An approach for finite element modeling of one-dimensional thaw consolidation

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ABSTRACT
Thawing of permafrost in the North could cause many issues such as the settlement in roads, foundations, pipelines, and slope instability. For a given above-zero temperature at the boundary (e.g., at the ground surface), the excess pore water pressure generates at the thaw front that moves with time depending upon the thermal properties of the soil. The excess pore water pressure generated in the thawed soil dissipates with time, and thaw consolidation occurs. This study presents a finite element (FE) modeling technique that can simulate heat transfer, including the thawing of ice-rich frozen soil, and generation of excess pore water pressure at the thaw front and its dissipation. The FE analyses are performed for a one-dimensional condition, and the results are compared with a simplified analytical technique. The developed FE method could be used for complex boundary value problems such as thawing of frozen soil around a warm buried pipeline.

RÉSUMÉ
La décongélation du pergélisol dans le Nord pourrait poser de nombreux problèmes, tels que le règlement des routes, des fondations, des pipelines et l'instabilité des pentes. Pour une température donnée au-dessus de zéro à la limite (par exemple à la surface du sol), la pression interstitielle en excès génère au front de dégel qui se déplace avec le temps en fonction des propriétés thermiques du sol. L'excès de pression interstitielle généré dans le sol décongelé se dissipe avec le temps et il se produit une consolidation par le dégel. Cette étude présente une technique de modélisation par éléments finis (FE) permettant de simuler le transfert de chaleur, notamment la décongélation d'un sol gelé riche en glace, la génération d'une pression interstitielle excessive dans le front de dégel et sa dissipation. Les analyses FE sont effectuées pour une condition unidimensionnelle et les résultats sont comparés à une technique analytique simplifiée. La méthode FE développée pourrait être utilisée pour résoudre des problèmes complexes liés aux valeurs limites, tels que la décongélation de sol gelé autour d'un pipeline enterré à chaud.

1 INTRODUCTION

Thaw settlement is a major geotechnical design consideration of many infrastructures in cold regions such as highways, airfields and energy pipelines. Ice-rich frozen soils might melt due to seasonal warm temperatures in the spring or by ground warming due to human activities such as flow of warm hydrocarbon in a pipeline.

Upon thawing near the thaw front, the volumetric ice in the forms of pore ice and ice lenses transfers to pore water, which experiences an increase in excess pore water pressure due to the weight of soil and other external loads. The hydraulic conductivity of frozen soil (k_f) is generally several order smaller than that of thawed soil (k_t) (Williams and Burt 1974); therefore, the excess pore water pressure dissipation primarily occurs through thawed soil. In addition to thaw-consolidation settlement, the generated excess pore water pressure could reduce the shear strength of thawed soils which might cause the failure of slopes and foundations (Morgenstern and Nixon 1971; Nixon and Morgenstern 1973).

The dissipation of thaw-induced excess pore water pressure depends on the rate of thawing and hydraulic conductivity of the soil. When thawing occurs at a faster rate, especially in the vicinity of above-zero temperature boundary, the generated excess pore water pressure may not dissipate completely during thawing. The pore pressure dissipation can be modeled using the consolidation theory for unfrozen soil; however, the moving boundary (thaw front) makes the problem complicated. In other words, proper modeling of heat transfer, excess pore water generation and dissipation are required to calculate thaw settlement.

Morgenstern and Nixon (1971) developed an analytical solution for one-dimensional consolidation of thawed soils by extending Terzaghi's one-dimensional consolidation theory. The compressibility of the frozen soil and flow of water through it were neglected. The location of the moving thaw front boundary was modeled using the Neumann solution (Carslaw and Jaegar 1947). Analyses were performed using a small strain approach adopting a linear void ratio (e)–effective stress (σ') relationship together with a constant hydraulic conductivity of thawed soil.

Foriero and Ladanyi (1995) developed a finite-strain approach by extending Gibson's large-strain consolidation theory which also accounts the varying compressibility and hydraulic conductivity during consolidation. Similar to Morgenstern and Nixon (1971), an uncoupled heat transfer analysis was considered to determine the thaw front depth prior to consolidation analysis.

