A CONSTITUTIVE MODEL FOR PARTIALLY SATURATED SOILS

S. Ghorbel, Université Laval, Ste-Foy, Québec, Canada
S. Leroueil, Université Laval, Ste-Foy, Québec, Canada

ABSTRACT

The paper presents an elasto-plastic model describing the behaviour at yielding of partially saturated soils. After a short introduction presenting the existing elasto-plastic models for unsaturated soils, a new model, based on Leroueil & Barbosa (2000) model is proposed. This model assumes that the soil remains saturated until the air entry value is exceeded and its behaviour is controlled by effective stresses. Beyond this particular point, the soil becomes unsaturated and the model then makes use of two independent stress variables: “modified net stress” $\sigma^* = \sigma - u_a + \chi^*(u_a - u_w)$ and matric suction. The model represents, in a consistent and unified manner, most of the fundamental features of the behaviour of partially saturated soils which had been considered separately in previously proposed models. An experimental programme was performed in suction-controlled apparatus on reconstituted clay from Sainte-Rosalie, Canada. The agreement between observed and computed results is satisfactory and confirms the possibilities for the model to reproduce the most important features of partially saturated soil behaviour.

RÉSUMÉ

L’article présente un modèle élasto-plastique pour décrire le comportement à l’état limite des sols partiellement saturés. Après une brève introduction des modèles élasto-plastiques existants pour les sols nos saturés, un nouveau modèle basé sur le modèle de Leroueil & Barbosa (2000) est présenté. Ce modèle suppose que pour une succion matricielle inférieure ou égale à la pression d’entrée d’air le sol reste saturé et son comportement est contrôlé par les contraintes effectives. À des succions matricielles supérieures à la pression d’entrée d’air le sol est partiellement saturé et le modèle utilise alors deux variables de contraintes: la contrainte nette modifiée $\sigma^* = \sigma - u_a + \chi^*(u_a - u_w)$ et la succion matricielle. Le modèle représente la plupart des caractéristiques fondamentales du comportement des sols partiellement saturés qui ont été considérées séparément dans les modèles existants. Un programme expérimental a été réalisé, en utilisant des appareils à succion contrôlée, sur une argile reconstituée de Sainte-Rosalie, Canada. Une concordance satisfaisante entre les résultats expérimentaux et ceux obtenus par le modèle confirme les possibilités de ce dernier de reproduire les caractéristiques les plus importantes du comportement des sols partiellement saturés.

1. INTRODUCTION

Given that experimental studies on unsaturated soils are generally costly, time-consuming and difficult to conduct, the majority of these studies are limited to only one aspect of behaviour such as deformability, swelling or shear strength. As a result, until recently, and in spite of the efforts made by researchers, there is no model which integrates the main characteristics describing the behaviour of unsaturated soils.

2. EXISTING MODELS

On the basis of literature review, the existing models can be divided into three categories. The distinction between them is discussed in the following sections.

2.1 Models for the prediction of shear strength

Several formulations have been proposed to study the shear strength of unsaturated soils. One of the most important is reported by Fredlund et al. (1995) who proposed to describe the shear strength of unsaturated soil at any suction as follows:

$$\tau_f = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\Theta^{\kappa}\tan\phi' \qquad [1]$$

where c' is the effective cohesion of the saturated soil, ϕ' is the effective angle of shearing resistance of the saturated soil, $(\sigma_n - u_a)$ is the net normal stress on the plane of failure at failure, $(u_a - u_w) = s$ is the matric suction of soil, κ is a fitting parameter and Θ is the normalized volumetric water content of the soil that is defined as follows:

$$\Theta = \frac{\theta}{\theta_s} \qquad [2]$$

In which θ and θ_s are the volumetric water contents at the considered suction and at saturation respectively.

The normalized volumetric water content relates the shear strength of unsaturated soil to the soil-water characteristic curve.

Extending the same concepts, another equation was proposed by Vanapalli et al. (1996) for predicting the shear strength without using the fitting parameter, κ. The equation is given below:

$$\tau_f = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)\tan\phi' \qquad [3]$$

where θ_r is the residual volumetric water content.

2.2 Barcelona-type models

These models present an elasto-plastic framework generally relating the compressibility of soils to the matric suction by the Loading-Collapse curve, called LC curve, and that will be described hereunder.

