TRANSIENT SEEPAGE ANALYSIS OF RAINFALL INFILTRATION USING A NEW CONJUNCTIVE SURFACE-SUBSURFACE FLOW MODEL

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ABSTRACT
Rainfall infiltration is one of the major triggering factors leading to slope failures. As rainwater infiltrates into an initially unsaturated soil slope, it would gradually reduce negative pore water pressure in the slope and weaken the slope stability. Most of subsurface flow models require the specification of infiltration amount (i.e., coefficient of infiltration) in advance. In reality, variations of infiltration amount during a storm event cannot be estimated easily unless interactions between rainfall, infiltration, and surface runoff processes are properly considered and modeled. In this paper, a new conjunctive surface-subsurface flow model is developed to consider the interaction between surface and subsurface flows to analyse transient seepage in initially unsaturated soil slopes. Comparisons between computed pore water pressure distributions from uncoupled and coupled procedures are presented and discussed. The implications of the differences in pore water distribution on slope stability are illustrated.

RESUME
Analyse d'écoulement transitoire de l'infiltration de la pluie en utilisant un nouveau modèle d'écoulement traitant conjointement l'écoulement de surface et l'écoulement sous la surface.

1. INTRODUCTION

Landslides cause enormous economic losses and threaten public safety in many countries around the world. Among various mechanisms causing landslides, rainfall infiltration is one of the major triggering factors leading to slope failures. Infiltration is a process by which rainwater moves downward through the earth surface to the soil underneath. As rainwater continues to infiltrate into an initially unsaturated soil slope, it would gradually reduce negative pore water pressure (suction) in the slope and weaken the slope stability. Therefore, a realistic modeling and predictive tool is essential to analyse transient seepage and pore pressure distributions in both saturated and unsaturated soil slopes. Many conventional approaches in modeling subsurface flow are to uncouple infiltration and runoff processes during a storm event. For simplicity, runoffs are assumed to be discharged instantly and the surface water depth can be thought as zero. Lam et al. (1987) is among the earliest researchers to apply finite element method to solve saturated-unsaturated seepage problem in geotechnical engineering. Ng and Shi (1998) conducted the parametric study on stability of unsaturated soil slope subjected to transient seepage using the two-dimensional finite element program SEEP/W. Furthermore, Tung et al. (1999) and Ng et al. (2001) performed a three-dimensional numerical investigation of rainfall infiltration in an unsaturated soil slope subjected to various rainfall patterns. In most of these subsurface flow models infiltration amount (or coefficient of infiltration) is specified in advance. In reality, variations of infiltration amount during a rainstorm event cannot be estimated easily unless interactions between rainfall, infiltration, and surface runoff processes are properly considered and modeled.

The coupled process of surface-subsurface flow is initially investigated by hydraulic and hydrological engineers. Akan and Yen (1981) developed a conjunctive one-dimensional surface flow and two-dimensional subsurface flow model. Recently, Morita and Yen (2002) further presented a conjunctive two-dimensional surface flow and three-dimensional subsurface flow model. Regarding surface flow modeling in hydraulic engineering, channel slopes are rather small as compared with the slope angle of natural, fill, and cut slopes encountered in geotechnical engineering. Hence, the surface flow governing equations should be modified to accommodate the steep slope condition. Kwok (2003) investigates the coupled surface-subsurface flow in steep slopes using finite difference method. While hydraulic engineers mainly concern with water balance in the flow analysis, geotechnical engineering applications are more interested in the spatio-temporal distribution and variation of pore water pressure.

In this paper, a new conjunctive surface-subsurface flow model is developed to consider the interaction between surface and subsurface flows in transient seepage analysis of initially unsaturated soil slopes. Comparisons between computed pore water pressure distributions from uncoupled and coupled analysis will be presented and discussed. The implications of the differences in pore water distribution on slope stability will be illustrated.

