# UNCERTAINTY EVALUATION OF UNDRAINED SHEAR STRENGTH

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ABSTRACT: This paper examines the influence of spatial variability of soils and uncertainty evaluation on the undrained shear strength clay. The paper presents a statistical approach to evaluate the scale of fluctuation and coefficient of variation of cone penetration test parameters. The uncertainty involved in a standard conversion model which is commonly used for transferring field measurements into design soil properties is also addressed. Results obtained from the proposed approach are used for estimating more reliable undrained shear strength.

RÉSUMÉ
Ce papier examine l'influence de variabilité spatiale d'évaluation de sols et incertitude sur la undrained force de cisailles de Macao argile marine. Le papier présente une approche statistique pour évaluer l'échelle de variation et le coefficient de variation de paramètres de test de pénétration de cône. L'incertitude a impliqué dans un modèle de conversion standard qui est ordinairement utilisé pour transférer de mesures de champ dans les propriétés de sol de conception sont aussi adressées. Les résultats ont obtenu de l'approche proposée sont utilisé pour estimer plus fiable undrained la force de cisailles.

## 1. INTRODUCTION

In recent years, field tests based design has become relevant for foundations because these tests can provide large amounts of low cost repeatable data. In general, the analysis and design processes use largely empirical models to transform field measurements into geotechnical design properties. Unfortunately, the processes involve considerable uncertainties due to improper accounting for the inherent soil variability and the conversion uncertainties of empirical conversion models. In early studies, (Lumb 1971, Vanmarke 1977, Orchant et al, 1986, Kay 1995, Fung 1998), the authors have addressed fundamental levels of uncertainty involved in soil exploration and foundation design problems. However, the utilization of the spatial variability parameters of field measurements such as the scale of fluctuation associated with the coefficient of variation in the analysis and design process has not been considered. In order to obtain a more reliable design of foundations compared to the previous studies, a quantitative approach based the cone penetration test (CPT) data to examine the inherent soil variability and uncertainty involved in the determination of design soil property has been proposed. In this approach, a method has been developed to evaluate the spatial variability parameters. Results obtained from the proposed approach are used for estimating more reliable undrained shear strength of clays in order to obtain a more reliable foundation design.

## 2. QUANTIFYING SOIL VARIABILITY

In the analysis of geotechnical problems, it is common to model the soil profile at a site in terms of homogeneous layers with constant soil properties. It is assumed that the idealised profile is characterized by a set of average values and the fluctuation about these values is neglected. However, being naturally formed materials, engineering soil properties may exhibit considerable variation from point to point. In a probabilistic soil profile, at least one of the characteristics is treated as a random function of one or more of the co-ordinates. Recent approaches based on a random field model (Baecher 1984, Tang 1984, Fung 1998) proposed by (Vanmarke 1983) provide statistical procedures for capturing the variable nature and interdependence of soil properties. Practical interests in the use of average soil properties for design that is common in geotechnical engineering makes those statistical procedures particularly useful. In this investigation, soil profiles are modelled by a random field model and the values of CPT, cone tip resistance $q_c$ along each soil profile are treated as separate random variables. Consideration is limited to the variation of CPT values in the vertical and horizontal direction within the clay and silty clay layers. In order to have an adequate description of the spatial variability of the soil profiles, two parameters, namely, the coefficient of variation $V$ and the scale of fluctuation $\delta$ based on CPT data have to be evaluated.

### 2.1 Coefficient of Variation

The coefficient of variation $V$ of a soil property is an useful indicator which is not only for characterizing the inherent variability of soils but also for quantifying individual components of uncertainty and variability involved in the testing program and design procedures. It is a dimensionless parameter and is expressed by the ratio of the standard deviation to the mean. The coefficient of variation provides a more stable measure of consistency than its constituents represent the variability of soil properties. Once the coefficient of variation of a soil property is estimated, the data can provide a potential benefit in the design of geotechnical projects and in the evaluation of their reliability effectiveness.

2.2 Evaluation of scale of fluctuation

The similarity in value for soil property at closely neighbouring locations can be described by using the scale of fluctuation $\delta$. The scale of fluctuation gives an indication of the degree of variability of a soil profile. Figure 1 illustrates the meaning of this parameter:

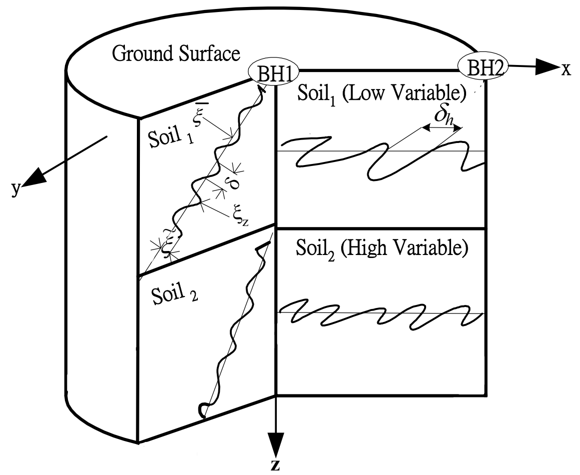


Figure1. Scale of fluctuation of a soil property

where $\delta$ represents the scale of fluctuation varied with depth and $\delta_h$ represents the horizontal scale of fluctuation. It shows that the soil property $\xi_z$ fluctuate about its mean value $\bar{\xi}$ with its standard deviation $\tilde{\xi}$ of the entire layer. A highly variable profile will have a low $\delta$ while a slowly varying profile will result in high $\delta$. The scale of fluctuation of any stratum is referred to as the distance within which the soil property shows relatively strong correlation. In view of this fact, $\delta$ is defined as correlation distance in this study.

The correlation distance $\delta$ is derived from the variance function $\Gamma^2$ (Vanmarke 1977) which adequately explains the effects of spatial averaging and is defined as follows:

$$\Gamma^2 = \left( \frac{\tilde{\xi}_z}{\tilde{\xi}} \right)^2 \qquad [1]$$

where $\tilde{\xi}_z$ is the standard deviation of spatial averages of sub-layers of different thickness, $z$. For the current situation, the CPT data are analyzed along a borehole, a linear spatial average case, the data are first considered in pairs ($n$ = 2) and a moving average series for the data are obtained where the length of averaging will be equal to the spacing of data points, ($Z_2$). The standard deviation $\tilde{\xi}_2$ of this series is also calculated. $\tilde{\xi}_2$ is lower than the standard deviation of the original data set $\tilde{\xi}$ due to the cancel out of fluctuations due to spatial averaging. The above procedure is extended to the case $n$ = 3, and the corresponding standard deviation of the series $\tilde{\xi}_3$ is calculated with the spacing $Z_3$ being equal to twice the spacing of the original data points. This procedure is repeated for $n$ = 4, 6, 8….. until $n$ approached the total number of data, $M$. The effect of spatial averaging will be more significant with increasing $n$ with $\tilde{\xi} > \tilde{\xi}_2 > \tilde{\xi}_3 \ldots > \tilde{\xi}_M$ . Therefore, for each $n$, the variance function of cone tip resistance can be predicted from:

$$\Gamma^2(Z_n) = \frac{\tilde{q}_{cn}^{\,2}}{\tilde{q}_c^{\,2}} \qquad [2]$$

where $\tilde{q}_{cn}^{\,2}$ is the variance of the derived moving average series of degree $n$, and $\tilde{q}_c^{\,2}$ is the variance of the original data. If the spacing of the data is $\Delta z$ , for large values of n, these predicted values had approached the theoretical values given by (Vanmarke 1983):

$$\Gamma_n^2 n\Delta z \cong \delta \qquad [3]$$

Equation 3 was solved graphically by plotting the $\Gamma_n$ versus $n$ (Vanmarke 1977). However, the method of determining the $\delta$ proposed by Vanmarke is not straightforward and it involves trial and error procedures, hence considerable uncertainty will be involved. In view of this fact, a more reasonable method is proposed in this investigation for better estimation of $\delta$. Equation 3 can be written as:

$$\gamma_n = \delta - \Gamma_n^2 n\Delta z \qquad [4]$$

where $\gamma_n$ is the residual value and $\Gamma_n^2$ is the experimental values and $\delta$ is a deterministic constant. Therefore, $\delta$ can be estimated by minimising the sum of the squared errors:

$$\sum \gamma_n^2 = \sum_{n=1}^{M} \left( \delta - \Gamma_n^2 n\Delta z \right)^2 \qquad [5]$$

where $M$ is the total number of data points and $\Delta z$ is the depth interval, which satisfies:

$$\frac{d \sum \gamma_n^2}{d\delta} = 0 \qquad [6]$$

and leads to

$$\delta = \sum_{n=1}^{M} \left( \frac{\Gamma_n^2 n\Delta z}{M} \right) \qquad [7]$$

Table 1 presents the statistical evaluation of $\delta$, its uncertainty level $V(\delta)$ and the coefficient of variation of cone tip resistance $V(q_c)$ obtained from the proposed and Vanmarke's methods based on the CPT data obtained in Macau.

