MONTECARLO SIMULATION TO EVALUATE SEISMIC SLOPE STABILITY

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ABSTRACT
In this research, deterministic equation of Bishop was improved by adding seismic loads. Based on a Monte Carlo simulation, the distribution of each input parameter was used with the deterministic performance equations to produce a probability distribution of the result of the analysis. The probabilistic method developed in this paper involved the application of the modified Bishop method for a finite slope with circular failure surface, using response surface analysis method combined with a Monte Carlo simulation. Seepage and seismic analysis were undertaken. Cohesion, friction angle and soil unit weight were considered as random variables. As far as sensitivity analysis is concerned, the friction angle proved to be a principal variable in seismic slope stability problems, in the absence of water table. A change in the probability of failure was measured for correlation coefficient $\rho_{c\phi}$ between –0.2 and –0.8.

RÉSUMÉ
Dans cette recherche, l'équation déterministe de Bishop est améliorée en ajoutant les charges sismiques. En se basant sur la simulation de Monte Carlo, la distribution de chaque paramètre d'entrée est utilisée avec les équations de comportement déterministes pour produire une distribution de la probabilité du résultat de l'analyse. La méthode probabiliste développée dans cet article implique l'application de la méthode modifiée de Bishop pour une pente finie, avec une surface de rupture circulaire, en utilisant la méthode d'analyse de surface de réponses, combinée avec une simulation de Monte Carlo. Les analyses comprennent les effets de l'eau et des séismes. La cohésion, l'angle de frottement et le poids volumique du sol sont pris comme variables aléatoires. Pour la sensibilité, l'angle de frottement apparaît comme une variable principale dans les problèmes sismiques, en l'absence d'une nappe phréatique. Un changement dans la probabilité de rupture est observé pour des coefficients de corrélation $\rho_{c\phi}$ allant de –0.2 à –0.8.

1. INTRODUCTION

Slope instability is a subject of major interest for geotechnical engineers in many parts of the world. Areas likely to be affected by land sliding are sometimes zoned by government authorities as being unsuitable for development for residential or industrial use. Assessment of landslide hazard is an important component of mitigation strategy because slope instability accounts for big economical loss as well as human casualties. Definitions for risk and hazard applicable to landslide risk assessment are established (Fell 1994). In Lebanese mountains, dealing with slope stability problems may be very frequent. Many landslides have recently occurred, resulting in destructive damage for the transportation infrastructure, agricultural territories, water supply, and irrigation facilities (Rahhal et al. 2003). As far as earthquakes are concerned, Lebanon is very known for its tectonic activity due to the fact that it is crossed in the east by the major fault of Yammouneh, extension of the Dead Sea fault, which continues northward to join the Taurus region in Turkey. No major landslide has yet been triggered by an earthquake; nevertheless, the risk remains present.

Most geotechnical systems are designed based on the fundamental concept of Capacity and Demand, using either the Safety Factor or Safety Margin. Variables involved in the analysis are: the physical characteristics of the soil, and the slope geometry. Geotechnical engineers are responsible for the analysis of slope stability, and their ability to exactly model slope performance is compromised by theoretical and practical factors. Traditional methods use principles of static equilibrium to evaluate the balance of driving and resisting forces. The engineers resort to an approach by factor of safety to reduce the risk of failure. This factor of safety is defined as the resisting forces divided by the driving forces or alternatively as the shear strength of the soil divided by the calculated shear stresses induced on the potential failure surface. However, the approach of the safety factor cannot measure the probability of failure taking into consideration the statistical scatter of geotechnical as well as geometrical properties of slopes (Duncan 2000, Christian et al. 1994). Probabilistic slope stability analyses are rational means to quantify and incorporate uncertainty into the design process (El Ramly et al. 2002, Li et al. 1993, Li and Lumb 1987).

