

THE PARAMETER EVALUATION METHOD (PEM) OF ELASTIC-PLASTIC MODEL PROPERTIES FOR UNSATURATED HIGH-PLASTIC CLAY

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ABSTRACT

The behaviour of unsaturated expansive clay soil is a study of considerable importance. The stress-volume change behaviour of high-plastic clay can be forecast by means of experimental testing or numerical modelling. The numerical model must be calibrated using results of experimental testing. Some constitutive models require many parameters to describe the behaviour of high-plastic clay. However, in some cases the existing laboratory results are not those required for determining model parameters. This paper proposes a Parameter Evaluation Method (PEM) to determine parameters of a constitutive model. The method uses the Barcelona Basic Model (BBM) (Alonso et al. 1990) to model the response of four triaxial high-plastic clay specimens subjected to four different stress paths. Evaluation of the BBM performed by means of the PEM results in several recommendations to improve the performance of the BBM for high-plastic clay soils and three criteria to evaluate the performance of a constitutive model. The results show that the combination of the PEM and the proposed criteria can be used as a tool for selecting constitutive models in numerical modelling.

RÉSUMÉ

Le comportement des sols expansibles insaturés d'argile est une étude très importante. Le volume entrepris par la tension change le comportement d'une haute matière de plastique argileuse peut être prévu soit par des tests expérimentaux soit par des modèles numériques. Le modèle numérique doit être calibré en utilisant les résultats des tests expérimentaux. La méthode (Parameter Evaluation Method – PEM) est suggérer pour déterminer les paramètres d'un modèle constitutif dont il est exécuté en utilisant le (Barcelona Basic Model – BBM) (Alonso et al. 1990) pour prédire la réaction de quatre spécimens, chacun de trois axes, qui vont suivre quatre différents chemins stressants. Les évaluations du (BBM), qui a été exécuté par le (PEM), donnent plusieurs recommandations pour améliorer la performance du (BBM) des sols fait d'haute matière de plastique argileuse. De plus, elles donnent trois critères pour évaluer la performance d'un modèle constitutif. Les résultats démontrent que la combinaison du (PEM) et des critères proposées peuvent être utilisé comme des outils de sélection du modèle constitutif trouvé enfin dans le modèle numérique.

1. INTRODUCTION

Much of the world's infrastructure is founded on or in expansive high-plastic clay that is sensitive to changes in water content. Expansive clay subsoils comprise one of the most damaging factors for structures, particularly for shallow foundations and pavements (Jones and Holtz 1973). The expansive nature of high-plastic clays can also be beneficial in some applications, for examples as engineered barriers in underground disposal vaults.

Numerical models provide an important tool in the design process when used in conjunction with laboratory testing. Numerical models must be calibrated and validated using laboratory testing and/or field measurements before confidence can be gained in their use for predicting prototype behaviour. After calibration and validation a numerical model is a powerful tool that can be used to evaluate and forecast the behaviour of a system subjected to anticipated conditions in the field application.

During design involving unsaturated expansive clay, one of several currently available constitutive models must be chosen to represent soil behaviour for all anticipated

stress and environmental loadings. Usually the choice of a model must be made prior to laboratory testing, since each model has been developed from a specific set of tests. Another difficulty is that individual models may only represent the soil types used during their development.

Traditionally, a number of laboratory tests are performed for calibrating a chosen model. The results are then summarized and parameters for the constitutive model evaluated using engineering judgment. This approach has several drawbacks.

- At least one laboratory test is required to determine each parameter of the model. If more than one laboratory test is performed, there can be variation in the results. Proper choice of model parameters is a difficult process that relies heavily on the experience of the engineer.
- A model with a large number of parameters requires many laboratory tests that can be costly and time-consuming. A model with fewer parameters may be more convenient to use. Typically, fewer parameters mean that the model captures fewer features of behaviour and its applicability may therefore be limited.

- Each model was originally developed from a specific program or laboratory tests on a specific soil type. If a project involves different loading paths or different soils, the engineer may need to choose another model. This new model may have parameters that cannot be obtained from previously completed tests.