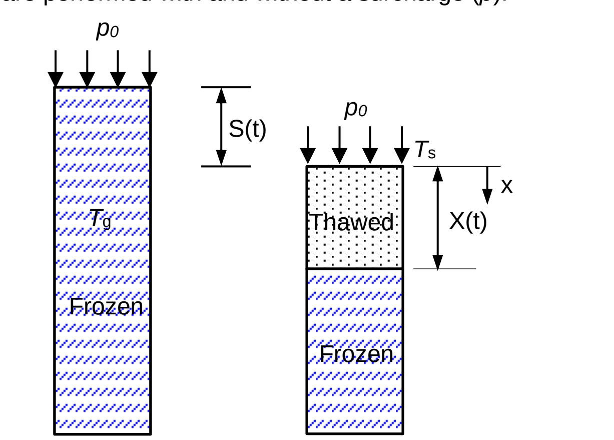
After addressing some of the limitations of the above studies, Dumais and Konrad (2018) developed a coupled large-strain thaw consolidation model in Lagrangian coordinates. Nonlinear e–σ' and e–k_t relationships were implemented which also considered the volume change when ice transfers to water. Assuming the one-dimensional analysis is valid, Dumais and Konrad (2019) analyzed the thaw consolidation around the Inuvik warm-oil pipeline using the consolidation model developed by Dumais and Konrad (2018).

The objective of this study is to present a finite element modeling approach for one-dimensional thaw consolidation. The generation and dissipation excess pore water pressure obtained from FE analyses are compared with the analytical solution of Morgenstern and Nixon (1971).

2 PROBLEM DEFINITION

Figure 1 shows the one-dimensional soil sample considered in this study. Initially, at time t = 0, the soil column is frozen having a constant temperature $T = T_g$ (< 0°C) (Fig. 1(a)). A step increase in temperature $T = T_s$ (> 0°C) is applied at the top surface. With time, the thaw front (T = 0°C) moves downward to X(t), and two regions are formed (frozen and thawed), as shown in Fig. 1(b). The frozen region (x > X(t)) is almost incompressible and impermeable as compared to the thawed region. The top surface of the soil domain (x = 0) is considered as a free drainage boundary. When the pore ice and ice lenses melt, the weight of the soil above the thaw front is carried by water that creates an excess pore water pressure (u(x,t)) at the thaw front. The thaw consolidation occurs if the pore water dissipates, which results in settlement of the soil layer (S(t)). In the present study, the thaw depth is measured from the top surface of the soil domain at a given time, which moves with settlement.

In the field, thaw consolidation might occur either under the self-weight of thawed soils (e.g., spring warming of the frozen ground at the surface) or under combined effects of self-weight and external loads (e.g., frozen ground warming by buried warm oil pipelines). To investigate thaw consolidation under these loading conditions, FE analyses are performed with and without a surcharge (p).



a) Initial, time t = 0 b) At time t

Figure 1. One-dimensional soil domain used in FE analysis

3 FINITE ELEMENT MODELING

Abaqus/Standard FE software is used for modeling coupled pore fluid diffusion and stress analysis along with conduction mode of heat transfer. An initially frozen soil sample of 250 mm high is thawed by a step increase in temperature at the top surface as shown in Fig. 1.

The FE program used in this study allows only linear brick, first-order axisymmetric, and second-order tetrahedron elements for modeling coupled heat transfer with pore fluid diffusion and stress analysis. In this study, the soil column is created using the 8-node linear brick reduced integration fully coupled temperature–pore pressure element (C3D8RPT) available in the software. The soil domain is discretized into 1 mm cubical elements. In order to perform one-dimensional analysis, only one element is used in all horizontal directions.

All the vertical faces are restrained to lateral movement, while the bottom boundary is restrained to vertical movement. For heat and fluid flow, zero heat flux and zero pore fluid flux boundary conditions are applied to all surfaces except for the top surface. A free drainage (zero pore pressure) boundary condition is applied at the top surface.

The initial conditions such as void ratio, degree of saturation and initial ground temperature are defined, and then the numerical analysis is divided into two steps. The geostatic load is applied in the first step to bring the soil domain to the in-situ stress condition. The pore fluid weight is excluded in this step to perform the analysis in terms of excess pore water pressure. In the second step, a step increase in temperature is applied at the top surface. To avoid numerical instability, the temperature is ramped from the initial frozen ground temperature (-5°C) to the targeted value within 1 minute.