Realistic estimates of the variability of soil parameters are needed for the development and application of the reliability based design (Phoon and Kulhawy 1999, Halim and Tang 1993). In geotechnical engineering, sources of uncertainty result from: spatial variation of soil properties, systematic error in the estimate of these properties at a point, and error in the analytical model (Lacasse et al. 2003, Whitman 2000, Christian et al. 1994). Scatter in the data, related to random error testing in the measurement, can be reduced by increasing the number of measurements (Mostyn and Li 1993). Systematic error consists of statistical ones, due to sampling process and bias in the measurement process. The variability of soil parameters should be evaluated as a function of inherent soil variability and measurement error (DeGroot 1996).

Model errors are another source of uncertainties that was treated by Whitman (2000).

The objective of this paper is to contribute to the understanding of Monte Carlo simulation applied to modified Bishop's method under seismic loading. A sensitivity analysis is undertaken and the effect of correlation between cohesion and friction angle on probability of failure is tested. Finally some conclusions are proposed.

2. THEORETICAL BACKGROUND

There are several techniques that can be used to evaluate geotechnical situations involving soil variability; the main probabilistic evaluations include: point estimate method, reliability assessment, and Monte Carlo simulations. These three methods will be discussed in the theoretical background.

First, the point estimate method is an approximate numerical integration approach to probability modelling. The evaluation of the point estimate method results in a single number for the sample data. This single number is representative of the sampled population. For slope analysis, input parameters are assumed to be normally distributed. The application of this methodology requires defining the acceptable level of risk.

Second, Reliability theory applied to slope stability analysis has been thoroughly explained by Christian et al. (1994). With the introduction of the concept reliability theory to limit state equilibrium, the conventional deterministic global factor of safety F_s lost its physical signification as far as risk evaluation is concerned. In other terms, it becomes impossible to realize a convincing comparative security study between different slopes based only on F_s. There are many definitions of a reliability index. The two most commonly used definitions are the first-order second moment reliability index β, and the Hasofer-Lind reliability index β_{HL}. The reliability index β_{HL} of Hasofer and Lind (1974), which ensures the invariance property, is directly related to the probability of reaching the limit state corresponding to failure in case of soils. This limit state representing failure state, is described by a performance function or a limit state function given by the equation F_s=1 or F_s-1=0, the mathematical expression of which is implicit, requiring hence iterations for its resolution. The relation between reliability index and factor of safety has been established. Reliability appears to be an important way to appraise risk, and hence is useful in any economical analysis. Reliability index is becoming more popular in geotechnical engineering (Rahhal and Moubarak 2000, Rahhal and Sherfane 2001, Rahhal and Germani 2003).

To avoid the difficulty of evaluating multidimensional integral, the probability of failure is estimated using Monte Carlo simulations. Probabilistic slope stability analysis based on Monte Carlo simulations has the advantage of being simple and not requiring a vast statistical and mathematical background (Chandler 1996). The simulation is a method by which a distribution of possible results is built by calculation of a deterministic model several times and using at each time random values of each input parameter. With more executed loops, the results become more precise. The slope problem is defined, available data is examined, and uncertainties in variables are described statistically by representative probability distributions. Random variables are described by probability distributions, with a wide available description in the modern software packages such as the uniform, triangular, normal, lognormal distributions. Statistical analysis of this distribution allows estimating the mean and the variance of the factor of safety and the probability of the factor of safety being less than 1.

The sampling is the process by which one of the values is chosen randomly from the probability distribution by the two most known methods: Monte Carlo and Latin Hypercube. Monte Carlo is the traditional method by which the values are chosen randomly on the whole distribution. This method needs a big number of loops to regenerate exactly the entry distributions and special problems arise when distributions are strongly biased. The Latin Hypercube is a stratified manner of sampling by which the cumulative distribution for each entry variable is divided in equal intervals. Then, samples are taken randomly, forcing each interval thus the sampling of the entire distribution. This approach requires a small number of loops for regenerating the entry distributions and is therefore a more effective method.

The simulations are run with computer softwares like @risk, which is an add-in for Microsoft Excel. This add-in permits treating a big quantity of data, and allows easy generations of the models. Even so, distributions for geotechnical variables must be assumed and consequently the results are as much exact as these initial hypotheses are.