The Parameter Evaluation Method (PEM) proposed in this paper evaluates the multiple parameters for a chosen constitutive model using one laboratory test. The test conditions can be chosen to reflect conditions in the application instead of “model-specific” paths. In this way, important aspects of the application can be captured in the laboratory. The PEM can reduce the number of laboratory tests required to determine model parameters because it generates multiple parameters from one test.

Several elastic-plastic constitutive models are available for unsaturated soils such as those considered in this paper. Examples include those proposed by Alonso et al. (1990, 1999), Wheeler and Sivakumar (1995), Tang and Graham (2002) and Blatz and Graham (2003). There is considerable debate as to which model is the most reliable. The ability of the PEM to determine various model parameters based on the same laboratory test results can be helpful in assessing the applicability of the various models without undertaking additional laboratory tests. This paper applies the PEM to a series of tests using one constitutive model and examines its capabilities. In future work, the PEM will be applied simultaneously to a series of tests using several models.

Here, the PEM is used to evaluate parameters for the Barcelona Basic Model (BBM) from tests performed on unsaturated high-plastic clay at the University of Manitoba. The BBM has been selected for this study since it is considered by many to be a first-choice model for unsaturated soils. For example, it has been implemented in Finite Element Method (FEM) computer programs such as CODE_BRIGHT (1996). Although the BBM was initially developed for low to moderately-plastic unsaturated clays, it is used in practice to design systems incorporating swelling clay behaviour. The paper shows how model parameters generated from the PEM can be used to evaluate the applicability of the BBM for unsaturated high-plastic clay soil. The paper also proposes three criteria for selecting a constitutive model for high-plastic clay.

2. THE PARAMETER EVALUATION METHOD (PEM)

The PEM is illustrated in the flow chart in Figure 1. It is based on the premise that numerical models must be calibrated using laboratory test results. The PEM starts by selecting stress or environmental conditions that will be applied in laboratory tests to replicate conditions in the proposed engineering application. Design of the testing program is intimately associated with the choice of a suitable constitutive model.

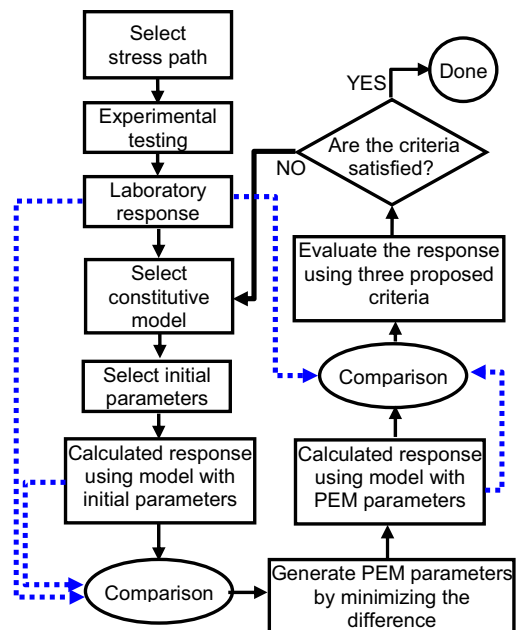


Figure 1. Flow chart of Parameter Evaluation Method (PEM).

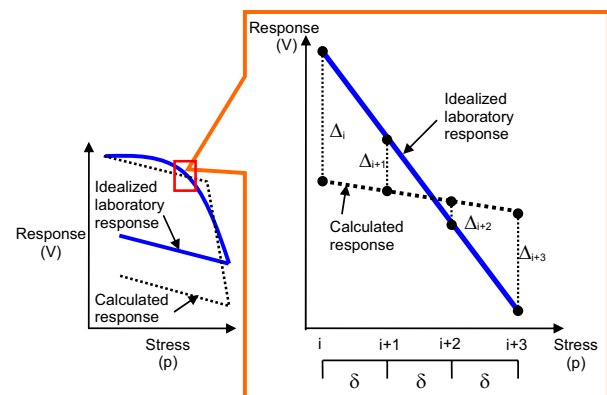


Figure 2. The method to assess the absolute difference for application of the PEM.