The concept of Loading-Collapse (LC) is depicted in Fig. 1 and can be explained as follows: if a saturated soil isotropically consolidated has an isotropic yield stress such as p^*_o, its limit state curve is Y_{sat} in Fig. 1b. If suction is increased, the isotropic yield stress increases along the LC curve and the entire limit state curve increases in size, as shown in Fig. 1b. It results in an enlargement of the elastic zone.

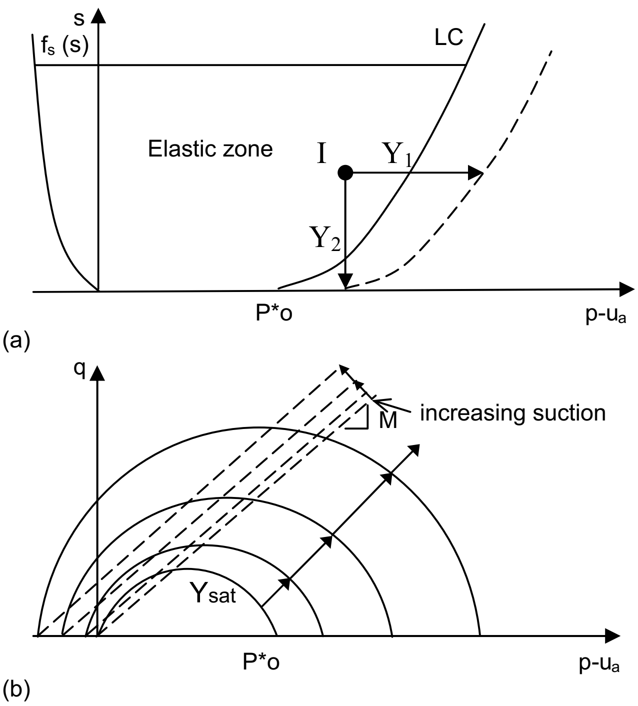

Figure 1. Limit state curves and elastic domains for unsaturated soils-BBM (after Alonso et al., 1990).

To understand the influence of the LC curve or the behaviour of unsaturated soil, for an element of soil with initial conditions at a point such as I, figure 1a can be considered. A decrease in suction under constant net total stress is associated with a collapse at Y2 (Fig. 1a); on the other hand, an increase in net total stress under constant suction first results in an elastic behaviour and then development of plastic volumetric strains after yielding at point Y1 (Fig. 1a). The volumetric strains generated in both cases cause a movement of the LC curves towards larger net total stresses.

Alonso et al. (1987, 1990) proposed a critical sate framework involving two independent set of stress variables, namely the net stress and the matric suction. The proposed model, called Barcelona Basic Model (BBM), is an extension of the modified Cam-clay model developed for saturated clays (Roscoe & Burland, 1968).

The BBM model presents a simple and powerful conceptual framework to describe and predict the behaviour of unsaturated soils, but does not consider the possible effects of anisotropy and microstructure. It also considers that the isotropic yield stress increases as soon as there is some matric suction (Fig. 1), and this is probably not true when suction does not exceed the air entry value and soil remains saturated.

2.3. Physical model of Leroueil & Barbosa

Leroueil and Barbosa (2000) assumed that yield curves for saturated soil can be schematised by four segments (see fig. 2): two corresponding to the strength envelopes in compression and in extension; and two corresponding to $\sigma'_a = \sigma^*_{ayo}$ and $\sigma'_r = \sigma^*_{ryo} = K_{AL}\,\sigma^*_{ayo}$. In these equations, σ^*_{ayo} is the axial yield stress, σ^*_{ryo} is the radial yield stress and K_{AL} is the anisotropy ratio. In a stress diagram, the line reflecting the anisotropy is called the anisotropy line (AL) and is characterised by $K_{AL} = \sigma^*_{ayo} / \sigma^*_{ayo}$. Leroueil and Barbosa (2000) consider that K_{AL} reflects the distribution of contacts between particles or aggregates in the soil.

For unsaturated soils, the model considers that matric suction generates a resistance to slippage at the contacts between particles or aggregates. Its global effect should thus reflect the distribution of these contacts, and the anisotropy line (AL) should be the same as for the saturated soil. Then , if a soil in saturated conditions has a yield curve such as $OB_oA_oD_o$ in Fig. 2a, a given suction should extend the cap $B_oA_oD_o$ to $B_sA_sD_s$, with an increase in axial net yield stress from σ^*_{ayo} to σ^*_{ays} and an increase in radial net yield stress from σ^*_{ryo} to σ^*_{rys}. The variation of the axial and radial yield stresses with suction defines two Loading-Collapse curves, LC_a and LC_r respectively (Fig. 2b). However, the anisotropy line, AL, remaining the same independently of suction, the ratio $\sigma^*_{rys} / \sigma^*_{ays}$ at any suction is constant and equal to K_{AL}.

