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Application of Random Set Finite Difference Method into a deep urban excavation

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ABSTRACT

Engineers have been attempting to design geotechnical structures in such a way to achieve the best performance with the minimum budget. To satisfy this demand, reliability methods have been proposed to increase the performance and serviceability; but, decrease the probability of failure in geotechnical structures by considering parameter uncertainties in design. Among various reliability methods, in the present study, Random Set theory in combination with finite difference method (RS-FDM) was used to investigate its applicability and efficiency in geotechnical practice. The 27-meter-deep urban excavation located in Tehran, Iran was chosen as a case study. The excavation was modeled numerically using FLAC3D software. The crown horizontal displacement and factor of safety were considered as desired performance functions. To find the most influential parameters on the desired performance functions, a sensitivity analysis was conducted for 19 different input parameters. The RS-FDM results fall well within the range of site measurements. The probability of failure was estimated to be 0.00002, which indicates that the excavation is safe against failure.

RÉSUMÉ

Les ingénieurs ont essayé de concevoir des structures géotechniques de manière à obtenir les meilleures performances avec un budget minimum. Pour satisfaire cette demande, des méthodes de fiabilité ont été proposées pour augmenter les performances et la facilité de maintenance; mais, diminuez la probabilité de défaillance des structures géotechniques en tenant compte des incertitudes de paramètres dans la conception. Parmi les différentes méthodes de fiabilité, la présente étude utilise la théorie des ensembles aléatoires en combinaison avec la méthode des différences finies (RS-FDM) pour étudier son applicabilité et son efficacité dans la pratique géotechnique. Les fouilles urbaines de 27 mètres de profondeur situées à Téhéran, en Iran, ont été choisies comme étude de cas. L'excavation a été modélisée numériquement à l'aide du logiciel FLAC3D. Le déplacement horizontal de la couronne et le facteur de sécurité ont été considérés comme des fonctions de performance souhaitées. Pour rechercher les paramètres les plus influents sur les fonctions de performance souhaitées, une analyse de sensibilité a été réalisée pour 19 paramètres d'entrée différents. Les résultats RS-FDM se situent bien dans la plage des mesures sur site. La probabilité de défaillance a été estimée à 0,00002, ce qui indique que l'excavation est sans danger.

1 INTRODUCTION

In recent years, the number of deep excavation has risen sharply mainly due to urbanization and lack of space for the construction of buildings. As excavation failure leads to fatal and monetary consequences, stability analysis of deep excavation should be done with utmost caution. A wide array of stability methods is available in the literature, which can be categorizes into two groups; deterministic and non-deterministic methods. Soils are typically heterogeneous in nature. In this case, the non-deterministic method is preferable over deterministic method for risk and reliability analysis considering uncertainties in soil parameters.

The non-deterministic approaches have been used in various studies to analyze reliability of geotechnical structures; for example, random field theory (Luo et al. 2011, fuzzy method, first order second moment, imprecise or interval probabilities (Walley and Fine 1982), possibility theory (Dubois and Prade 1987), Dempster-Shafer or evidence theory (Shafer 1992), Fuzzy set theory (Zadeh 1965), random set theory (Kendall 1974) and convex model (Ben-Haim 1995). Among them, Random Set (RS) method has gained more popularity due to its simplicity and less computational efforts.

Random Set (RS) theory uses the range of data as input parameters in case where no exact value for input parameters are available (likely distribution curve of the parameters cannot be obtained) followed by the calculation of probability of failure and other desired performance functions in the form of intervals. Matheron (1975) was the first who systematically developed RS method. Less computational efforts and simplicity in calculating the probability of the occurrence of performance functions make RS theory attractive. Details of RS method including mathematical and statistical concepts are available in Momeni et al. (2018). The RS method was used by several researchers including Robbins (1945), Dempster (1967), Kendall (1974), and Matheron (1975). The main advantages of RS are that i) it can be coupled with numerical methods and ii) reliably results can be produced with even limited soil data available. Several researchers used RS Finite Element Method (RS-FEM) to perform reliability analysis of geotechnical structures (Tonon et al. 2000, Griffiths and Fenton 2004, Peschl 2004, Schweiger et al. 2007, Nasekhian and Scheweiger 2011, Poormousavian and Fakher 2017, Momeni et al. 2018). Shen (2013) proposed Random Set Finite Difference Method (RS-FDM) using FLAC 2D to perform rock slope stability. Sekhavatian and Choobbasti (2019) developed

RS-FDM in reliability analysis of excavation using FLAC 2D.

