). Predictions compare favorably to laboratory results, though certain disparities are still evident.

An important point to note with regard to these behaviour predictions is that no particular effort was made to modify the anisotropic NorSand model input parameters to obtain a better fit in comparison to laboratory test results. Parameters obtained from triaxial test calibrations were directly used to model static simple shear tests. This approach relies on the fact that the constitutive law is based on sound physic principles that remain valid even when boundary conditions change.

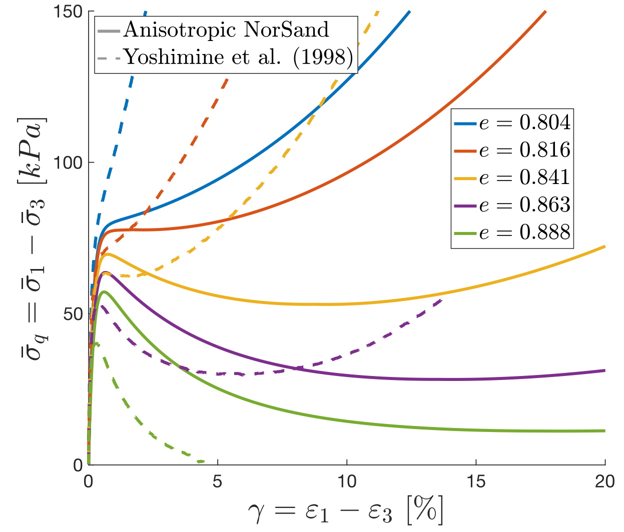

Figure 8. Isotropically consolidated – Anisotropic NorSand predictions – Stress-strain behaviour

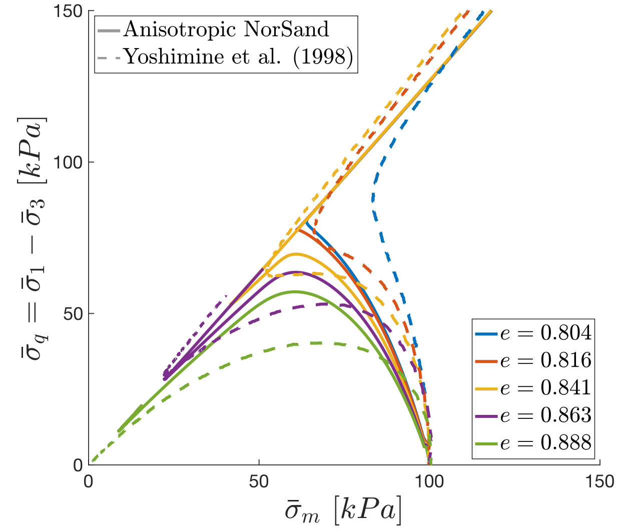

Figure 9. Isotropically consolidated – Anisotropic NorSand predictions – Stress path

4.2 Anisotropically consolidated simple shear tests

While isotropic NorSand predictions did not match laboratory results for anisotropically consolidated simple shear tests, the anisotropic NorSand model performs much better. Strongly contractive behaviour is well predicted by the model in Figure 10 (especially for $e = 0.880$). However, strongly dilative behaviour is less evident in predictions than it is in laboratory test data (especially for $e = 0.835$). Predicted stress paths shown in Figure 11 are in good agreement with measured laboratory data, especially for $e = 0.858$.

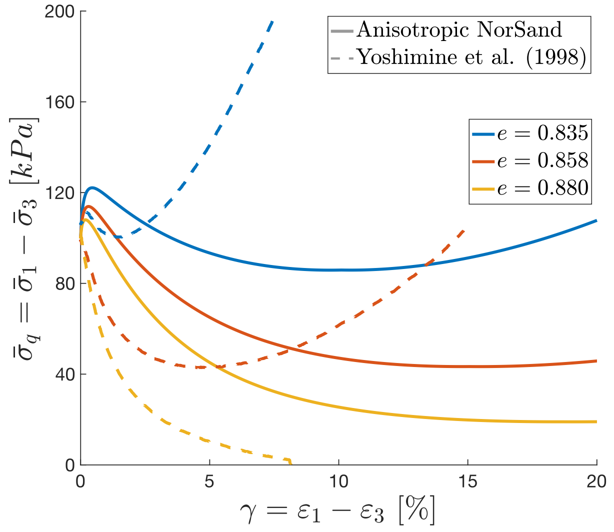

Figure 10. Anisotropically consolidated – Anisotropic NorSand predictions – Stress-strain behaviour

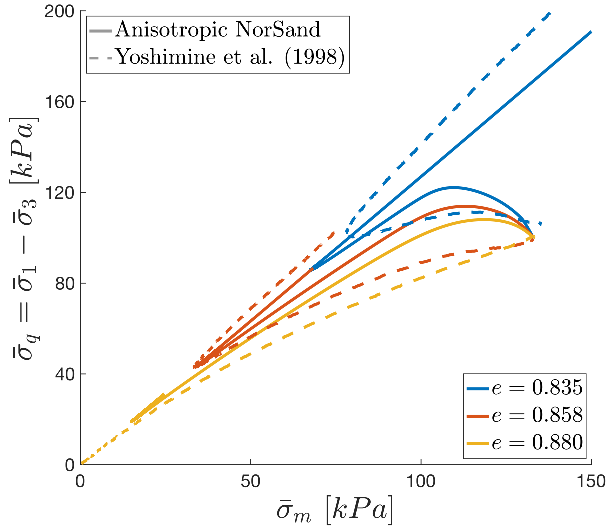

Figure 11. Aniosotropically consolidated – Anisotropic NorSand predictions – Stress path

5 CONCLUSION

As was demonstrated in this paper, the static simple shear test is a strongly anisotropic loading. As principal stress direction rotates at the beginning of the test to quickly reach an inclination of $\alpha = 45°$, loading direction is no longer perpendicularly aligned with the soil's stronger fabric orientation that was developed through consolidation. This effect of induced anisotropy needs to be accounted for to accurately predict soil behaviour.

Using the original isotropic NorSand model to simulate such a complex anisotropic loading process inevitably leads to an underestimation of the contractive response measured in laboratory tests. Such contractive response dictates the overall soil behaviour and cannot be left out of modelling. Modelling undrained static simple shear tests using the isotropic NorSand model would necessitate caution since overly dilative responses would consistently be obtained. By implementing the anisotropic critical theory within NorSand, satisfactory behaviour predictions were achieved for isotropically and anisotropically consolidated undrained static simple shear tests.

6 ACKNOWLEDGEMENTS

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