After the constitutive model is selected, reasonable initial parameters are estimated and put into the constitutive model. These values can be approximated using available databases of previous test results or engineering judgment. Using these initial parameters, the general shape of the calculated response of the model should be similar to the measured laboratory response. At this stage, the calculated and laboratory responses will usually not match well. Figure 2 shows a way of assessing the difference between the calculated and measured responses. First, the laboratory response is idealized and then divided into very small increments ($\delta \approx 0$). The

calculated response using the model parameters is determined using the same increment interval. The absolute difference between the idealized laboratory result and the response using the initial parameters is calculated for each increment and then summed for the entire test. All relevant model parameters are then altered systematically within physically meaningful ranges until the total absolute difference is minimized. Here, these optimized parameters will be called 'PEM parameters'. The responses using the PEM parameters and the initial parameters are plotted with the laboratory test results for comparison. Depending on the availability of data, PEM parameters should be generated for several stress paths. The modeled response is validated using three criteria that include comparison of test responses, comparison of yield surfaces and variability of PEM parameters generated using different stress paths. If the criteria are satisfied, the process is complete. If not, a further process can be undertaken to evaluate alternative constitutive models.

3. IMPLEMENTATION OF THE PARAMETER EVALUATION METHOD FOR THE BARCELONA BASIC MODEL

3.1 Stress Paths

Laboratory tests used in this paper include results of triaxial tests with controlled suction (Blatz and Graham 2000, Anderson et al. 2003, and Siemens and Blatz 2004). Figures 3 and 4 show stress paths from four specimens that have been used for evaluating the BBM. Specimens JB104 and JB105 are taken from Blatz (2000), DA007 is from Anderson (2003) and GS041 is from Siemens and Blatz (2004).

All specimens consisted of a 50:50 mixture (by dry weight) of well-graded sand and bentonite. All specimens have an initial 'as-compacted' suction of approximately 3 - 4 MPa. Specimens JB104 and JB105 contained Avonlea bentonite, while the specimens DA007 and GS041 contained Wyoming bentonite. Although there are differences in mineralogical properties of the Avonlea and Wyoming bentonites, Anderson et al. (2002) showed that compacted specimens with initial suctions higher than 3 MPa exhibited only small differences in mechanical behaviour when they were dried (increasing suction), isotropically loaded, or sheared. The compressibility of compacted specimens of Avonlea and Wyoming bentonite is relatively similar and they show similar shear responses at suctions greater than 10 MPa (Anderson et al. 2002). This paper evaluates the use of PEM in the BBM with specimens which all had suctions are higher than 4 MPa.

Specimens JB104 and JB105 (see Figure 3) follow stress paths a-b-c-d-e and a-b-c-f-g respectively in isotropic p' - S stress space. They were initially loaded isotropically ($q = 0$) along a-b until yielding was observed, indicating that they had exceeded their as-compacted loading collapse (LC) line (Alonso et al. 1990). Figure 3 shows that increasing mean stress is accompanied by decreases in suction (see also Tang et al. 2002). After further plastic hardening occurred, specimens were unloaded from b-c to

achieve an overconsolidation ratio of 2.0. The specimens were then dried while still under constant pressure in the triaxial cell to a target suction value. This produced increasing suctions from c-d and from c-f respectively. Once equilibrium was achieved, the specimens were again isotropically loaded (d-e and f-g) to measure their compressibility and suction response.