Scheweiger and Peschl (2005) were the first who implemented RS on the hypothetical excavation. Ghazian and Fakher (2016), Arabani and Fakher (2016) and Poormousavian and Fakher (2017) used RS-FEM to calculate the probability of failure in deep excavations. Sekhavatian and Choobbasti (2019) analyzed the stability of excavation using RS-FDM. Table 1 summaries recent reliability analyses carried extending RS for different excavations. These previous studies have been performed by assuming a plane strain condition. However, no attempt was made to analyze the reliability of geotechnical structure using RS-FDM in 3-dimensional condition.

Table 1. Excavation projects analyzed by RS

Project Name	Excavation Depth (m)	Method	Reference
Soheil	21	RS-FDM	Sekhavatian and Choobbasti (2019)
Saba	27	RS-FEM	Poormoosavian and Fakher (2017)
Niavaran	20	RS-FEM	Poormoosavian and Fakher (2017)
Iran Zamin	40	RS-FEM	Ghazian and Fakher (2016)
Haghani	23	RS-FEM	Arabani and Fakher (2016)

The main purpose of this paper is to investigate the applicability of RS-FDM in 3D space. A 27-meter-deep urban excavation in Iran was chosen as a case study to check its feasibility.

2 RANDOM SET FINITE DIFFERENCE METHOD

The procedures for adopting RS-FDM in the analysis can be summarized in 7 steps as follow:

[Step 1]: The first step is defining the geometry of the problem. The finite difference model is prepared during this step and the deterministic soil and structural elements properties are determined (More details of numerical modelling of the problem are available in the following sections).

[Step 2]: The second step is to collect model parameters that have more significant influence on the displacement and stability of the problem. These parameters are considered as basic variables for RS-FDM. In this step, different ranges of information obtained from site investigation and similar projects are imported. Weights are also assigned to each range to show the confident level. For example, in case where two ranges of information are available and each of them are equally reliable, the probability of 0.5 is given to each range. All sources of uncertainties are considered in this step.

[Step 3]: The third step is to address the spatial variability of the soil into RS. A variance reduction technique proposed by Vanmarcke (1983) can be used to consider spatial variability of the soil into RS. In this regard, the correlation length needs to be defined. Correlation length expresses the distance between two points in the soil domain in which a specific variable value is repeated between those two points.

[Step 4]: The fourth step is to find the most influential parameters on the desired performance functions such as factor of safety and displacement. For this purpose, a sensitivity analysis is performed. The number of analysis required to perform the sensitivity analysis is 4N+1 (where N is number of input parameters).

[Step 5]: In this step, 3 parameters are taken based on the sensitivity analysis and then 64 combinations are generated using those 3 parameters. Subsequently, each combination is formulated into the finite difference model and the factor of safety and desired displacement is computed. The probability or weight for each output is also assigned. Since three parameters were found to have the most influential on the performance functions. In the present case study, the probability of happening of each output is 0.125 (i.e. 0.5 × 0.5 × 0.5).

[Step 6]: The sixth step is to sort the results and define the boundaries. After that, the distributions are fitted on them.

[Step 7]: The last and the most critical step of RS-FDM procedure is calculating the probability of failure and probability of excessive displacement. These probabilities correspond to a specific value (i.e. targeted value) on the fitted distribution. The probability of failure is determined as the probability of the factor of safety less than 1 on the cumulative density function graph.

3 PROJECT OVERVIEW

A deep urban excavation is chosen to check the applicability of RS-FDM in geotechnical practice. The case study chosen for this paper is a 27-meter-deep urban excavation located in the vicinity of Chitgar lake, Tehran, Iran. The southern and eastern sides of the excavation are near Chitgar park. The western side is adjacent to several residential buildings. The northern side of the project is adjacent to a bus station and a street. Figure 1 shows a plan view of the deep excavation project.