3.1 Excess pore water pressure generation

Upon thawing, excess pore water pressure is generated at the thaw front because of the weight of the thawed soil and other external loads. In order to simulate the pore water pressure generation, the volume compressibility and hydraulic conductivity are ramped from the frozen state to the thawed state over a temperature range of -0.5°C to 0°C. In other words, the soil compressibility is very small at a temperature below -0.5°C; however, it becomes a compressible thawed soil at a temperature above 0°C. Similarly, hydraulic conductivity increases from k_f to k_t in this temperature range.

3.2 Model parameters

Table 1 shows the thermal, mechanical and hydraulic parameters used in this study. The hydraulic conductivity of thawed fine-grained soils is generally in a range of 10^{-10} m/s –10^{-7} m/s (Morgenstern and Smith 1973; Nixon and Morgenstern 1974; Konrad 1989). Therefore, a range of hydraulic conductivities from10^{-10} m/s to 10^{-8} m/s is used.

The soil is modeled as a linear elastic material. One-dimensional volumetric compression occurs after thawing. The Young's modulus (E) is obtained from the volume compressibility (m_v) using Eq. (1) for FE input.

$$E = \frac{(1+\nu)(1-2\nu)}{m_v(1-\nu)} \quad [1]$$

where ν is the Poisson's ratio. In this study, $\nu = 0.25$ is used for both frozen and thawed soil. A very small volume compressibility is used for frozen soil (m_{vf} = 6.94x10^{-13} m^2/kN (E = 1200 MPa)), which yields a negligible settlement of frozen soil. On the other hand, the volume compressibility of the thawed soil (m_{vt}) is 3–4 order higher than that of frozen soil (E = 0.1 MPa–1.0 MPa), which implies that settlement mainly occurs in thawed soil (Table 1 and 2).

A total of six cases are simulated for a varying hydraulic conductivity and volume compressibility of the thawed soil, and the temperature boundary conditions. In S1–S3, only the top surface temperature is varied to create different thaw front penetration rates. On the other hand, the hydraulic conductivity and volume compressibility of thawed soils are varied in S4–S6. In all cases, simulation is continued for 58 hours. FE analyses are performed without any surcharge (i.e., only self-weight) and with a surcharge p_0 = 15 kPa.

3.3 Governing heat transfer equations

The heat transfer by conduction is more significant than convective heat transfer during freezing and thawing of fine-grained soils (Nixon 1975). Therefore, the convective part is neglected in this study. The energy balance in conduction mode can be written as:

$$\rho c \frac{\partial T}{\partial t} - L\rho_i \frac{\partial \theta_i}{\partial T} - \nabla(\lambda \nabla T) = 0 \quad [2]$$

where c is the heat capacity of soil by mass, ρ is the density of soil, L is latent heat of fusion of water per unit mass, ρ_i is the density of ice, λ is thermal conductivity and T is the temperature.

Table 1 shows the thermal properties of the multiphase system (soil skeleton and pore fluid). The phase change of pore water in soil generally occurs over a range of temperature rather than at a specific temperature of 0°C. This range depends on several including salt content, applied load and grain sizes (Lunardini 1988; McKenzie et al. 2007; Gholamzadehabolfazl 2015). In this study, a typical range of -3 °C to 0 °C is considered to model the latent heat.

Table 1. Parameters used in finite element modeling

Soil Skeleton	
Thermal conductivity, λ (W/m°C)	2.1
Specific heat, c (J/kg°C)	1381
Dry density, ρ_s (kg/m^3)	1317
Volume compressibility of frozen soil, m_{vf} (m^2/kN)	6.94x10^{-13}
Volume compressibility of thawed soil, m_{vt} (m^2/kN)	10^{-10}–10^{-9}
Hydraulic conductivity of frozen soil, k_f (m/s)	1x10^{-14}
Hydraulic conductivity of thawed soil, k_t (m/s)	10^{-10}–10^{-8}
Pore fluid	
Thermal conductivity, λ (W/m°C)	0.58
Specific heat, c (J/kg°C)	4186
Density, ρ (kg/m^3)	1000
Latent heat, L (J/kg)	3.34x10^5
Initial conditions	
Void ratio, e	2.6
Degree of saturation, S_r	1.0