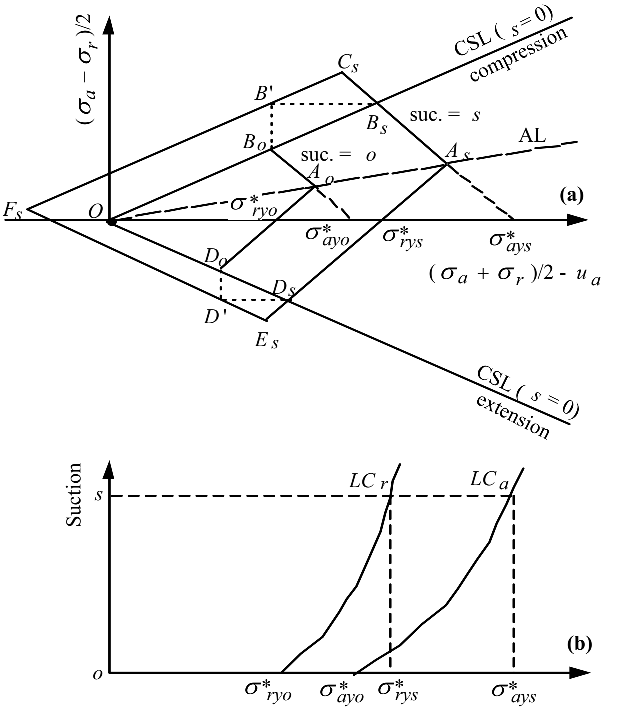

Figure 2. Description of Leroueil and Barbosa (2000) model.

By increasing the axial yield stress from σ^*_{ayo} to σ^*_{ays}, matric suction increases the shear strength of the soil in compression from point B_o to point B_s, or from B_o to B', in Fig. 2a. Leroueil and Barbosa (2000) demonstrated that this compression strength increase is equivalent to a cohesion in compression due to suction, $c_{c,suc}$, defined as follows:

$$c_{c,suc} = \left(\sigma^*_{ays} - \sigma^*_{ayo}\right)\frac{\sin\phi' \cos\phi'}{1+\sin\phi'} \qquad [4]$$

This equivalent cohesion depends only on suction and on the distribution and number of contacts.
In extension, the shear strength increase due to a given suction is associated with to an increase in radial yield stress from σ^*_{ryo} to σ^*_{rys} (Fig. 3a). This corresponds to an equivalent cohesion in extension, $c_{e,suc}$, equal to:

$$c_{e,suc} = \left(\sigma^*_{rys} - \sigma^*_{ryo}\right)\frac{\sin\phi' \cos\phi'}{1+\sin\phi'} \qquad [5]$$

This model is certainly idealised and approximate but well represents the behaviour of partially saturated anisotropic soils. It could also easily take into account the effects of structure and viscosity (Leroueil and Ghorbel, 2004).

The weaknesses and limitations of existing models have led the authors to attempt to develop a general elasto-plastic model which would take into account the most important features of the behaviour of partially saturated soils. The purpose of this paper is summarized in the following diagram:

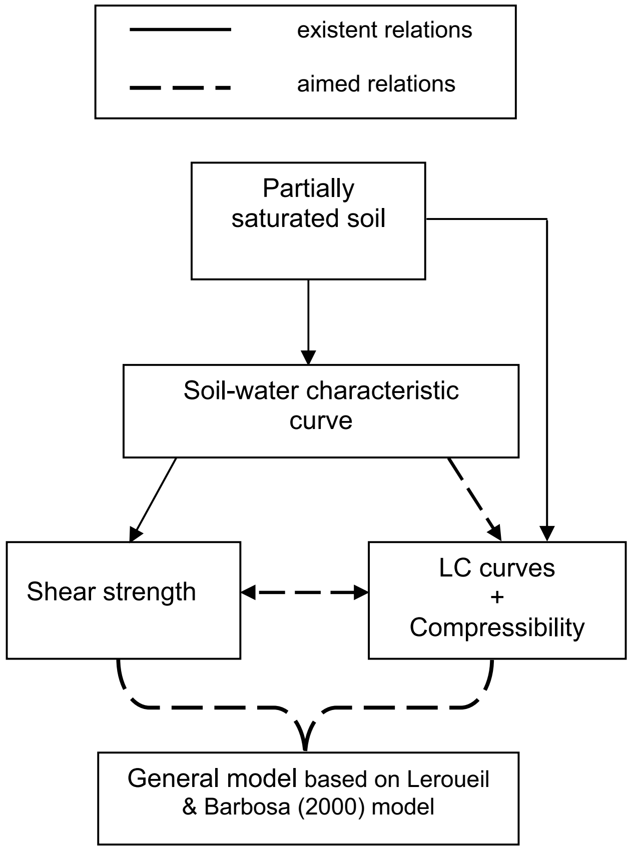

Figure 3. Purpose of this paper

The diagram shows that the main purpose of this paper is to relate the Loading-Collapse (LC) curve to the soil-water characteristic curve and then to the behaviour of soils in strength and volumetric strain within a generalized elasto-plastic model based on Leroueil and Barbosa (2000) model (see Fig. 2).

3. TESTED MATERIAL AND TESTING PROGRAMME

The tested soil is an agricultural clayey soil from Sainte-Rosalie, Québec, Canada, that has been passed on through a 1.18 mm sieve. The grain size distribution indicates 53% clay, 27% silt and 20% sand. The specimens were statically compacted at water content of 22% and at a void ratio of 0.76. Several of the specimens were used to obtain the soil-water characteristic curve using a pressure plate apparatus and a desiccator. The other soil specimens were subjected to 3 types of tests, in saturated conditions and at different matric suctions up to 400 kPa: constant rate of strain (CRS) oedometer tests; drained triaxial compression (CID) tests: an $K = (\sigma_r - u_a) / (\sigma_a - u_a) = cst$ triaxial compression tests.

4. EXPERIMENTAL RESULTS

4.1 Soil water characteristic curve

The soil-water characteristic curve obtained from the specimens compacted at an initial water content of 22% and void ratio of 0.76 is shown in Fig. 4 in terms of water ratio e_w defined as follows:

$$e_w = e\,S_r \qquad [6]$$

where e is the void ratio and S_r is the degree of saturation. It presents an air-entry value approximately equal to 70 kPa, which means that, until this value, the soil remains essentially saturated.

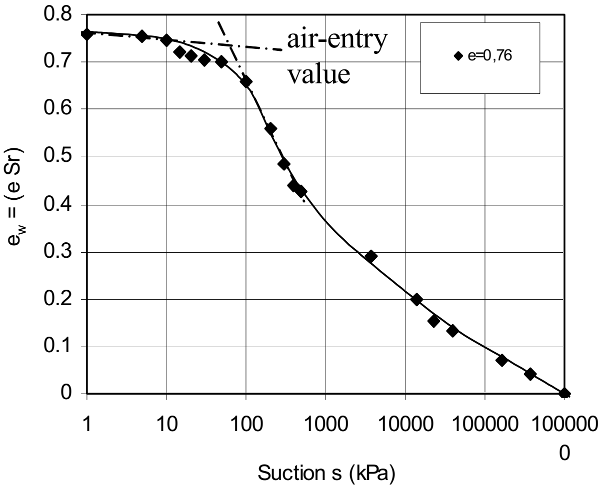

Figure 4. Soil water characteristic curve of Sainte-Rosalie Clayey soil.

4.2 Oedometer test results

The compression curves obtained at different matric suctions on the specimens of soils trimmed horizontally (as for conventional oedometer tests) and vertically are shown in Figs. 6 and 7 respectively. It can be seen that the initial void ratio is not affected by the increase in suction; only from 0 to 50 kPa a slight decrease of void ratio is produced. The yield stresses are well defined for all tests and generally increase with suction (except for zero suction value).

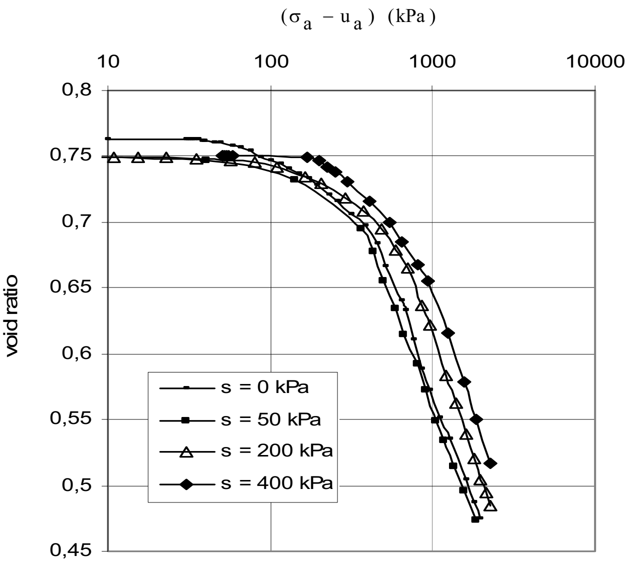

Figure 5. One-dimensional compression curves for specimens of Sainte-Rosalie soil trimmed horizontally.