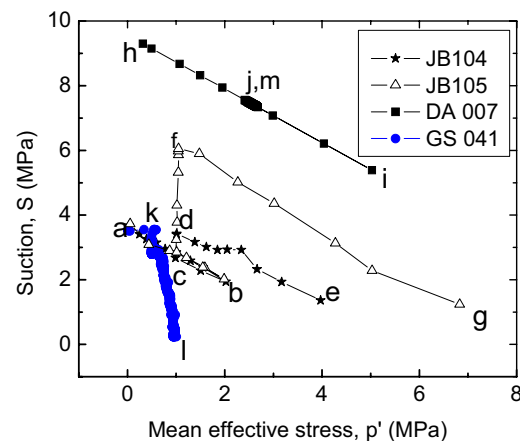


Figure 3. Stress paths in p' - S stress space

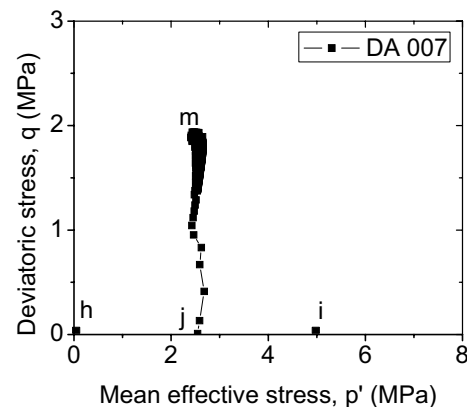


Figure 4. Stress paths in p' - q stress space

Before being installed in the triaxial cell, specimen DA007 (see Figures 3 and 4) was first subjected to an increase in suction from its 'as-compacted' value in a glass dessicator using an acid solution set to a target suction. The specimen was then loaded isotropically to surpass its isotropic yield stress. This is shown as h-i in the p' - S and p' - q spaces in Figures 3 and 4 respectively. The specimen was then unloaded to a target overconsolidation ratio along i-j. Finally specimen DA007 was sheared at a constant mean stress and suction (j-m).

The final specimen examined is GS041 which was subjected to a much different stress path than the other three specimens. This is shown as a-k-l in Figure 3.

GS041 was initially loaded isotropically to 0.5 MPa. It was then given access to water (wetting) under controlled constant volume boundary conditions until equilibrium.

3.2 Barcelona Basic Model (BBM)

The BBM was first presented in qualitative form by Alonso et al. (1987) and then as a mathematical formulation by Alonso et al. (1990). The model employs the three stress state variables: mean effective stress p' , deviatoric stress q , and suction S . In the p' - S plane, the yield surface is controlled by the LC (loading collapse) yield curve, the SI-line (suction increase) and the k-line (increase in tensile strength). In p' - q space, the yield surface is represented by ellipses that increase in size with increasing suction. The gradient M of the Critical State Line (CSL) in p' - q space is independent of suction. At stress states within the yield surface, the soil behaves elastically. When a stress path reaches a position on the yield surface, non-recoverable plastic hardening occurs and the yield surface grows in size as a result of permanent densification. During yielding, the strain increment consists of elastic and plastic strain components. When the stress path decreases, the size of the yield surface remains at the position corresponding to the maximum stress state achieved along any given stress path.

3.3 Selection of the Initial Parameters

Application of the BBM model requires nine parameters plus the initial specific volume and initial reference stress state that define the as-compacted position of the yield surface. Laboratory tests at the University of Manitoba (Blatz 2000) have produced physically meaningful ranges of BBM parameters that can be used as starting points for the PEM. One important aspect of the PEM process is that final values of the parameters (after PEM) must be within physically reasonable ranges. For this study, initial values were selected as follows: $p_o = 0.5$ MPa; $\lambda(0) = 0.12$; $\kappa = 0.004$; $\kappa_s = 0.002$. Parameters p_o^* and $\lambda(0)$ are yield stress at zero deviatoric stress ($q = 0$) for zero suction ($S = 0$), and slope of normal compression line in V - $\ln(p')$ plot for zero suction ($S = 0$) respectively. The κ and κ_s are elastic swelling indices with respect to p' and S respectively. The initial values of shear modulus G , and slope of the critical state line M , were evaluated as $G = 200$ MPa and $M = 1.0$ respectively. The parameter describing the increase in cohesion with suction was taken as $k = 0.0375$.

There is no direct database available for defining the physically meaningful ranges of the parameters r , β and p^c . After fitting the LC curve to yield points interpreted from laboratory testing (Blatz 2000) gave $r = 0.6823$, $\beta = 0.2327$ and $p^c = 0.0100$. The parameters r , β , and p^c are parameters defining maximum soil stiffness, the rate of increase of the slope of normal compression line with suction, and the reference stress respectively. The other two additional BBM parameters s_{0i} and λ_s are not required for predicting the stress paths in this paper because yielding in suction is not considered.