Table 2. Soil properties and boundary conditions used in the parametric study

Parameter	S1	S2	S3	S4	S5	S6
T_g (°C)	-5	-5	-5	-5	-5	-5
T_s (°C)	1	5	20	20	20	20
k_t (m/s) x10^{-10}	1	1	1	10	10	100
m_{vt} (m^2/kN) x10^{-10}	8.33	8.33	8.33	8.33	83.33	83.3

4 RESULTS

The present FE model provides the thaw depth (i.e., the location of the 0 °C isotherm), excess pore water pressure and resulting settlements. Therefore, it is not required to determine the thaw depth using an idealized equation as used in the development of previous analytical solutions for thaw consolidation (Morgenstern and Nixon 1971). Thaw consolidation is simulated for both self-weight and a combination of self-weight and applied loads. The FE results are compared with the simplified Morgenstern and Nixon (1971) analytical solution, as described below.

4.1 Analytical solution

Morgenstern and Nixon (1971) developed the thaw consolidation model based on the thaw depth measured from the initial position of the top surface. However, the top surface settles considerably with the progress of consolidation, as presented later in this paper and also shown in other studies (Dumais and Konrad 2018). Therefore, for a better comparison, the FE calculated thaw depth measured from the moving top surface (X(t)) is used. Note that Morgenstern and Nixon (1971) used the Carslaw and Jaegar (1947) solution to calculate the thaw depth as a function of $\sqrt{t}$, which is also valid for FE calculated X(t) as

$$X(t) = \alpha\sqrt{t} \quad [3]$$

where α is a constant determined from heat transfer solution.

Morgenstern and Nixon (1971) also showed that the excess pore water pressure $u(x,t)$ in the thawed soil at a location x at time t is

$$u(x,t) = \frac{p_0}{\mathrm{erf}(R) + \frac{e^{-R^2}}{\sqrt{\pi}R}} \times \mathrm{erf}\left(\frac{x}{2\sqrt{c_{vt}t}}\right) \times \mathrm{erf}\left(\frac{x}{2\sqrt{c_{vt}t}}\right) + \frac{\gamma' x}{1 + \frac{1}{2R^2}} \quad [4]$$

where c_{vt} is the coefficient of consolidation of the thawed soil, p_0 is the surcharge (Fig. 1), γ' is the submerged unit weight of the thawed soil, and R is the thaw consolidation ratio, which is given by

$$R = \frac{\alpha}{2\sqrt{c_{vt}}} \quad [5]$$

The coefficient of consolidation is related to volume compressibility and hydraulic conductivity as

$$c_{vt} = \frac{k_t}{m_{vt}\gamma_w} \quad [6]$$

where γ_w is the unit weight of water.

4.2 Simulations with a surcharge (p_0 = 15 kPa)

In this section, the FE simulations of the six cases listed in Table 2 with a surcharge p_0 = 15 kPa is presented. Figure 2 shows the FE calculated settlement (S(t)) of the soil layer with time. The surcharge causes additional settlement, as compared to only the self-weight cases, as discussed later in Section 4.3.

Figure 2 shows that the maximum settlement during the simulation period is the highest in S6 and the lowest in S1, which is due to the higher surface temperature, hydraulic conductivity and volume compressibility in Case S6 than those compared to Case S1. A higher surface temperature results in a larger thawed zone, while the higher hydraulic conductivity and volume compressibility causes more settlement due to the dissipation of pore water pressure. Note that the excess pore water pressure is not completely dissipated during the simulation period shown in Fig. 2. The post-thaw dissipation is discussed later in Section 4.4.

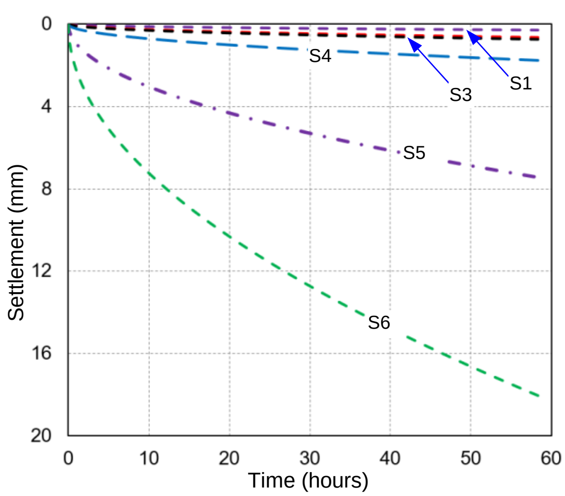


Figure 2. Surface settlement in the simulations with a surcharge p_0 = 15 kPa

The settlement of the soil sample, as compared to the initial height, is not negligible for all the cases. For example, the soil sample settle ~18 mm (8% of the initial height) in case S6 after 58 hours. Therefore, the small-strain assumption, as used by Morgenstern and Nixon (1971), may not be always valid.