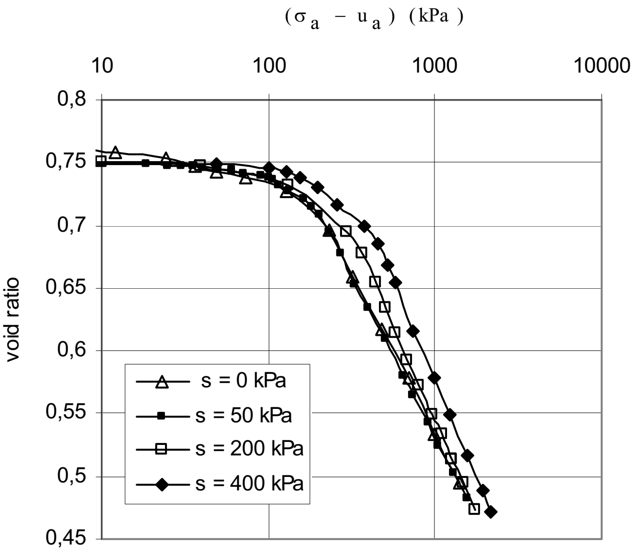

Figure 6. One-dimensional compression curves for specimens of Sainte-Rosalie soil trimmed vertically.

4.3 Triaxial test results

Figures 7 shows the shear strength envelopes obtained at different matric suctions from drained triaxial compression (CID) tests performed at cell pressures of 20 and 40 kPa. We can notice that, for both cell pressures, the maximum deviatoric stress increases with increasing suction. The envelopes obtained at different matric suctions are almost parallel and corresponding to a friction angle of 28°.

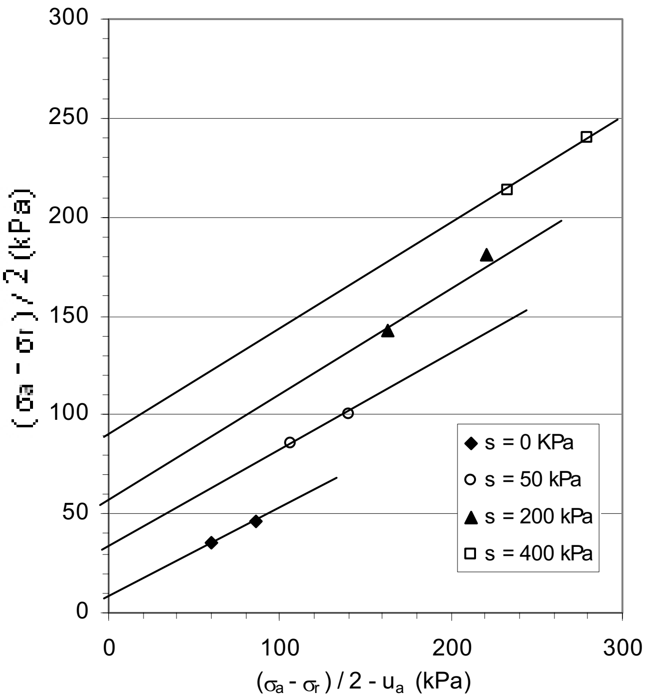

Figure 7. Shear strength envelopes at different matric suctions

5. PROPOSED MODEL BASED ON LEROUEIL & BARBOSA (2000) MODEL

5.1 Proposed stress variables

Several combinations of stress state variables have been proposed to model the behaviour of unsaturated soils. The most common choice is to use net stress $(\sigma - u_a)$ and matric suction $(u_a - u_w)$ as two independent stress state variables. This approach was used to develop the most important models during the last 40 years, in particular the BBM type models (see section 2.2).