2. CONJUNCTIVE SURFACE-SUBSURFACE FLOW MODEL

2.1 Surface flow submodel

The basic equations describing gradually varied unsteady shallow water flow are commonly known as the Saint Venant equations, which are developed for mild slopes. In order to describe the behavior of surface flow on steep slope, Yen (1977) modified governing equations to accommodate the rapid varied unsteady flow condition using a new coordinate system as shown in Figure 1.

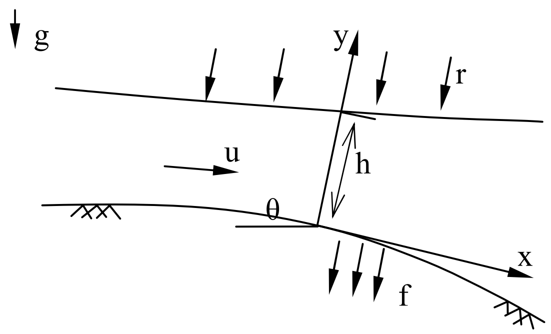

Figure 1. Description for variables in Saint Venant equations

For one-dimension surface flow, the modified governing equations are as follows:

$$\begin{aligned} &\frac{\partial h}{\partial t}+\frac{\partial (uh)}{\partial x}=r-f \\ &\frac{\partial (uh)}{\partial t}+\frac{\partial (u^2 h)}{\partial x}+gh\cos\theta\frac{\partial h}{\partial x}=gh(S_0-S_f) \end{aligned} \qquad [1]$$

where positive x-axis direction is along the ground surface downwards; y-axis is perpendicular to x-axis as illustrated in Figure 1; t is time; h is the surface water depth; u is the mean velocity along *x*-axis; θ is the slope angle measured from the horizontal plane; S_0=sinθ, the bottom slope; S_f is the friction slope; r is the rainfall intensity or lateral inflow; f is the infiltration rate; and g is the gravitational acceleration.

To determine the friction slope S_f in Equation 1, Darcy-Weisbach equation is used by Morita and Yen (2002), i.e.,

$$S_f = f_d \frac{u^2}{8gh} \qquad [2]$$

where f_d is the frictional resistance coefficient, which is a function of Reynolds number, R_e=(uh)/ ν, with ν being the kinematic viscosity, and can be determined by

$$f_d = \left[24+660\left(\frac{r}{\sqrt[3]{g\nu}}\right)^{0.4}\right] \Big/ R_e \qquad [3]$$

for laminar flow with R_e<500;

$$f_d = \frac{0.223}{R_e^{\;0.25}} \qquad [4]$$

for transitional flow with 500<R_e<30000; and

$$f_d = \frac{1}{4}\left[-\log\left(\frac{k_s}{12R}+\frac{1.95}{R_e^{\;0.9}}\right)\right]^{-2} \qquad [5]$$

for turbulent flow with R_e>30000, where k_s is the equivalent sand grain roughness size and R is the hydraulic radius of the flow.

2.2 Subsurface flow submodel

Saturated-unsaturated subsurface flow is governed by Richards' equation. Two-dimensional Richards' equation with total head as the main variable can be expressed as:

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial H_w}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial H_w}{\partial y}\right)+Q=m_w\gamma_w\frac{\partial H_w}{\partial t} \qquad [6]$$

where m_w= $\partial\theta_w / \partial u_w$ representing the slope of soil-water characteristic curve; u_w is the pore water pressure; θ_w is the volumetric water content; γ_w is the unit weight of water; k_x and k_y are the hydraulic conductivity in x- and y-direction, respectively. In saturated zone, k_x and k_y are the saturated hydraulic conductivity in each direction, θ_w is the saturated volumetric water content, and m_w is equal to zero under the assumption of incompressible of both water and soil structure. While in unsaturated zone, k_x, k_y, θ_w and m_w all vary with pore water pressure u_w. Given the boundary and initial conditions, Richards' equation can be solved numerically by finite element method or finite difference method.

2.3 Conjunctive surface-subsurface flow model

Conjunctive surface-subsurface flow model consists of surface flow governing equations (Equation 1) and subsurface flow governing equation (Equation 6) through interface infiltration. Two sets of equations are solved separately and iteratively during the same time step to determine infiltration following the algorithm shown in Figure 2.