Figure 3 shows the pore pressure variation with depth for three time intervals (8h, 16 h and 58 h) for Case S6. In this figure, the vertical axis represents the depth measured from the top surface of the soil sample before settlement (i.e., initial state). As the settlement of the soil sample is considered in the present FE analyses, there is a discrepancy between the analytical solution of Morgenstern and Nixon (1971) and FE analysis in terms of thaw depth. Therefore, to compare the results obtained from FE and analytical solutions, the thaw depth measured from the top of the settled soil sample is used.

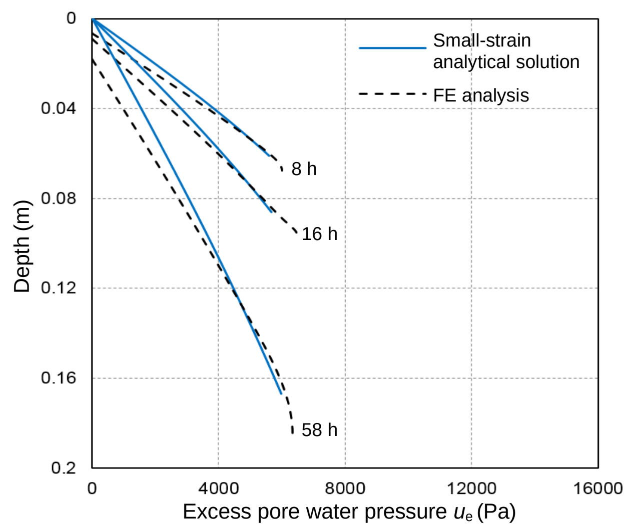


Figure 3. Comparison of excess pore pressure in Case S6 obtained from FE analysis and a small-strain analytical model

Morgenstern and Nixon (1971) presented the thaw consolidation results using the dimensionless thaw consolidation ratio (R) (Eq. 5). The parameters R incorporate the effects of several factors including the coefficient of consolidation (c_{vt}) and heat transfer (α). Plotting the frost front penetration depth in FE analyses with time and fitting the results as Eq. (3), the value of α is obtained for each case (S1–S6 in Table 2). Table 3 shows the calculated values of α. Now, using the values of α and c_{vt}, the thaw consolidation ratio (R) is calculated, as shown in Table 3. Note that the higher the value of R the lower the pore pressure dissipation (i.e., higher excess pore pressure accumulation during thawing).

Table 3. Calculated parameters for analytical solution

Calculated parameters	S1	S2	S3	S4	S5	S6
α (m/s$^{1/2}$) x10^{-5}	5.07	16.4	36.6	36.6	36.3	36.1
c_{vt} (m^2/s) x10^{-8}	1.22	1.22	1.22	12.2	1.22	12.2
R	0.23	0.74	1.66	0.52	1.64	0.52

Figure 4 shows the FE simulated excess pore water pressures(u_e) (dashed lines) with time up to the thaw depth at that time for the six simulation cases. The calculated excess pore pressure using the Morgenstern and Nixon (1971) analytical solution (solid line) is also plotted in this figure. Note that, x = 0 represents the current position of the top surface of the soil sample at a given time for both FE analysis and analytical solution.