The experimental results presented in this paper and several results reported by other researchers show that soils remain essentially saturated for matric suctions lower than the air entry value and thus have a behaviour controlled by effective stresses. Beyond this particular point, the soil becomes unsaturated and can be controlled by two independent variables. In this latter domain, we are proposing to use the "modified net stress" $\sigma^* = \sigma - u_a + \chi^*(u_a - u_w)$ and matric suction as stress variables, rather than the net stress $(\sigma - u_a)$ and matric suction. The main reason is to insure a continuous transition from saturated conditions to unsaturated conditions.

The effective stress can be written as $\sigma' = (\sigma - u_a) + (u_a - u_w)$ and has therefore to be the modified net stress for suctions lower than or equal to the air entry value $(u_a - u_w)_b$. This would correspond to $\chi^* = 1.0$. For matric suctions larger than air entry value, it appeared, on the basis of experimental results that will be presented later on interesting to define $\chi^* = \frac{(u_a - u_w)_b}{(u_a - u_w)}$. Consequently, the modified net stress σ^* is defined as follows:

$$\sigma^* = \sigma - u_a + \chi^*(u_a - u_w) \qquad [7]$$

where

$$\chi^* = 1 \quad \text{for} \quad (u_a - u_w) \le (u_a - u_w)_b \ \text{and} \ S_r \cong 1.0 \qquad [8]$$

and

$$\chi^* = \frac{(u_a - u_w)_b}{(u_a - u_w)} \quad \text{for} \quad (u_a - u_w) \ge (u_a - u_w)_b \ \text{and} \ S_r < 1.0 \qquad [9]$$

5.2 Expressions for yield stresses

The following model is based on Leroueil and Barbosa (2000) model that was named GFY. Hence, the present model is called here GFY-2 and Leroueil and Barbosa model will be henceforth called GFY-1.

The behaviour in compression of soil in GFY-2 model is controlled by two yield stresses: the axial yield stress and the radial yield stress, as in GFY-1 model (see section 2.3).

The GFY-2 model considers that the soil remains saturated for suctions lower than the air-entry value. Consequently, the axial and radial modified net yield stresses, σ^*_{alo} and σ^*_{rlo}, are written as follows:

$$\sigma^*_{alo} = (\sigma_{al} - u_a) + (u_a - u_w) = \sigma_{al} - u_w \qquad [10]$$

$$\sigma^*_{rlo} = (\sigma_{rl} - u_a) + (u_a - u_w) = \sigma_{rl} - u_w \qquad [11]$$

where σ_{al} and σ_{rl} are respectively the axial and radial total stresses at yielding. It can be considered that σ^*_{alo} and σ^*_{rlo} are constant when the soil remains saturated.

According to the concept proposed by Leroueil and Barbosa (2000), when the soil becomes unsaturated, its strength increases (see Eqs. 4 and 5). In the GFY-2 model, the modified net yield stresses and thus the strength would increase only in comparison with values at the air entry value. In compression, the strength increase would be:

$$\Delta\tau_{fc} = \frac{\sin\phi' \cos\phi'}{1 + \sin\phi'} \left[\sigma^*_{al} - \sigma^*_{alb}\right] \qquad [12]$$

In extension, the strength increase would be:

$$\Delta\tau_{fe} = \frac{\sin\phi' \cos\phi'}{1 + \sin\phi'} \left[\sigma^*_{rl} - \sigma^*_{rlb}\right] \qquad [13]$$

In Eq.12 σ^*_{al} and σ^*_{alb} are the axial modified net yield stresses at the considered suction and at suction equal to the air-entry value respectively. Similarly, σ^*_{rl} and σ^*_{rlo} are the radial modified net yield stresses at the considered suction and at suction equal to the air-entry value respectively. $\Delta\tau_{fc}$ and $\Delta\tau_{fe}$ are the contribution of strength due to suction, in excess of the air –entry value, respectively in compression and in extension. That will be also considered as "cohesion".

Equation 3 can be written in an equivalent manner, using the water ratio e_w rather than the volumetric water content. It gives:

$$\tau_{fc} = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\left(\frac{e_w - e_{wr}}{e_{ws} - e_{wr}}\right)\tan\phi' \qquad [14]$$

The water ratio at the considered suction e_w, at saturation e_{ws} and at residual condition e_{wr} are determined from the soil-water characteristic curve.