Table 1. Statistical evaluation of $\delta$ and $V$ for $q_c$

| Soil Types | Methods | $\delta$(m) | | | |
|---|---|---|---|---|---|
| | | $n$* | Range | *Mean* | $V(\delta)$ |
| Clay | - Vanmarke's | 42 | 0.34~0.48 | 0.46 | 0.35 |
| $V(q_c)$ = 0.12 | - Proposed | 42 | 0.46~0.50 | 0.48 | 0.11 |
| Sitly Clay | - Vanmarke's | 38 | 0.32~0.49 | 0.43 | 0.47 |
| $V(q_c)$ =0.28 | - Proposed | 38 | 0.42~0.45 | 0.44 | 0.15 |

* n means the number of cases(A case refers to the set of results from one particular soil layer within a site).

It is observed that the variability of $\delta$ obtained from the proposed method is relatively low and its values generally lie in a narrow range whereas the level of uncertainty of $\delta$ obtained from Vanmarke's approach is especially high and the mean values lie in a wider range compared to the proposed approach for both types of soils. Based on Equation 7 the correlation distances of soils can be evaluated more conveniently and uncertainty level of $\delta$ can be reduced compared to Vanmarke's approach.
Similarly, the horizontal correlation distance $\delta_h$ has also been estimated from the same CPT data derived by placing a set of CPT soundings at constant spacing perpendicular to the vertical sampling direction as shown in Figure 2.

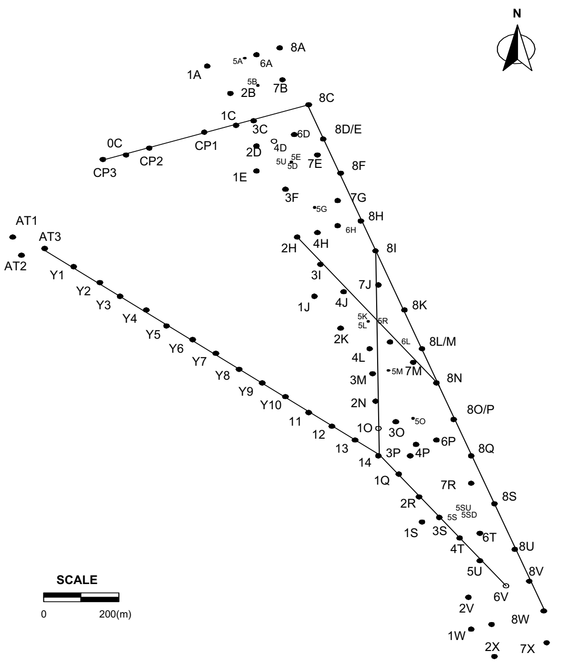


Figure 2. Typical planning for horizontal analysis

Estimation is proceeded to successive elevations separated by a predetermined depth layer by layer within the same profile and each soil profile is analyzed in the same way. The results of the horizontal analysis are shown in Table 2.

Table 2. Variability Parameters-Horizontal Analysis

| Soil Types | Clay | | Silty clay | |
|---|---|---|---|---|
| Depth (m) | $\delta_h$ (m) | $V_h(q_c)$ | $\delta_h$ (m) | $V_h(q_c)$ |
| 9-12 | 285 | 0.12 | 245 | 0.19 |
| 12-15 | 286 | 0.12 | 224 | 0.18 |
| 15-18 | 282 | 0.13 | 212 | 0.20 |
| 18-21 | 279 | 0.11 | 218 | 0.22 |
| Average | 283 | 0.12 | 225 | 0.20 |

It is observed that the values of $V_h(q_c)$ in each depth level are very close to each other although there is a gentle decrease in $\delta_h$ with depth. It shows that the variability of both types of soils in the horizontal direction is very small with depth.

2.3 Application of correlation distance

According to the fundamental requirement of sampling, in the statistical sense, is random selection of samples. This means that every element of the population has an equal chance of being sampled and therefore individual elements of the sample must be independent of each other. This selection mode is usually called representative sampling. Otherwise the information obtained will be somewhat redundant and therefore will not be informative as that obtained from the same number of independent samples. In this case, the correlation distance can give an indication of choosing how far the individual elements of the sample can be regarded as independent from each other's. Therefore, it is proposed that in order to avoid redundancy of information obtained, the sampling distances should be chosen at least equal or larger than the correlation distances such that the uncertainty involved in the estimation of a design soil property can be minimized. It should be pointed out that the values of correlation distance vary with different geological conditions and is assumed that if the geological conditions of the sites are similar, the correlation distance obtained from any location can be applied to other locations. Otherwise the correlation distance should be determined specifically by conducting a field test program. However, it may not be necessary to perform a specific field test program for the sake of the determination of correlation distance in different geological regions, if sufficient prior information of the field tests such as the CPT data obtained from previous project reports is available.