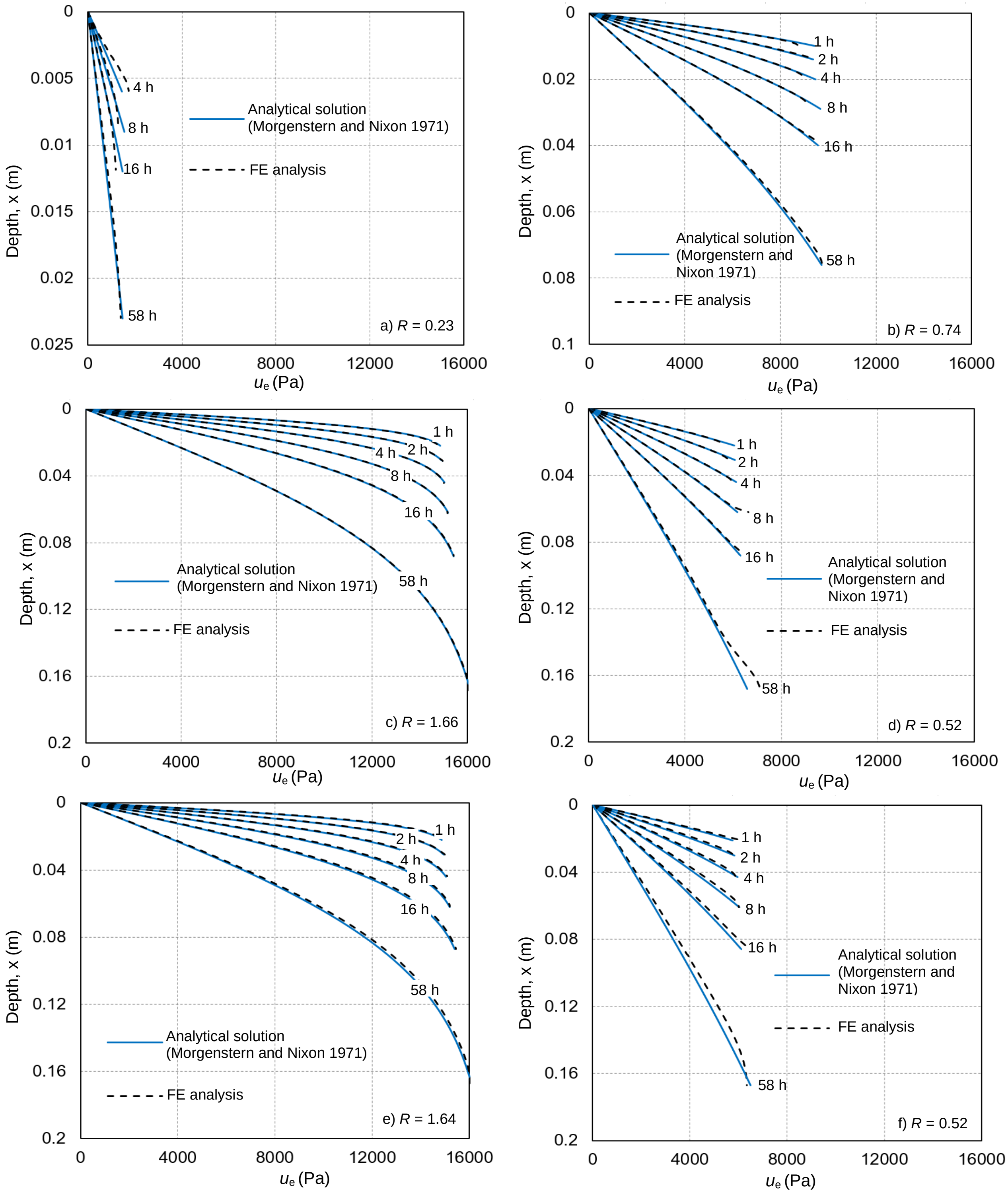


Figure 4. Excess pore water pressure in the soil sample under a surcharge

The FE calculated excess pore water pressure closely matches with the analytical solution, which implies that the FE approach presented in this study can be used successfully for modeling thaw consolidation. Some discrepancies in Case S1 at the early stage of consolidation ($\leq$ 4 h) is primarily due to very small thaw front penetration depth (only few elements) under a low surface temperature (T_s = 1 °C).

4.3 Simulations for only self-weight (p_0 = 0)

In this set of analysis, no surcharge is applied to the top surface; therefore, the consolidation occurs only due to the self-weight of the soil. The other parameters used in these analyses are the same as before (Tables 1 and 2). Figure 5 shows the FE simulated excess pore water pressure (u_e) (dashed lines) for the six simulation cases listed in Table 2. Note that, the surface settlement in this case during thawing is less than that of with a surcharge as discussed in Section 4.2.