3.4 Minimizing the Differences

Calculating changes in specific volume resulting from stress changes employs explicit integration of the constitutive relations. Here, the values of the parameters listed in the previous section were used to define the yield surface of the BBM for the materials being tested. Macari et al. (2003) presented an explicit procedure for step-by-step integration of the BBM for a drained (constant- S) conventional triaxial compression test. Further modification of the algorithm has been developed by the authors for other stress paths. Stress paths from the laboratory tests (Figures 3 and 4) were idealized and then divided into small increments. Up to 500 points were used in the simulations presented here. The stress state values (p' , q , S) at each point were determined and inserted into the program. The corresponding laboratory strain responses were then idealized and used to calculate the absolute difference between the calculated and laboratory responses at each point. The individual differences were then summed to calculate the total difference for the entire test. The spreadsheet program used an algorithm that coupled the model parameters and the total difference so that the differences could be minimized. The process used the 'Solver' tool built into the MS-Excel program, which tests sequential combinations of the parameters. Constraints are applied so that the parameters are maintained within a previously established physically meaningful range.

During difference minimization, the calculated total difference is specific to the stress path being examined. For isotropic loading ($q = 0$), the specific volume from the laboratory test is considered, while for shearing stress paths, shear strains are used for calculating total differences. For isotropic loading, the parameters p_o , $\lambda(0)$, κ , r , β , κ_s and p_o^* are all potentially subject to alteration. Since the others parameters do not affect the isotropic loading response they are not included. For constant- p shearing paths, parameters G , M , and k are varied while the other parameters remain constant.

Figure 5 shows a typical comparison of the modeled response using the initial and PEM parameters for specimen JB104. Using the PEM parameters, there is considerable improvement in the calculated response compared with using the initial parameters, particularly in the region after initial yielding. It is important to note that this is a calibration exercise as opposed to a validation or prediction exercise. Values of the PEM parameters for the BBM are summarized in Table 1. The PEM was applied to each specimen's results separately to obtain the parameters.

4. DISCUSSION

The applicability of the BBM for unsaturated high-plastic clay can be evaluated by the following three criteria:

- 1) comparison of laboratory responses and calculated responses using the model with PEM parameters,

- 2) evaluation of the location and the shape of the yield surface generated for the model,
- 3) the variability of the PEM parameters for different stress paths using the same constitutive model.

Table 1. The PEM parameters of the BBM model for four different specimens (JB104, JB105, DA007, and GS041)

Sample	JB-104	JB-105	DA-007	GS-041
Isotropic	$\lambda(0)$	0.13	0.11	0.11
	κ	0.020	0.011	0.005
	r	0.68	0.68	0.68
	β (MPa ⁻¹)	0.23	0.23	0.23
	p^c (MPa)	0.01	0.02	0.01
	κ_s	0.0179	0.0171	0.0055
	p_0^* (MPa)	0.51	0.50	1.20
Shearing	G (MPa)	N/A	N/A	200
	M	N/A	N/A	0.71
	k	N/A	N/A	0.04

N/A: not applicable

4.1 Comparison of Laboratory Tests Results and Calculated Response Using PEM Parameters

Figures 5 to 8 present responses simulated using the BBM with the PEM parameters for specimens JB104, DA007 and GS041. The laboratory observations are also plotted in the figures to allow direct comparison. The V - $\ln(p')$ responses during isotropic loading of JB104 and DA007 are shown in Figures 5 and 6, respectively. Comparing simulations using the initial and PEM parameters shows that the optimization method implemented in this paper improves the fit between the predicted and measured responses. For specimen JB104 (see Figure 5), the BBM gives a more reliable prediction for the first isotropic loading-unloading sequence than for the second loading, which took place after the suction had been increased.

Figure 6 plots volume changes during the isotropic loading and unloading (A-B) and the constant- p shearing stress paths (B-C and B-D) for specimen DA007. The volume changes due to isotropic loading show good agreement between the calculated and the experimental results. Figure 8 plots deviatoric stress versus shear strain for specimen DA007. During shearing, the simulation shows opposing results to the experimental data after yielding. The simulation shows decreases in specific volume while the experimental results show increases, that is, dilatation. Using the PEM to develop optimized parameters, the model produced a reasonable simulation of shear strains (Figure 8). However, it did not reproduce the strain softening behaviour seen in the experimental results in Figure 6.