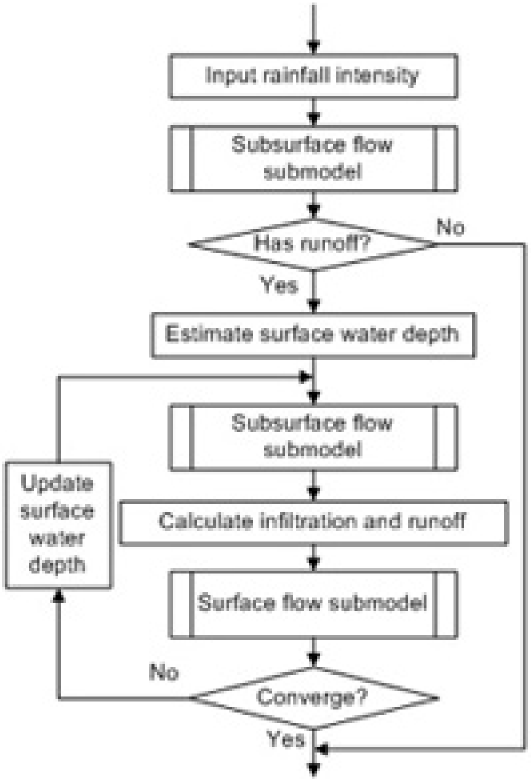

Figure 2. Flow chart of solving conjunctive surface-subsurface flow model

Firstly, apply full rainfall intensity at a time step to slope surface as flux boundary with which the subsurface flow submodel solves for pore water pressure distribution. If the positive water pressure does not occur on slope surface, the slope surface is not saturated and all rainwaters infiltrate into the slope. Otherwise, the slope surface is saturated and not all rainwaters can enter the slope, then surface runoff will occur under such circumstance. To estimate surface water depth, Equation 7 is used (Morita and Yen, 2002).

$$Y = h + r\Delta t \qquad [7]$$

where h is the surface water depth at the previous time step; r is the rainfall intensity; Δt is the time increment. Before runoff generation, surface water depth h is set to be zero initially.

Secondly, use Y as head boundary to calculate subsurface flow once again. Infiltration rate through this head boundary interface can be back calculated. Rainfall intensity subtracting calculated infiltration rate yields corresponding runoff rate.

Then, by applying estimated runoff rate above, surface flow submodel is used to calculate the corresponding surface water depth. Comparing calculated surface water depth and previously applied head boundary, if they are not consistent with the given precision requirement, update the head boundary according to newly estimated surface water depth and re-solve subsurface flow again. The process is repeated until convergence is reached before switching to the next time step.

3. NUMERICAL SIMULATION

3.1 Introduction of the programs

3.1.1 Conjunctive surface-subsurface flow analysis program

Based on the conjunctive one-dimensional surface and two-dimensional subsurface flow model presented above, a Fortran program is written to solve coupled processes of rainfall, infiltration and surface runoff on steep and mild slopes. In the program, one-dimensional surface flow governing equations (Equation 1) are solved by finite difference method. To achieve numerical stability and convergence, a four-point implicit finite difference scheme, known as Preissmann scheme (Chaudhry, 1993), is adopted. Two-dimensional subsurface flow governing equations (Equation 6) are solved by Galerkin finite element method. For numerical stability, lumped mass matrix method is adopted. Weighted spline is used to fit the nonlinear soil-water characteristic curve and hydraulic conductivity function. In addition, the program can automatically adjust the boundary conditions for different surface flow types, i.e., for supercritical flow, two upstream boundary conditions will be given; while for subcritical flow, one upstream boundary and one downstream boundary condition will be given.

3.1.2 Two-dimensional saturated-unsaturated seepage analysis program SEEP/W

SEEP/W is a commercial two-dimensional finite element seepage analysis program which is an uncoupled model to solve rainfall-infiltration problem. It has only a subsurface flow analysis model which neglects the interactions between rainfall, infiltration, and surface runoff processes.