Following the same approach described in Section 4.2, the thaw consolidation ratio (R) is calculated from FE results for this case, as shown in Figs. 5(a)–5(f). The excess pore water pressure is then calculated using the Morgenstern and Nixon (1971) analytical solution for these values of R.

Figures 5(a)–5(f) show that the FE calculated u_e increases almost linearly with depth and reaches the maximum at the thaw front. Below this depth where the soil is still frozen, FE analysis calculates some pore water pressure which gradually decreases to zero. This is attributed to the pore fluid diffusion into the frozen soil from the thaw front, although it has a low permeability as compared to the thawed soil (Table 1 and 2). Note that, an impermeable boundary is considered in the analytical solution (Morgenstern and Nixon 1971).

For a case without surcharge (p_0 = 0), the analytical solution for the excess pore water pressure (Eq. (4)) becomes a time-independent linear function of depth (x), where the slope of the linear line depends on R.

$$u(x,t) = \frac{\gamma' x}{1 + \frac{1}{2R^2}} \qquad [7]$$

The solid lines in Fig. 5 shows the excess pore water pressure in the thawed soil during thawing, which match with the FE calculated results. Moreover, the higher the value of R the larger the excess pore water pressure. For example, the maximum u_e is 1400 Pa for R = 1.65 (Fig. 5(c) while it is 550 Pa for R = 0.52 (Fig. 5(d)). Therefore, it can be concluded that the FE modeling approach presented in this study can simulate the process of thaw consolidation.

4.4 Post-thaw consolidation under self-weight

Figure 5 shows that the generated excess pore water pressure does not dissipate completely during thawing. Therefore, consolidation settlement will be continued even if the thaw front does not move further. This process is known as post-thaw consolidation.

FE analysis is performed for the case S2 to calculate the post-thaw consolidation. In this simulation, the heat transfer analysis is stopped after 58 hours, when the thaw front moves ~75 mm; however, the analysis for pore pressure dissipation is continued. Figure 6 shows the pore pressure in the thawed zone with time factor $T = c_{vt}.t/H^2$, where c_{vt} = 1.22×10^{-8} m^2/s and H is the drainage length which is equal to the thaw depth in this case (= 0.075 m). The pore pressure dissipates with time, and the pore pressure isochrones are very similar to the self-weight consolidation of a soil layer having an impermeable bottom boundary.

5 CONCLUSIONS

Thaw consolidation settlement is one of the major concerns in the cold climates. The effectiveness of an FE modeling approach for thaw consolidation is presented in this study. The FE simulations are performed for a soil sample in one-dimensional condition. Coupled heat transfer, pore water pressure generation and its dissipation are successfully simulated. An idealized soil behaviour—linear void ratio-effective stress relation, constant hydraulic conductivity and heat transfer only by conduction—is considered in order to compare the FE result with the analytical solution developed by Morgenstern and Nixon (1971).

The analyses are performed with and without a surcharge at the top surface of the soil sample in order to simulate effects of self-weight and external loads. It is shown that the present FE approach can successfully simulate the thaw consolidation process.

In terms of practical implication, the FE approach can be modified and used for thaw consolidation with complex boundary conditions, such as moving thaw front boundary or thawing around a buried pipeline.

One of the limitations of this study is that a linear void ratio–effective stress relationship and constant permeability is used for the thawed soil. It is understood that e–σ′ relationship is highly nonlinear, especially at a low-stress level, and hydraulic conductivity decreases with void ratio change during consolidation.