## 3. UNCERTAINTY EVALUATION OF UNDRAINED SHEAR STRENGTH

When designing a pile foundation, it is necessary to obtain an estimate of the undrained shear strength $s_u$ within a soil stratum. The correlation distance provides a useful guideline for estimating the undrained shear strength of soils in which the uncertainties can be minimized. In common practice, in the case of using in-situ measurements for evaluating the undrained shear strength of soils, empirical models are often used for transforming the in-situ measurements to the design parameters. Under this situation, additional uncertainty exists in those conversion models. This uncertainty associated with the conversion of test measurements to design parameters, refers to as conversion uncertainty. The conversion of a test measurement $\xi_m$ to design parameter $\xi_d$ derived from a reference test can be represented by a linear model as follows:

$$\xi_d = \psi\xi_m + \varepsilon \qquad [8]$$

where $\psi$ is the constant multiplier for conversion and $\varepsilon$ is a random variable representing the uncertainty of the transformation model. Fung (1998) proposed a probabilistic approach for the evaluation of uncertainty of correlation in conversion. This approach considers the usually deterministic conversion factor having a probability distribution with mean $\mu_\psi$ and variance $Var(\psi)$; that is the random nature of the model error $\varepsilon$ is incorporated into the correlation factor $\psi$ itself. In most cases, in evaluating the performance of geotechnical structures, emphasis is placed on the determination of average soil properties instead of individual point properties. Fung (1998) suggested that the *V* of the mean soil parameters $\bar{\xi}_d$ can be obtained from:

$$V^2(\bar{\xi}_d) \approx \frac{V^2(s)}{n} + V^2(b) + V^2(\psi) \qquad [9]$$

where *s* is the scatter component that includes the natural inherent soil variability and random effects *b* is the bias component representing the equipment, procedures and/or operator effects and *n* is the number of independent tests.

The undrained shear strength estimated from $q_c$ is commonly obtained from the standard relationship

$$s_u = \frac{q_c - \sigma_{v0}}{N_k} \qquad [10]$$

where $q_c$ is the cone tip resistance, $\sigma_{v0}$ is the total overburden stress and $N_k$ is the cone factor which is a constant varies between approximately 5 and 70 (Schmertmann 1975). Equation 10 is the conversion model for transforming the measured CPT data to the design undrained shear strength. In this study, the inverse of $N_k$ is referred to the conversion factor ψ and the distributions for $N_k$ have been found using conventional statistical methods.

In order to obtain $N_k$ for the local situation, reference tests for undrained shear strength should be conducted. In this study, the field Vane Shear Test, VST and Unconsolidated Undrained Test, UU test results have been adopted for the determination of the in-situ $s_u$ and laboratory $s_u$ as the reference strength for correlations. In order to minimize the uncertainty, the reference tests data were chosen close to the CPT soundings at corresponding depth. The purpose of using the reference strength is to establish the $N_k$ values in order to evaluate the design parameter and its uncertainty level. The statistical results for $N_k$ values and the level of uncertainty of $N_k$ obtained from the corresponding reference tests are presented in Table 3.

Table 3. Statistical evaluation of $N_k$ values

| Soil Type | $s_u$ (ref.) | n* | Mean | $V(N_k)$ |
|---|---|---|---|---|
| Clay | VST | 6 | 13 | 0.14 |
| | UU | 6 | 13 | 0.16 |
| Silty Clay | VST | 4 | 12 | 0.15 |
| | UU | 4 | 12 | 0.16 |

* n means the number of cases(A case refers to the set of results from one particular soil layer within a site).

With the statistical results of $N_k$ values obtained from the reference tests in this study, the level of uncertainty of design $s_u$ can be evaluated by using the conversion model. It is suggested by the author that in order to have more reliable design $s_u$, each data chosen for analysis should be at a distance equal or larger than the correlation distance. This approach not only facilitates for planning the site characterization programme but also for reliability-based design of foundation.

## 4. CONCLUSIONS

A statistical approach has been performed for dealing with the influence of the natural soil variability and the conversion model uncertainty on undrained shear strength evaluation. In order to achieve this purpose, a method has been developed for determining the correlation distance. It is recommended that the in-situ test data used for analysis and design foundations should be selected at a spacing equal or larger than the correlation distance.

It appears that useful guidelines may be established to minimize the uncertainties involved in the foundation design processes through the proper use of the correlation distance, coefficient of variation and the proper account for the uncertainty evaluation. The evidence indicates that the use of correlation distance for analysis can provide a reliable $s_u$ determination. It is believed that the proposed approach is valuable for planning of more optimum CPT site characterization and for the purpose of reliability based design of foundations.

5. REFERENCES


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