3.2 Examples and discussions

3.2.1 Pore water pressure distribution

Comparisons between computed pore water pressure distributions from SEEP/W and the coupled model are presented and discussed. As shown in Figure 3, the length of the slope is 100m; the slope angle is 15°; and the depth of calculation zone is 11m. Total 2200 elements are divided for subsurface flow domain with the mesh length of each element is Δz=0.25m vertical and Δx=2m parallel to slope surface. The grid for surface flow modeling is Δx=2m, the same as that of subsurface mesh along slope surface direction. The initial groundwater table is illustrated in Figure 3 which is obtained from a steady state condition, where the depths of groundwater table at top and toe of the slope are fixed at 10m and 7m, respectively. The slope surface is a flux boundary subjected to rainfall while all other boundaries are impermeable.

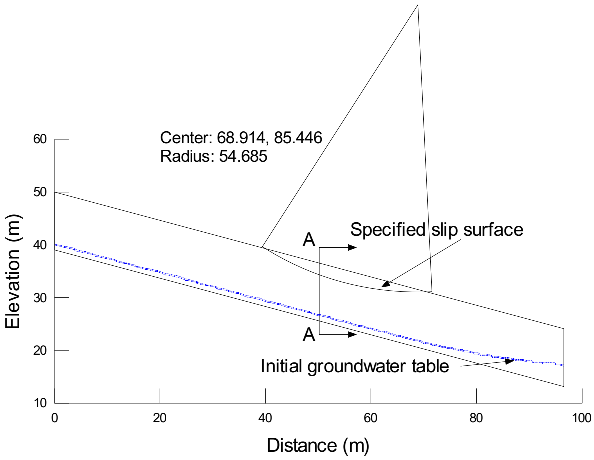

Figure 3. Calculation profile and initial groundwater table

The slope material is homogeneous fine sand with the saturated conductivity 4.3×10^{-6}m/s (15.5mm/hr), porosity 0.347, and soil-water characteristic curve and hydraulic conductivity function shown in Figure 4.

Table 1 lists the three cases of uniform rainfall intensity and duration used in the numerical simulation. The total rainfall applied in three cases is about 267mm which corresponds to a 10-year return period daily rainfall at Hong Kong Observatory

At the end of rainfall event, the surface water depths are different with various rainfall intensities. As shown in Figure 5, the surface water depth increases towards the downstream direction along the slope. In addition, given the same conditions except rainfall intensity, the surface water depth is deeper, as expected, under larger rainfall intensity. In Figure 5, the maximum surface water depths under 22, 45 and 89mm/hr rain are about 4, 6 and 10mm occurring at the end of storm, respectively. During rain storms, the surface water depth on a steep slope could vary from several millimeters to several centimeters depending on the rainfall intensity and slope angle.

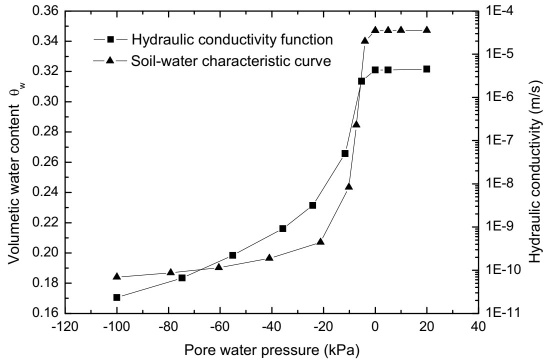

Figure 4. Soil-water characteristic curve and hydraulic conductivity function

Table 1. Rainfall characteristics of cases study

Cases	Rainfall intensity (mm/hr)	Duration (hrs)	Rainfall volume (mm)
1	22	12	264
2	45	6	270
3	89	3	267