According to the site investigation, the soil consists of three different layers. The first 2 m is poorly graded gravel (G) underlain by sandy layer (S1) (from 2 to 10 meter) and denser sandy layer (S2) (from 10 to 30). The observed ground water table was far below the ground surface. The excavation walls were stabilized by anchors. The northern wall was stabilized with 42 anchors in 6 rows with the average horizontal and vertical spacing of 4 m and 3 m, respectively. Figure 2 shows the longitudinal profile of the excavated northern wall. All the anchors were installed at 10° angles, which constituted of six strands. Each anchor is fixed by grout in partial lengths (6 m in this case), while the other part is free stressing zone. Figure 3 depicts the geometric configuration of the northern wall. The bar-grout contribution develops the bonding effect to the soil within the fixed zone whereas tension resistance is triggered within the free part by applying the tension to the anchor. The pre-applied tension forces to the anchors are 236.4 kN/m. The detailed information on the mechanical properties of anchors is shown in Table 2.

Figure 1. Plan view of the excavation [Momeni et al. 2018]

Figure 2. Longitudinal profile of the excavated northern wall [Poormousavian and Fakher 2017]

Table 2. Anchor properties

Properties	6 strands anchor @2.75 m	6 strands anchor @4 m
Behavior	Elastic	Elastic
Cross sectional area (mm^2)	840	840
Grout cohesion (kN/m)	100	100
Grout stiffness (N/m/m)	4x10^6	4x10^6
Elastic modulus (GPa)	200	200
Tensile capacity (kN/m)	341.5	234.6

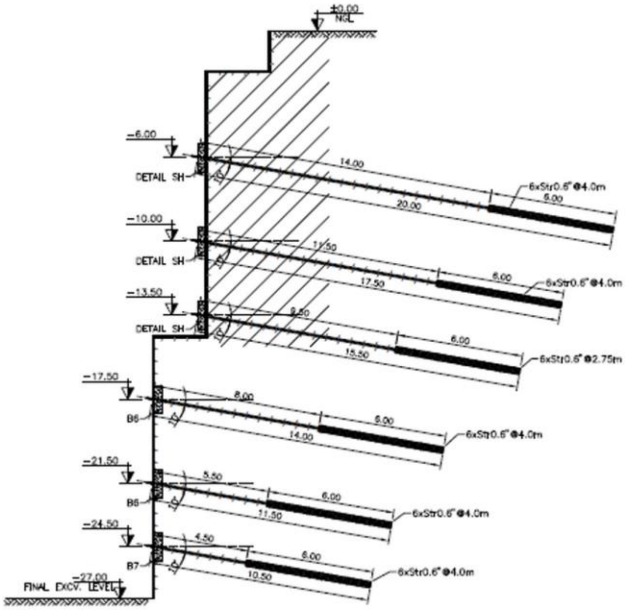


Figure 3. Geometric configuration of the northern wall

4 APPLICATION OF RS-FDM TO DEEP EXCAVATION

4.1 Soil Constitutive Model

Modified Duncan-Chang hyperbolic model was selected as the soil constitutive model. The effects of increasing stress level with the depth are considered by this nonlinear model. The hysteresis reaction of the soil can be simulated by this model as well. Hysteresis is dependent on the state history of soil. Tangent elastic modulus, E_t in Duncan-Chang hyperbolic model is expressed as Eq. [1].

$$E_t = 1 - \left[\frac{R_f (1-\sin\phi)(\sigma_1 - \sigma_3)}{2c\cos\phi + \sigma_3 \sin\phi} \right]^2 K p_a \left(\frac{\sigma_3}{p_a} \right) \qquad [1]$$

where σ_1 and σ_3 are principal stresses, p_a is atmospheric pressure, K is the initial tangent modulus factor, ϕ is friction angle of the soil, c is cohesion of the soil and R_f is the failure ratio.

The loading-unloading modulus, E_{ur} can be calculated using Eq. [2].

$$E_{ur} = K_{ur} p_a \left(\frac{\sigma_3}{p_a} \right)^n \qquad [2]$$

where K_{ur} is loading-unloading modulus factor and n is the stress influence exponent for the Young's modulus.