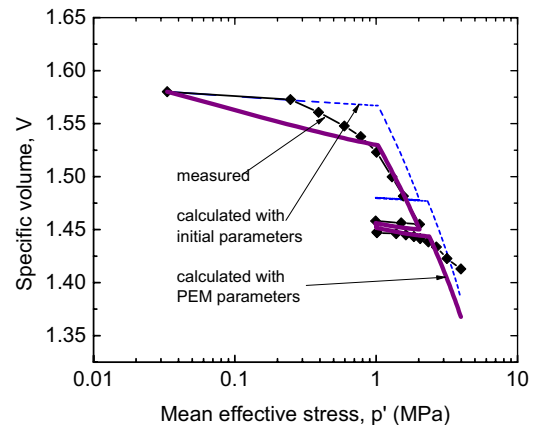


Figure 5. Experimental results and simulated V - $\ln(p')$ responses calculated using initial and PEM determined parameters for specimen JB104.

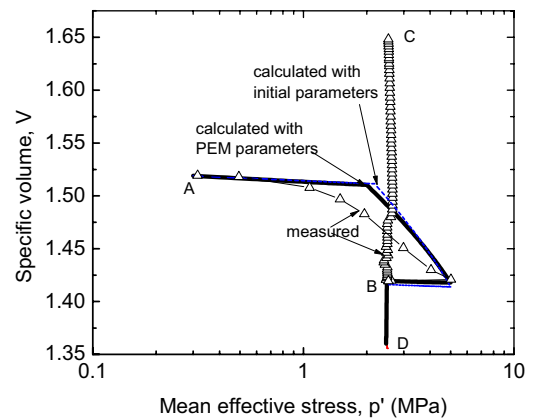


Figure 6. Experimental results and simulated V - $\ln(p')$ responses calculated using initial and PEM determined parameters for specimen DA007.

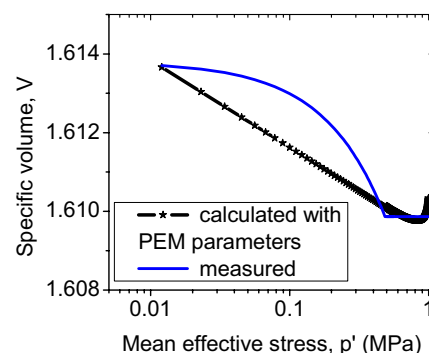


Figure 7. Experimental results and simulated V - $\ln(p')$ responses calculated using initial and PEM parameters for specimen GS041

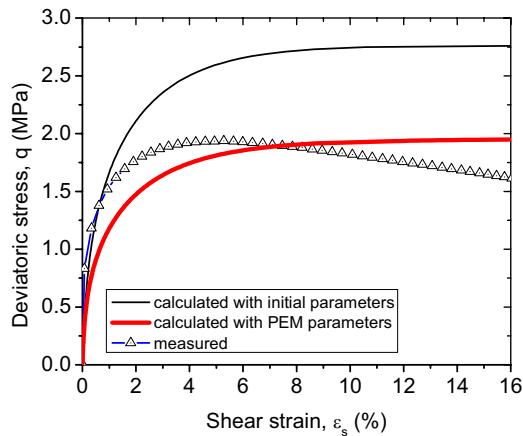


Figure 8. Experimental results and simulated q - ϵ_s responses calculated using initial and PEM parameters for specimen DA007

Figure 7 presents the V - $\ln(p')$ response for specimen GS041. Although the differences between the experimental response and the simulation using the PEM parameters are relatively small, the main features of soil behaviour in the specimen GS041 are not represented by the simulation. Unlike the experimental data that showed the volume of the specimen remaining constant during the last stages of the test, the simulation indicates continued increases of volume.