The contribution of strength increase due to suction in excess of the air –entry value, can be written as follows:

$$\Delta\tau_{fc} = \left[(u_a - u_w)\left(\frac{e_w - e_{wr}}{e_{ws} - e_{wr}}\right) - (u_a - u_w)_b\right]\tan\phi' \qquad [15]$$

Considerations of Eqs.12 and 15 give the following relation for the increase in axial modified net yield stress due to suction:

$$\sigma^*_{al} - \sigma^*_{alb} = \frac{(1+\sin\phi')}{(\cos\phi')^2}\left[(u_a - u_w)\left(\frac{e_w - e_{wr}}{e_{ws} - e_{wr}}\right) - (u_a - u_w)_b\right] \qquad [16]$$

Given that the model considers that radial yield stress is related to axial yield stress by the anisotropic parameter as follows (from Leroueil and Barbosa (2000)) :

$$K_{AL} = \frac{\sigma^*_{rlo}}{\sigma^*_{alo}} = \frac{\sigma^*_{rl}}{\sigma^*_{al}} \qquad [17]$$

Therefore, the combination between Eqs. 16 and 17 gives the following relation for the contribution of radial yield stress due to suction:

$$\sigma^*_{rl} - \sigma^*_{rlo} = K_{AL}\frac{(1 + \sin\phi')}{(\cos\phi')^2}\left[(u_a - u_w)\left(\frac{e_w - e_{wr}}{e_{ws} - e_{wr}}\right) - (u_a - u_w)_b\right] \qquad [18]$$

6. APPLICATION OF GFY-2 MODEL TO EXPERIMENTAL RESULTS

6.1 Application of GFY-2 model to experimental results obtained on Sainte-Rosalie clay

The axial and radial modified net yield stresses determined, at different matric suction, on the basis of experimental results obtained on Sainte-Rosalie clay (see section 4.2) and by using the theoretical equations (equations 10, 11, 16 and 18), are shown on the Fig. 8 as function of matric suction.

It can be seen that the LC curves obtained by the theoretical equations proposed in GFY-2 model (air enter value equal to 70 kPa) well fit the experimental data points obtained at yielding.

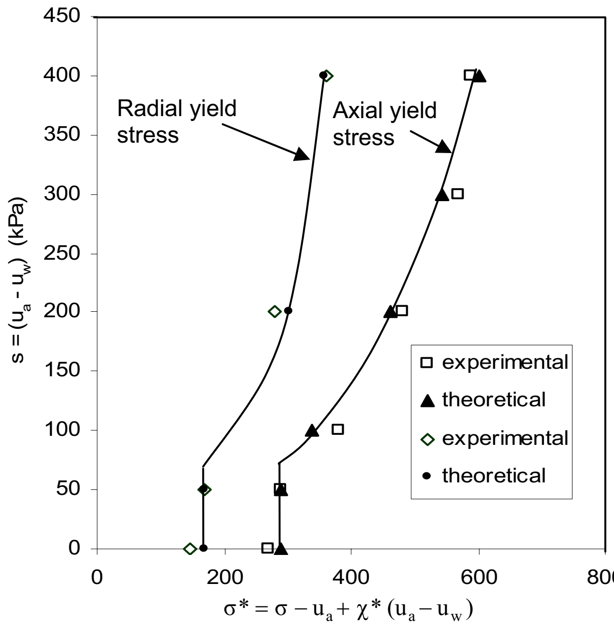

Figure 8. LC curves of Sainte-Rosalie clay

The yielding points obtained on Sainte-Rosalie clay, under suctions of 0, 50, 200 and 400 kPa, are shown in figure 9.

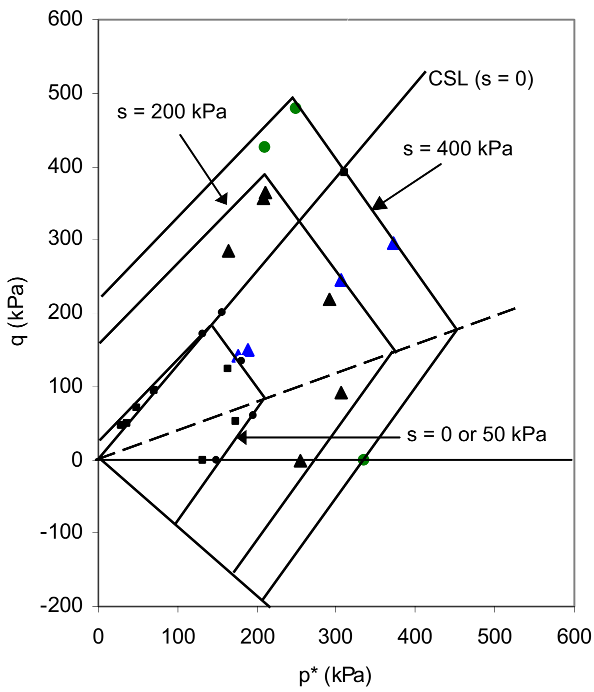

Figure 9. Yielding and GFY-2 curves for Sainte-Rosalie clay

The GFY-2 model previously described has been applied to the experimental results obtained, as shown in figure 9. It can be seen that the proposed model well fit the data points (K_{AL} appears to be close to 0.6 in this case).