3. METHODOLOGY DEVELOPMENT

A methodology is developed to apply risk analysis for slope stability. Deterministic equations are used to predict factor of safety. These equations are structured within a Monte Carlo simulation to perform the calculations. The modified Bishop method, applied to a finite slope with a circular failure surface, is considered. If r is the radius of the slip circle and L_n the arm lever arm between the centre of slip circle and the centre of the n^{th} slice, and if the resultant of tangential forces ΔT is considered equal to 0, then the proposed factor of safety is expressed by:

[1] $$FS=\frac{\sum_{n=1}^{p}\left\{\left[\left(1\pm K_v\right)\times W_n-u_n b_n\right]\times tan\phi+c\times b_n\right\}\times\frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{p}\left[W_n.r.sin\alpha_n+K_h.W_n.\frac{L_n}{r}\right]}$$

[2] with $$m_{\alpha(n)}=cos\,\alpha_n+\frac{tan\,\phi\times sin\,\alpha_n}{FS}$$

where c is the cohesion, ϕ the friction angle and where W_n, b_n and u_n are respectively the weight, the width and the pore pressure of the n^{th} slice. K_h and K_v are the horizontal and vertical pseudostatic seismic coefficients, and α_n is the angle formed by W_n and a perpendicular line to slip surface. The earthquake effects are taken into consideration by means of constants K_h and K_v. It is to be noted that that the term "Factor of safety" appears on both sides of the equation. To find the minimum value of FS, an iterative calculation is needed. The analysis of a finite slope with a circular surface of rupture by the method of Bishop demands iterative solutions. Consequently, the application of the Monte Carlo simulation is not direct, and it is necessary to use the response surfaces method. This is a tool that was chosen to overcome the incompatibilities between the method of Bishop and a Monte Carlo simulation (Wong 1985). For a slope and a given ground, 2^n combinations of entry parameters will be chosen to represent the extreme cases, n being the number of variable parameters. The form of the regression model is as follows:

[3] $FS = a_1 + a_2 X_1 + \ldots + a_{n+1} X_n + a_{n+2} X_1 X_2 + \ldots + a_{2^n-1} X_1 \ldots X_{n-1} + a_{2^n} X_1 \ldots X_n$

FS being the factor of safety, *a* being the regression constant, and finally *X* the variable. The entry parameters for a Monte Carlo simulation fall in two categories: the used deterministic parameters for a conventional analysis and the parameters that define the distribution of variables. For each of these parameters, the Monte Carlo simulation demands the definition of the descriptive statistics that characterize the distribution of the parameter. Following the nature of the distribution, this could include the maximum, the minimum, the average, and the standard deviation. The maximum and minimum are necessary to avoid very small or very large results in the factor of safety. It should be noted that, generally, a symmetrical distribution like the normal is assumed for the variable (Motsyn and Li 1993; El-Ramly 2002). However, a common approach is to assume that FS follows a lognormal distribution (Hassan and Wolff 1999, Duncan 2000). Assuming a lognormal distribution of the factor of safety does not mean that the values of the individual variables must be distributed in the same way. A lognormal distribution avoids negative factors of safety, because, in practice, the probability that a negative factor of safety will arise is insignificant. Christian and Baecher (2001) indicate their preference for the normal distribution of the factor of safety. Assuming a lognormal distribution of factor of safety can be either less conservative or more conservative than assuming a normal distribution, depending on the most likely value of factor of safety and the coefficient of variation.

4. ANALYSIS AND DISCUSSION

In this study, a typical clayey slope of 50% (20 m horizontally and 10 m vertically) is considered. The geotechnical properties of the slope are cohesion c'=28.5 kN/m^2, angle of internal friction ϕ'=20° and unit weight γ =19 kN/m^3. A phreatic surface exists in the slope, which yields to slightly different geotechnical properties under the water table. K_h and K_v are considered equal to 0.2 and 0.13 respectively, and K_v is a function of K_h. The cohesion, the friction angle, the soil unit weight and the horizontal pseudo static seismic force are considered variables, while all geometric parameters are supposed deterministic parameters. The adopted distributions of variables are listed in Table 1. The response surface method is adopted by applying the methodology presented earlier, and equation (1) is solved 16 times using spreadsheet programs. The constants a_i are then determined by linear regression, and the new equation to which Monte Carlo simulation is applied is expressed by:

[4] $FS = a_1 + a_2 c + a_3 \phi + a_4 \gamma + a_5 K_h + a_6 c\phi + a_7 c\gamma + a_8 cK_h + a_9 \phi \gamma + a_{10} \phi K_h + a_{11} \gamma K_h + a_{12} c \phi \gamma + a_{13} c \phi K_h + a_{14}\gamma K_h + a_{15} \phi \gamma K_h + a_{16} c \phi \gamma K_h$

In Table 2, both deterministic and probabilistic results are presented for different situations. First, it is clear that a higher slope (12 m instead of 10 m) proves to be less secure than a lower slope. As far as water effect is concerned, the depth of water table is considered to be at 4 m below surface. Also in Table 2, the water does not seem to affect the slope stability as much as seismic loading does: in fact, without seismic loading the probability of failure P_f is lower than 10^{-13} while it increases to a value higher than 3x 10^{-2} in the presence of seismic loads. On the other hand, it is interesting to note that, in parallel to this high change in P_f, the factor of safety varies only from 1.75 to 1.12, hence proving the importance of considering probabilistic analysis. In the presence of seismic loading, deterministic calculations yield values of FS close to 1 (associated to high probabilities of failure) implying a failure condition. A larger scatter in the variables will induce a larger difference in P_f for almost the same factor of safety

Table 1. Distributions and Statistic Parameters of Variables

Parameter	Distribution	X_{min}	μ[X]	X_{max}	σ [X]	Units
Cohesion	Normal	24	28.5	33	1.5	kPa
Friction Angle	Normal	16	20	24	2	Degree
Unit Weight	Normal	16	19	22	1	kN/m^3
Horizontal Seismic Coefficient	Exponential	0	0.2	1	-	-

Table 2. Reliability Index, Probability of Failure, and Deterministic Factor of Safety

Situation	μ[FS]	σ[FS]	β	P_f Normal distr. of FS	P_f Lognormal distr. of FS	FS
Without water, Without seismic loading (h=12m)	1.566	0.0883	6.417	$6.9 * 10^{-11}$	0	1.51
Without water, Without seismic loading (h=10m)	1.832	0.1003	8.297	10^{-15}	0	1.75
Water table, Without seismic loading (4m)	1.706	0.0966	7.311	$1.3 * 10^{-13}$	0	1.71
Without Water table, seismic loading (0.2g)	1.489	0.2655	1.841	0.0327	0.0153	1.12
Without Water table, seismic loading (0.3g)	1.448	0.2774	1.615	0.0530	0.0317	1.02
Water table, seismic loading (0.2g)	1.322	0.2817	1.142	0.1267	0.1114	0.86

Figure 1 shows a sample distribution of FS in the case of the presence of water without seismic loading. The probabilities of failure are calculated with the assumption of a normally distributed factor of safety. Using log-normally distributed results leads to the same conclusions. During the analysis, the level of the phreatic surface was changed. The authors, to understand the water effect on slope stability, are carrying out more work.

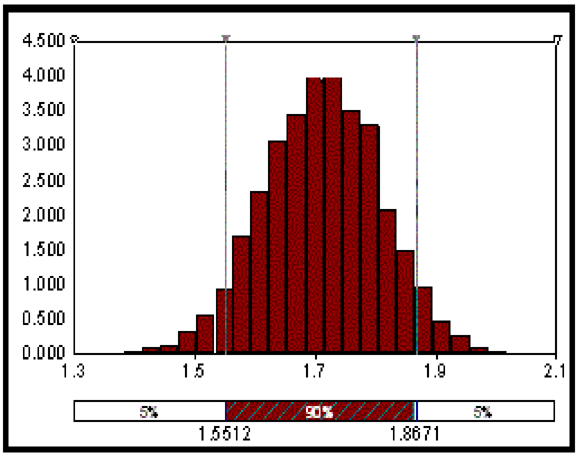

Figure 1. Example of Factor of Safety Distribution in the Presence of Water.