4.2 Evaluation of Location and Shape of Yield Surface

Because the stress paths generally start with isotropic loading ($q = 0$), the yield surfaces generated by the model in this paper are generally controlled by the LC curve of the BBM. (An exception is DA007.) The LC curves generated by the BBM parameters in Table 1 are shown in Figure 9. The figure shows LC curves calculated using both the initial and the optimized PEM parameters. Since the laboratory tests were performed on materials with essentially the same properties (Anderston et al. 2002), the LC curves should be similar for all four specimens. The figure also shows the stress paths in the simulation for direct comparison. The LC curves from the BBM calculated using initial parameters and the optimized PEM parameters have been labeled the 'initial LC curve' and the 'final LC curve' respectively. The two curves are essentially indistinguishable for specimens JB104, JB105 and DA007. However, the curve for GS041 calculated using PEM parameters is noticeably different. This is the specimen in Figures 6 and 8, where the shear strain behaviour could be simulated quite well, but the volume strains could not.

Considering that the initial parameters are determined by fitting the LC curve based on selected yield points from laboratory tests (Blatz 2000), the expected location of the final LC curve should be consistent with the initial LC curve. The initial and final LC curves are very similar for

specimens JB104, JB105, and DA007. Although specimen DA007 and specimens JB104 and JB105 have different types of Na-bentonite (Wyoming and Avonlea) their LC curves are similar indicating that the type of bentonite is not notably affecting the preconsolidation pressure or LC curve. Comparison of the stress paths and the final LC curve for these three specimens in Figure 9 shows that the final LC curves are reasonable, because they pass through the preconsolidation pressure points of the specimens. The final LC curve for specimen GS041 is shifted significantly to the right, compared to the initial LC curve. This indicates that the PEM parameters generated from specimen GS041 are only in the elastic range, since the stress path for this specimen is located inside the LC curve. Since all specimens are compacted using the same procedure, their LC curves should be at relatively the same location and this is clearly not the case.

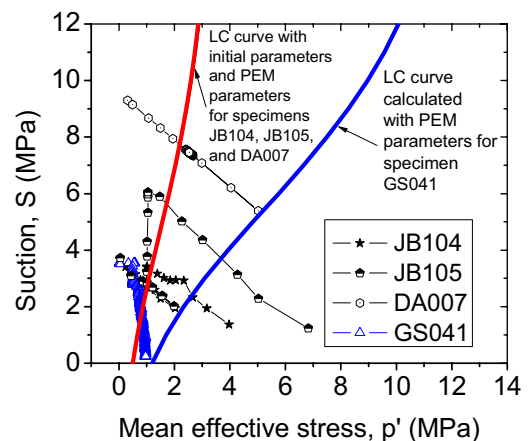


Figure 9. Experimental stress paths and LC curve of Barcelona Basic Model (BBM) calculated with initial and PEM parameters.

Figure 10 shows the initial and final LC curves for suctions of 0 – 12 MPa plotted with experimentally determined yield points (Blatz 2000, Anderson 2003). For suctions in the range of 0 – 8 MPa (Figure 9), the initial and final LC curves for the specimens JB104, JB105, and DA007 show reasonable agreement with the experimentally determined yield points. However, for specimen GS041, the final LC curve is located far from these yield points as discussed previously. The LC curve for a higher range of suction (0 – 140 MPa) is illustrated in Figure 11. This figure shows that the model predicts constant yield mean effective stress when the suction is greater than approximately 20 MPa. In contrast, the experimental data show that the mean effective stress at yielding increases only slightly with increasing suction. The LC curve for the suction range of approximately 10 – 60 MPa (see Figure 11) is noticeably higher than the experimentally determined yield points. Evaluation of the LC curve with final BBM parameters shows that the final parameters presented in Table 1 are only valid for specimens JB104, JB105, and DA007 for the

suction range of 0 – 8 MPa. The PEM parameters for specimen GS041 presented in Table 1 are significantly different from the other specimens. This leads to a yield surface that does not agree with experimental yield points generated from other similar specimens.