6.2 Application of GFY-2 model to some other experimental studies

The GFY-2 has been applied to the sets of data provided by Machado and Vilar (1999) on the figure 10 and Cui & Delage (1996) on the figure 11.

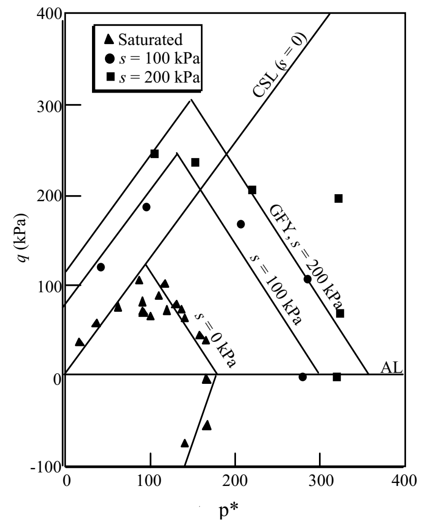

Figure 10. Yielding and GFY-2 curves for a natural residual soil from Brazil (after Machado and Vilar, 1999, 2002)

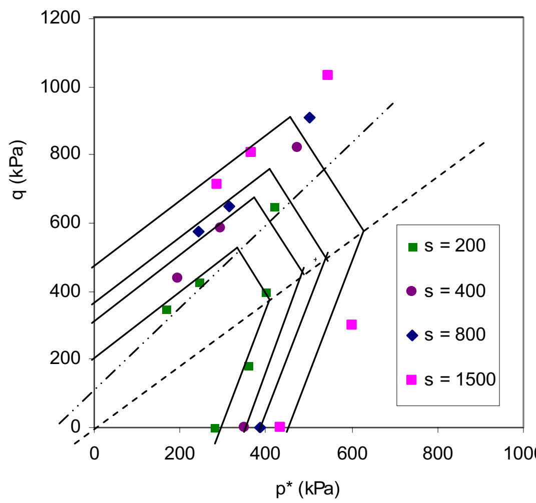

Figure 11. Yielding and GFY-2 curves for compacted Jossigny silt (after Cui and Delage, 1996)

It can be seen that the yield points both obtained by Machado & Vilar (1999) on the residual soil from Brazil and by Cui & Delage (1996) on compacted silty soils, are perfectly fitted by GFY-2 model.

7. DISCUSSION AND CONCLUSION

A model of yielding, which would apply to saturated or unsaturated, and structured or non-structured soils is proposed. This model (GFY-2 model) takes into account the anisotropy of soil and can be applied to cohesion or non cohesion soils. GFY-2 model is certainly idealised and approximate. In particular, yield curves of soils are certainly more rounded than those given by the model. It is thought, however, that it provides a rational physical framework for understanding yielding of soils.

GFY model assumes that the soil remain saturated until the air entry value is reached and the behaviour of soil, in this domain, is controlled by effective stresses (this fact is not takes into account by Barcelona type models). Beyond the air entry value, the soil becomes unsaturated and the model is controlled by the modified net stress $\sigma^* = \sigma - u_a + \chi^* (u_a - u_w)$ and the matric suction.

GFY-2 model shows that the increase of axial and radial yield stresses depend of water characteristic curve. The compressive behaviour can be related to the shear strength behaviour (as demonstrated by Leroueil and Barbosa, 2000) and both of them depend of water characteristic curve. The increase of shear strength in compression mainly depends of the increase of the axial yield stress. It still has to be confirmed that the increase of shear strength in extension mainly depends of the increase of the radial yield stress.

The input parameters of the GFY model can be defined by oedometer tests performed at different suctions to determine the axial Loading-Collapse (LC_a) curve and a few isotropic triaxial compression tests, or oedometer tests on specimens trimmed horizontally, at different suctions to define the fabric anisotropy characterised by K_{AL}.
The model gives representative results for saturated and unsaturated soils. It still has to be confirmed for strength envelopes in extension, mostly because of the lack of data in this domain.

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