A regression and correlation sensitivity study shows that, without water, the seismic pseudostatic coefficient K_h has the largest effect on the result followed by the friction angle effect (Figure 2). Unit weight of the soil and cohesion, have approximately the same importance. It should be noted that the seismic coefficient K_h and γ are negatively correlated with FS in opposition with c and ϕ. Figure 2 shows a sample tornado diagram obtained with a regression sensitivity analysis, when studying the defined clayey slope with a 0.2g pseudostatic seismic acceleration but without water table. Figure 3 shows a tornado diagram when studying the same clayey slope under 0.2g seismic acceleration, but with a water table. In the presence of water the effect of cohesion becomes more significant than the one of friction angle (Figure 3).

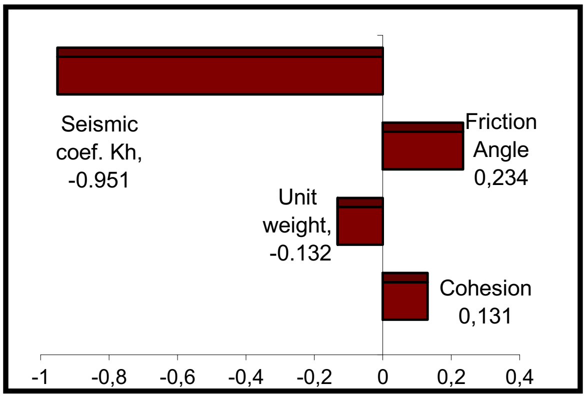

Figure 2. Regression Sensitivity Analysis in the Presence of Earthquake Acceleration of 0.2g without Water Table.

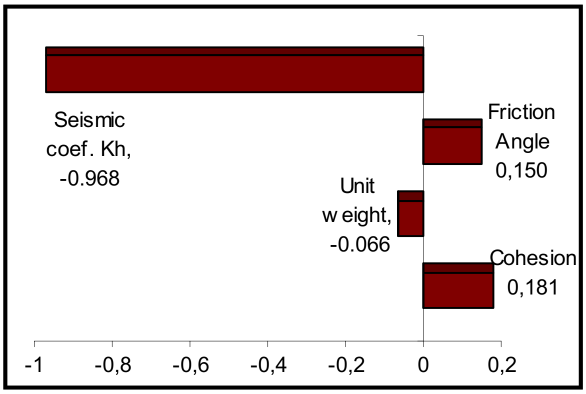

Figure 3. Regression Sensitivity Analysis in the Presence of Earthquake Acceleration of 0.2g with Water Table.

Table 3. Regression and Correlation Sensitivity of Input Parameters

Situation	Parameter	Regression	Correlation
Without water, Without seismic loading (h=12m)	Angle of Friction	0.7309736	0.7350132
	Soil Unit Weight	-0.499766	-0.4736193
	Cohesion	0.4573754	0.4459199
Without water, Without seismic loading (h=10m)	Angle of Friction	0.694035	0.679988
	Soil Unit Weight	-0.54394	-0.51712
	Cohesion	0.48794	0.448607
Water table, Without seismic loading (4m)	Angle of Friction	0.727753	0.721109
	Soil Unit Weight	-0.54437	-0.52319
	Cohesion	0.419225	0.402977
Without Water table, seismic loading (0.2g)	Seismic Coefficient	-0.951188	-0.9286533
	Angle of Friction	0.2338549	0.2412419
	Soil Unit Weight	-0.132638	-0.1605124
	Cohesion	0.1309694	0.123904
Without Water table, seismic loading (0.3g)	Seismic Coefficient	-0.946312	-0.9488386
	Angle of Friction	0.2169409	0.2306939
	Soil Unit Weight	-0.129417	-0.1679331
	Cohesion	0.1256122	0.1388151
Water table, seismic loading (0.2g)	Seismic Coefficient	-0.96846	-0.96062
	Angle of Friction	0.149842	0.149928
	Soil Unit Weight	-0.06617	-0.06739
	Cohesion	0.180781	0.195902