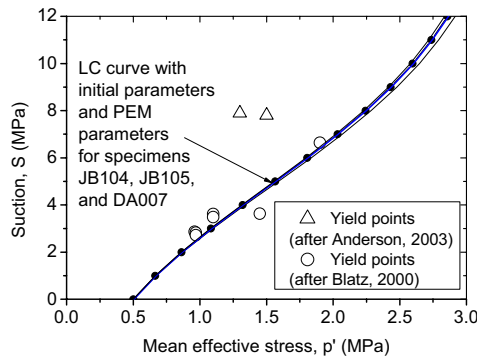


Figure 10. Experimental yield point and LC curve calculated with initial and PEM parameters for specimens JB104, JB105, and DA007 for suction $S = 0 - 12$ MPa

4.3 Comparison of PEM Parameters Generated From Different Stress Paths

Elastic-plastic constitutive models assume stress-path independence in the elastic region. For one material, the elastic parameters should be the same regardless of stress-path. Table 1 shows the BBM parameters generated by the PEM. The parameters $\lambda(0)$, κ , r , β , p_c , κ_s , and p_o are used for simulating isotropic loading. For the similarly loaded specimens JB104, JB105, and DA007, the parameters vary only slightly, while they are quite different for the different loading path in specimen GS041. For example, the parameter p_c for specimen GS041 is higher than that for the other specimens. The very small values of the parameters κ and κ_s for the specimen GS041 appear to be anomalous. Table 1 indicates that the BBM works well for stress paths for isotropic loading but is less successful in simulating shear loading. The parameters G , M , and k are involved in simulating shear loading. Evaluation of the variability of these parameters cannot be performed, because only one specimen (DA007) is being reported at this stage. This observation suggests that the variability of parameters generated with the PEM can be used to indicate the applicability of a given constitutive model.

The parameters generated using the PEM are only valid when the selected model can capture the soil behaviour of the associated with the chosen stress path. When this is not possible, the model parameters generated by PEM will vary with the type of test that has been done. Table 1 shows considerable variability of some of the BBM parameters for different specimens. This suggests that the BBM does not 'capture' all of the behaviour of the tested high-plastic clay, particularly in shear or along wetting paths. One needs, therefore, to be cautious in using the

BBM in a numerical model involving high-plastic clay. Combining the PEM with the three criteria presented earlier provides a tool for evaluating the applicability of a particular constitutive model.

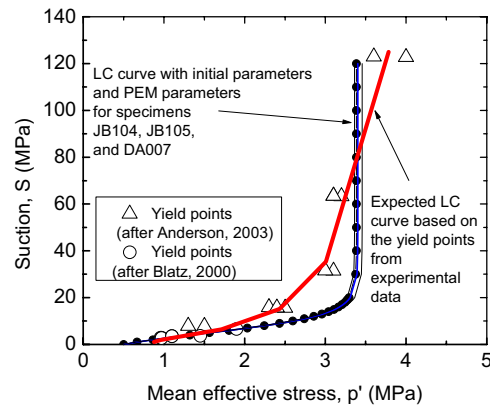


Figure 11. Experimental yield point and LC curve calculated with initial and PEM parameters for specimens JB104, JB105, and DA007 for suction $S = 0 - 140$ MPa.

5. CONCLUSIONS

The following conclusions can be drawn from the work described in this paper.

- Three criteria to evaluate the performance of a given constitutive model are proposed, namely:

- 1) location of the yield surface.
- 2) assessment of differences between results from model simulation and experimental results.
- 3) variability of parameters generated from tests along different stress paths.

- A combination of the PEM and the proposed criteria can assist in selecting a suitable constitutive model for numerical modeling.

- Application of the PEM is not affected by the number of the parameters used in a constitutive model and can reduce the number of tests required.

- The BBM for the 50:50 sand bentonite soil in this program is only valid for isotropic stress paths where the suction is less than 8 MPa. This is the range used in the tests that were available for calibrating the model. Modification of the model is required for higher suctions and more general stress paths such as strain softening and wetting.

- Further use of the PEM presented in the paper will be applied to evaluate various constitutive models for unsaturated high-plastic clay. This evaluation will allow detecting the limitation of each model and giving direction on how to modify such a model to simulate the behaviour of unsaturated high-plastic clay material.

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