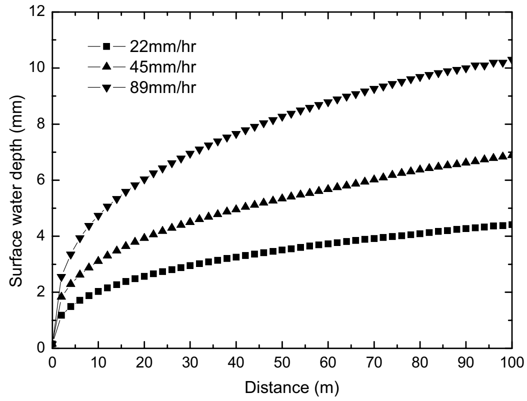

Figure 5. Surface water depth profiles under different rainfall intensities

Under rainfall intensity of 22mm/hr, comparison of pore water pressure distribution between coupled and uncoupled analysis at section A-A is shown in Figure 6. Without considering the interaction between surface runoff and infiltration, the wetting front from the uncoupled model reaches about 1.5m deep from the ground surface at the time rain stops. While by using coupled model, the wetting front reaches about 2.0m in depth, a half meter deeper than that of uncoupled model.

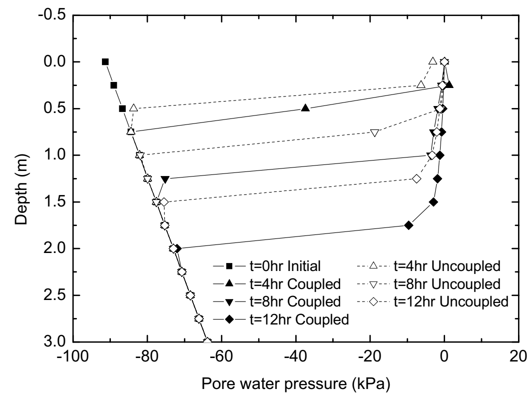

Figure 6. Vertical pore water pressure profile at section A-A under 22mm/hr rainfall intensity

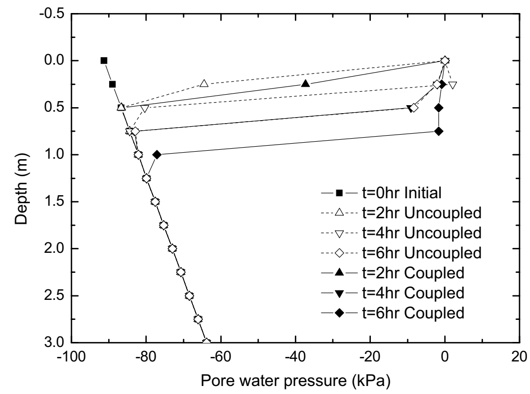

Figure 7. Vertical pore water pressure profile at section A-A under 45mm/hr rainfall intensity

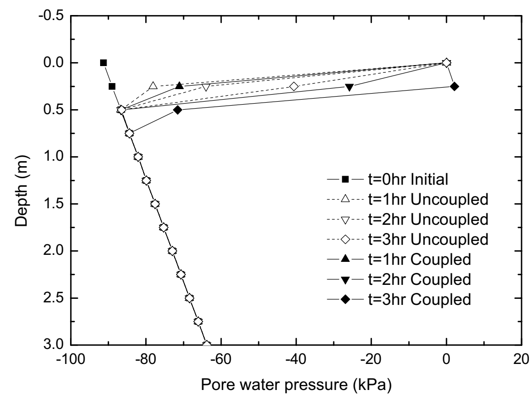

Figure 8. Vertical pore water pressure profile at section A-A under 89mm/hr rainfall intensity

As shown in Figure 7, the rainfall intensity of case-2 is twice as large as that of case-1. It is interesting to observe that the rainfall intensity increases, the runoff portion in total rainfall increases, whereas the corresponding portion of infiltration decreases. Under 45mm/hr rainfall intensity, the wetting front reaches about 1.0m deep from the ground surface according to the coupled model, which is about 0.3m deeper than the uncoupled procedure at the end of rainfall.