4.2 Modelling

FLAC 3D software was chosen as a finite difference program for modelling the excavation. To minimize the boundary effects on the horizontal displacements, an area of 95 m ×50 m × 30 m was defined as the geometry of the model. The depth of the excavation was set to 27 meters. The boundary conditions of the bottom of the model was fixed in all directions while the southern, northern, western and eastern sides of the model were allowed to move in all directions except normal to their surfaces (Roller fixities). The number of zones on the front side of the excavation was more extensive than the other zones. The brick command was used to generate the grids. The size of the grids in x and y-direction were 3 m and 1 m, respectively. In y-direction, it started with 1 m and then increased at the ratio of 1.1 (Figure 4(a)). The soil layers were defined in accordance with the soil profile. The water level was far below the ground surface; therefore, the effect of water was neglected. Hardening Soil (HS) model was selected as the constitutive model for the soil. The drained condition for soil was considered. As mentioned in Section 3, the soil was stabilized with anchors (See Table 2 for anchor properties) (Figure 4(b)).

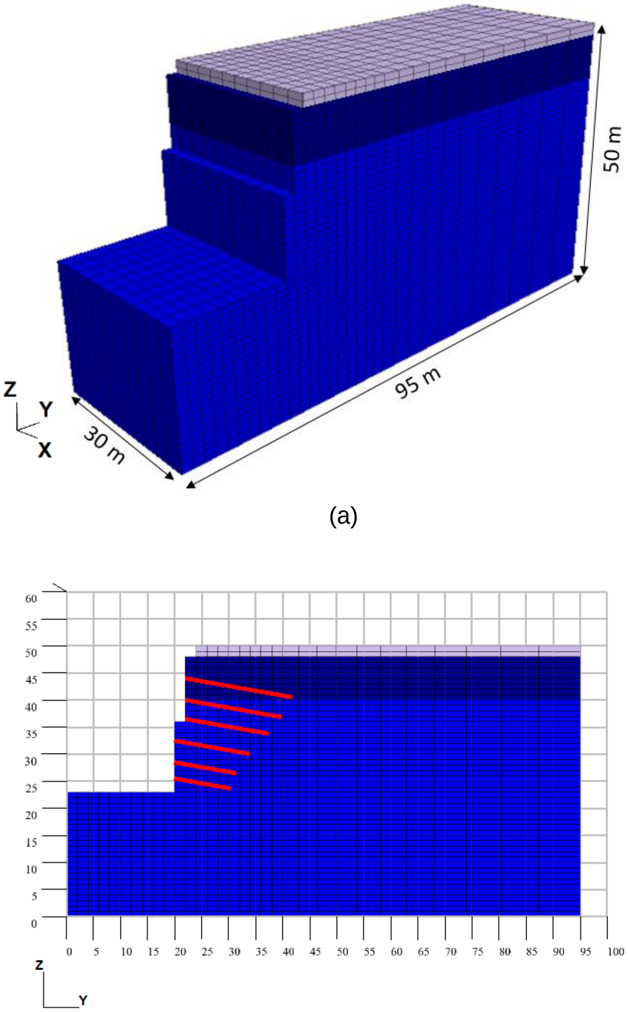
Staged construction method was employed to model the excavation. In-situ initial stress conditions were generated as the first step followed by the excavation of 2 m depth of the soil. The pretension force was then applied to the first row of anchors. The anchoring process continued with next 2 m of excavation and implementation of the second row of anchors. The same procedures were repeated until the entire excavate was completed (i.e. 27 m).

4.3 Step-by-Step Procedure

The first step in RS-FDM is the setup of the analyzed geometry. The 3D geometry of the analyzed model was prepared according to the previous sections.

The second step is determining the random and basic variables (or input parameters) that have influence on the performance functions. These parameters are chosen based on the engineering judgement. By considering these variables into analysis, variabilities of soil parameters are modelled. 19 parameters are considered as basic variables in this particular case, including soil cohesion (c), soil internal friction angle (ϕ), soil stiffness modulus (E), soil unit weight (γ), soil dilation angle (Ψ), stiffness exponent for minor stress formulation (m) for each layer and anchorages angle (α) (See Table 3 for more details).

The third step is determining different sources of information on each basic variable. There were two sources available; one from site investigation and laboratory tests, and another one from geotechnical reports on similar projects built near the excavation. The weight given to each source is equally 0.5. After determining basic variables and their related ranges, the variance reduction technique is utilized to perceive spatial variability of the soil. Spatial correlation length, θ and characteristic length (potential failure surface) values are required to perform this step. According to the Peschl (2004), spatial correlation length varies between 5 to 30. In this case study, correlation length is set to 15 because the variability of soil parameters obtained from two sources were normal.