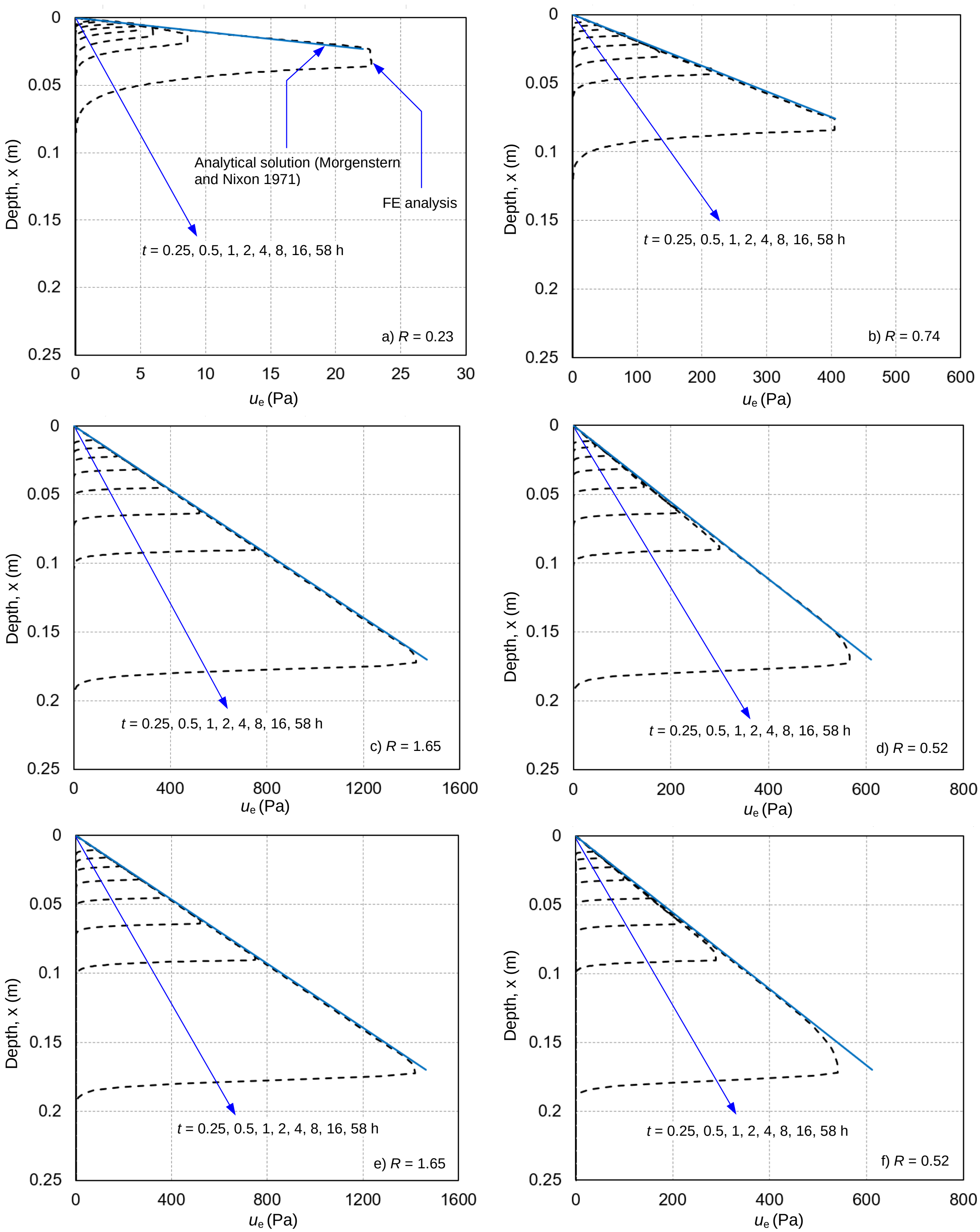


Figure 5. Excess pore water pressure under self-weight

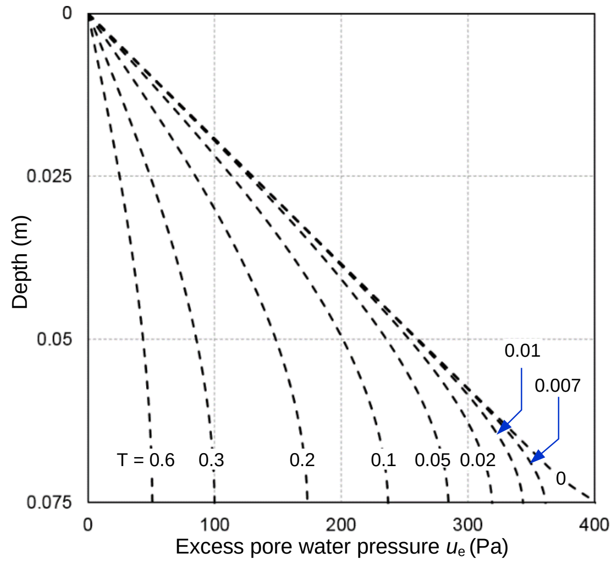


Figure 6. Post-thaw excess pore water pressure dissipation in S2

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