While under a heavy rainstorm of 89mm/hr, which is two times larger than that of case-2 and four times larger than that of case-1. As shown in Figure 8, most of the rainfall transforms to runoff and the remaining small portion infiltrates into the slope which affects the pore water pressure distribution only within 0.7m in depth. Regardless of interaction between surface and subsurface flow, the wetting front reaches about 0.5m in depth below the ground surface.

3.2.2 Slope stability analysis

As rainwater infiltrates into an unsaturated soil slope, pore water pressure increases with time and the corresponding matric suction decreases with the increase in water content. Based on the extended Mohr-Coulomb failure criterion for unsaturated soil (Fredlund et al., 1978), Equation 8 shows that the shear strength will decrease with loss in suction.

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \qquad [8]$$

To illustrate the variation of factor of safety with pore water pressure distribution change, a single circular slip surface is specified as shown in Figure 3, which is about 2.5m deep from the ground surface. The unit weight of soil is assumed to be 19.6kN/m^3, effective cohesion c'=0 kPa, effective friction angle φ'=20°, and the friction angle related to matric suction φ^b=8°. Using simplified Bishop method in SLOPE/W to perform the slope stability analysis, The results of are presented in Figure 9.

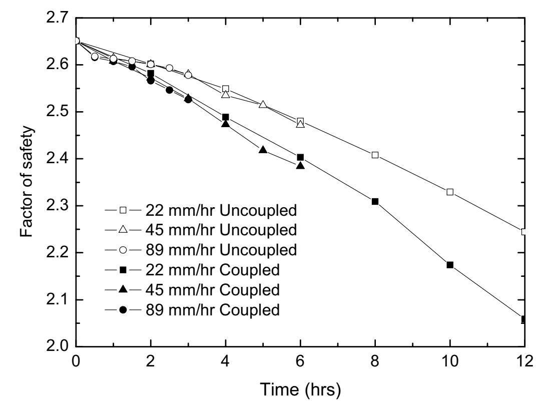

Figure 9. Variation of factor of safety with time under different rainfall intensities and models

The slope is initially dry and the factor of safety of the specified slip surface is high. As the rainwater infiltrates into the slope, the factor of safety decreases with decreasing in matric suction. Under heavy rainstorm with intensity of 89mm/hr, and rainfall of 3 hours, the wetting front only reaches 0.5-0.7m in depth, and the factor of safety reduces only about 0.1. The differences in pore water pressure between uncoupled and coupled models are not significant. Correspondingly, the difference in factor of safety between the two results after 3 hours rainfall is about 0.05. Thus, the factors of safety calculated by the uncoupled and coupled models have no significant difference either. When rainfall duration extends longer, the wetting front penetrates deeper and the difference in pore water pressure becomes more pronounced. Then, the factor of safety of slope drops more significantly as matric suction reduces. The differences in results between the uncoupled and coupled models become larger. After 12 hours rainfall with intensity of 22mm/hr, the factor of safety decreases by 0.5 and the differences in factor of safety between the uncoupled and coupled models are about 0.2.

4. CONCLUSIONS

A new conjunctive surface-subsurface flow model is developed to consider the interaction between surface and subsurface flows to analyse transient seepage for initially unsaturated soil slopes. Comparisons of computed pore water pressure distributions between uncoupled and coupled models are presented and discussed. The implications of the differences in pore water distribution on slope stability calculations are illustrated. Based on the studies above, the following conclusions can be drawn:

(1) Surface water depth on steep slope increases with the rainfall intensity, which is usually about several millimeters. Under heavy rainstorm, surface water depth can reach several centimeters.

(2) Interaction between surface-subsurface flow has significant effects on infiltration process. Considering the coupled surface-subsurface flow process, more rainwaters will infiltrate into the slope. Therefore, pore water pressure will change faster and the wetting front will move deeper into the soil.

(3) Under smaller rainfall intensity with prolonged duration, the differences in infiltration and pore water pressure between the conjunctive surface-subsurface flow model and uncoupled model are more significant.

5. ACKNOWLEDGMENTS

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