(a)

(b)

Figure 4. (a) 3D model and (b) Y-Z profile in FLAC 3D

The next step is conducting a sensitivity analysis that is one of the crucial parts of the RS-FDM. Although, in the second step, 19 parameters were chosen as the effective variables, performing analysis and making combinations with these 19 parameters are time consuming. Therefore, sensitivity analysis is necessary to decrease the number of analysis. Eighty-one deterministic analyses are performed. In each analysis, the impact of each parameter on the performance functions including the factor of safety and horizontal displacement are computed. Relative sensitivity is defined as the sensitivity of each performance function to each parameter and the total sensitivity is the sensitivity of each parameter to both performance functions at the same time. To find the most influential parameters via sensitivity analysis, there is a need to define a threshold on the total sensitivity. A threshold value of 10% is selected for the case of interest.

5 ANALYSIS RESULTS

5.1 Sensitivity Analysis

Sensitivity analysis results are summarized in Table 3. The results are ranked based on their influence. For example, in case of the cohesion of the first layer (Cohesion G), there are two sources of information that can be used to define its ranges. The first range is between 6 and 11 kN/m^2 (Set no. 1) and the second range is from 12 to 22 kN/m^2 (Set no. 2). The probability assigned to each range is equally 0.5. To calculate the sensitivity of the performance functions to Cohesion G, firstly, the cohesion of the first layer changes to 6 kN/m^2 and all other parameters of the model are remained unchanged (equal to their deterministic values). Then, the performance functions are calculating. This process is repeated for cohesion 11, 12 and 22 kN/m^2 and their corresponding performance functions are computed. With four values of performance functions' values in hands, the total sensitivity of the Cohesion G can be computed. Similar approach is performed to get the total sensitivity for all other variables. A parameter with the rank 1 out of 19 is considered as the most influential parameter while a parameter with the rank 19 has the lowest impact on the performance functions. By applying the previously defined threshold (i.e. 10%), it turned out that cohesion, friction angle and the elastic modulus of the lower sandy layer are RS basic variables. With identifying 3 important variables, 64 different combinations can be made. Two ranges are identified with the maximum and the minimum value for each parameter. In other words, 4 values in total are determined for each variable, which results in 64 combinations (i.e. 4 × 4 × 4). Each combination leads to different displacement and a factor of safety. The weight corresponding to each of them is 0.125 (i.e. 0.5 × 0.5 × 0.5). The calculated values are then sorted from the lowest to the highest.

5.2 RS Boundaries

RS boundaries are drawn using p-box representation. The performance function results with their corresponding weights (0.125) are cumulatively placed from the lowest value to the highest value. Figure 5 shows the RS boundaries for the horizontal displacement of the excavation. The y-axis is cumulative density function and the x-axis is horizontal displacement. The lower and the upper bounds show the ranges obtained from RS-FDM analysis. The red dash line shows the acceptable horizontal displacement (75 mm) which was reported in the geotechnical report of the project. The green dash line shows the horizontal site measurements at the excavation's crown, ranging from 20 mm to 40 mm. In other words, by the chance of more than 50%, the displacement will fall within the range of 38 mm to 60 mm. The acceptable displacement is far from the upper bound of the most likely values. This means that the probability of having excessive displacement is significantly low.

As can be seen, the lower bound of RS analysis falls within the site measurements. This indicates that the design of the excavation was conservative, which results in lower risk of failure but higher cost. The deterministic dash line shows that the average values of soil parameters were used in the design.

Similarly, the same procedure can be performed to get the most likely values for the factor of safety. To get the factor of safety, the Phi-C reduction method is done in FLAC3D software. Some factor of safety values that are most likely to happen are ranging from 1.2 to 1.5. It means that by the chance of more than 50 %, the factor of safety of the model will be between the aforementioned range. Also, a deterministic value of FOS, is 1.4, which falls well inside RS-FDM bounds.

5.3 Serviceability Limit Function

The main purpose of the RS analysis is finding the probability of failure and excessive displacement in geotechnical the structure. To find these values, there is a need to fit the distribution on the RS boundaries. Either Easy fit or Matlab software can be employed for this purpose. Fitted distributions on the RS bounds can be found in Figure 6. Probability of failure is defined as the probability of the occurrence of factor of safety less than one, as it can be seen in Eq. [3].

$$P_f = P(FS < 1) = \int_{FS<1} f_x(x)\,dx \qquad [3]$$

where $f_x(x)$ is the performance function of the safety factor.