The sensitivity analysis shown in Table 3, demonstrates the influence of the input parameters on the output of the Monte Carlo simulations. Two types of sensitivity analysis, regression and correlation are performed. For regression analysis, a normalized standard regression coefficient is determined for each input variable distribution, with a value varying from –1 to +1: a value of zero indicates no significant relationship between input and output distributions. A standard regression coefficient of one means there is a one standard deviation change in the output distribution for a one standard deviation change in the input distribution. As for the correlation analysis, it describes the strength of the relationship between input and output distributions. A correlation coefficient of zero indicates that the input and the output distributions are independent.

The sensitivity analysis conducted using regression and correlation analyses indicates that, in the absence of seismic loading, the friction angle becomes the principal variable in both the regression, and the correlation sensitivity analyses. In the presence of seismic loading, Table 3 shows that the horizontal seismic coefficient is the most predominant variable. As far as mechanical properties are concerned, under seismic loading, the friction angle has the most critical distribution in the absence of water, and the effect of cohesion becomes more significant in the presence of water table. The sensitivity analysis confirms the expected significance of each input parameter.

The effect of correlation between cohesion and friction angle on probability of failure is tested using the discussed methodology. Results are shown in Figures 4 and 5. For seismic loading without water table, a variation of $\rho_{c\text{-}\phi}$ between -0.8 and -0.4 leads to an increase of the probability of failure from 0.0284 to 0.0319, which means 12% of its value. For a seismic loading with water table, the variation of $\rho_{c\text{-}\phi}$ between -0.6 and -0.2 produces an increase of 2.3% of the probability of failure. When the correlation coefficient becomes more negative, the probability of failure decreases. The highest probability of failure is obtained for uncorrelated variables.

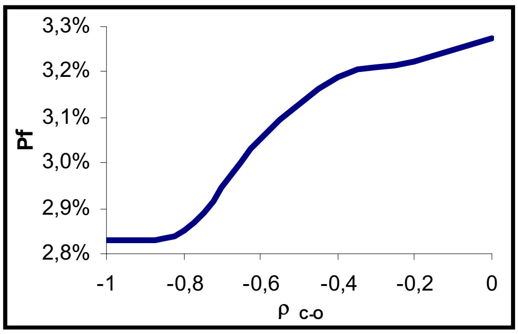

Figure 4. Probability of Failure as a Function of Correlation Coefficient $\rho_{c\phi}$ in the Presence of Earthquake Acceleration of 0.2g without Water Table.

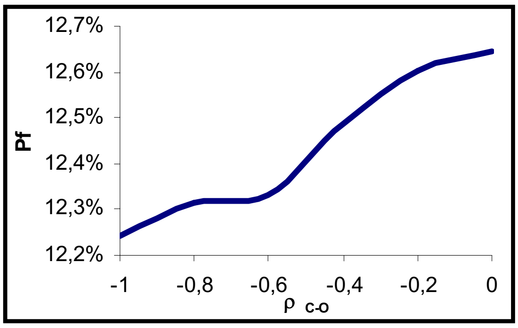

Figure 5. Probability of Failure as a Function of Correlation Coefficient $\rho_{c\phi}$ in the Presence of Earthquake Acceleration of 0.2g with Water Table.

5. SUMMARY AND CONCLUDING REMARKS

The aim of this paper is to understand a Monte Carlo simulation applied to a finite slope with circular failure surface, using a modified Bishop deterministic analysis incorporating the seismic load, in conjunction with a response surface method. The importance of considering uncertainties is outlined when comparing probabilities of failure of probabilistic and deterministic calculations. Results show the large effect of seismic loading in increasing drastically the probability of failure. This is not quite obvious when considering the factor of safety alone. In regression and correlation sensitivity analyses, the friction angle proves to be the most critical variable in the absence of seismic loading. Under seismic loading, friction angle remains a principal variable, but with the presence of a water table as well, cohesion becomes the most critical mechanical property.

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