The probability of excessive displacement is defined as the probability of displacement exceeding the allowance. In the present case study, the allowable horizontal displacement was 75 mm based on the distances of buildings and streets from the excavated wall. To obtain the probability of excessive displacement, lognormal distribution was accurately fitted on both bounds of RS using Easy fit software. The lower bound does not intersect the allowable displacement; therefore, probability of excessive displacement is 0 (i.e. optimistic probability). On the contrary, the upper RS bound intersects the threshold (75 mm), resulting in about 0.23 chance of exceeding the allowable displacement (i.e. pessimistic probability). According to Momeni et. al. (2017), the acceptable probability of excessive displacement is 0.1. Therefore, the serviceability limit function is not satisfactory for the worst-case scenario. Apart from that, by considering uncertainties, it can be concluded that there is always a chance of failure. Traditional stability approaches cannot simulate the effect of uncertainties into design.

Similar approach can be performed for calculating probability of failure by fitting statistical distribution on bounds of RS. The probability of failure obtained from fitting lognormal distribution on the lower bound of RS is 0.00002. According to US Army corps of engineer (1997), the expected performance level from the excavation is favorable. It means that the excavation is safe against failure and the risk of failure is significantly low. Typical acceptable probability of failure for geotechnical structures is 0.0001. The computed probability of failure is far below this threshold, which indicates high reliability in the structure.

Table 3. RS-FDM sensitivity analysis results

Variable	Set no.	Prob.	range	Total sensitivity ranking	Deterministic value
Cohesion G (kN/m^2)	1	0.5	6-11	12	5
	2	0.5	12-22		
Cohesion S1 (kN/m^2)	1	0.5	26-36	5	35
	2	0.5	30-41		
Cohesion S2 (kN/m^2)	1	0.5	48-71	2	65
	2	0.5	73-94		
Internal friction angle G (°)	1	0.5	24-32	13	27
	2	0.5	27-37		
Internal friction angle S1 (°)	1	0.5	26-33	6	34
	2	0.5	29-37		
Internal friction angle S2 (°)	1	0.5	31-41	3	38
	2	0.5	40-45		
Dilation angle G (°)	1	0.5	1-8	18	0
	2	0.5	4-11		
Dilation angle S1 (°)	1	0.5	1-8	17	4
	2	0.5	4-11		
Dilation angle S2 (°)	1	0.5	1-8	16	8
	2	0.5	4-11		
Secant modulus G (MPa)	1	0.5	10-22	11	15
	2	0.5	14-28		
Secant modulus S1 (MPa)	1	0.5	56-92	4	68
	2	0.5	79-110		
Secant modulus S2 (MPa)	1	0.5	105-135	1	130
	2	0.5	124-155		
Unit weight G (kN/m^3)	1	0.5	13.5-16.5	14	16.5
	2	0.5	15.5-18.5		
Unit weight S1 (kN/m^3)	1	0.5	18-21.6	7	19
	2	0.5	20-23.5		
Unit weight S2 (kN/m^3)	1	0.5	17-20.7	8	20.7
	2	0.5	19.4-23.3		
m G	1	0.5	0.4-0.6	19	0.5
	2	0.5	0.6-0.8		
m S1	1	0.5	0.4-0.7	10	0.78
	2	0.5	0.7-1		
m S2	1	0.5	0.3-0.6	9	0.5
	2	0.5	0.6-0.9		
α (°)	1	0.5	5-25	15	10
	2	0.5	15-35		

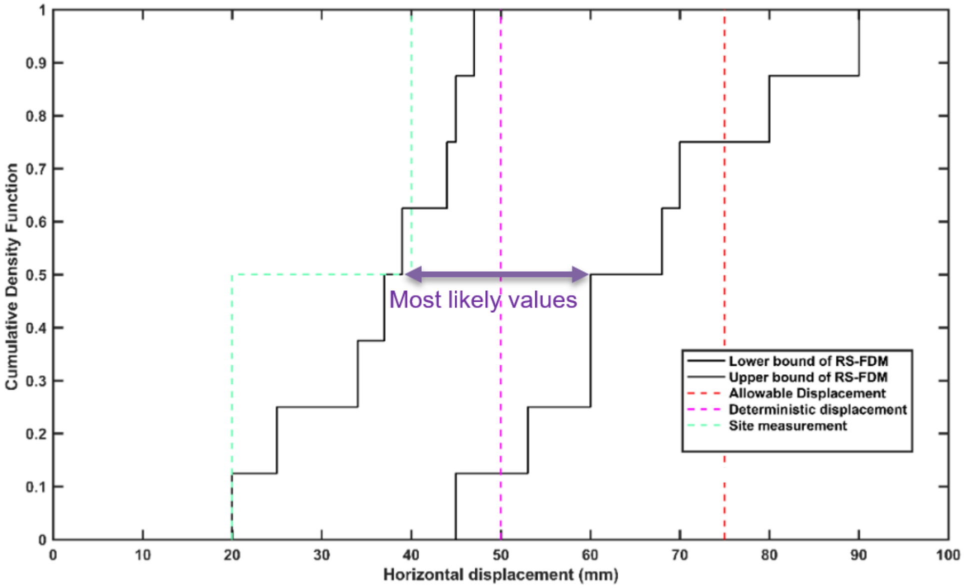


Figure 5. RS-FDM P-Box representation for horizontal displacement of the excavation

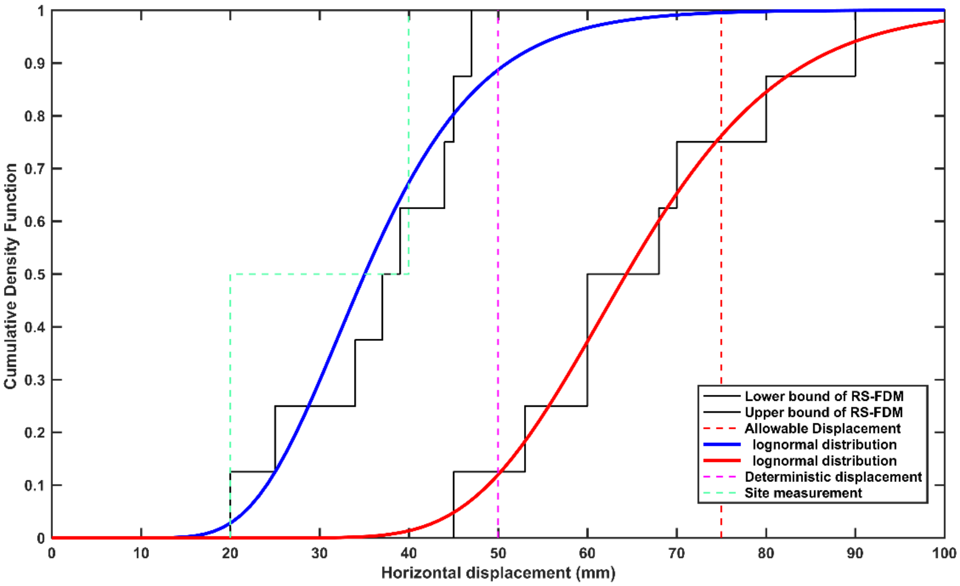


Figure 6. Fitted distribution on RS-FDM bounds

6 SUMMARY AND CONCLUSIONS

In recent years, many catastrophic landslides and excavation failures have been reported. One of the main reasons for these failures can be attributed to the deterministic methods conventionally used to determine factor of safety. In this paper, a non-deterministic approach, Random Set (RS) method is used in combination with Finite Difference (FD) method (i.e. RS-FDM) to conduct reliability analysis of a deep urban excavation in Iran. Conclusions obtained from the current study as follow:

(a) The main advantage of using RS-FDM is that maximum information concerning the risk of failure can be obtained with the minimum input data about the model.
(b) The sensitivity analysis shows that the soil cohesion, soil friction angle and soil elastic modulus of the bottom layer are the most influential parameters on the factor of safety and horizontal displacement.
(c) Site measurements are close to the lower bound of RS-FDM, which demonstrates that the design is conservative and safe against failure.
(d) The probability of failure calculated for the case of interest is 0.00002, which indicates he excavation is safe against failure. However, the probability of excessive displacement for the worst-case scenario is 0.23, which is higher than the acceptable threshold.

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