Evaluation of Footing Bearing Capacity via Energy Safety Factor

Xueqiang Yang Civil Engineering Department, Hubei Polytechnic University, Wuhan, China
Peijun Guo Civil Engineering Department, McMaster University, Hamilton, Canada
D.E.F.Stolle Civil Engineering Department, McMaster University, Hamilton, Canada

ABSTRACT

Based on the theory of limit analysis, this paper proposed a new approach to evaluate the bearing capacity of footing based on a factor of safety defined in terms of energy stability. Examples are presented to show the details of this approach. The results show that the energy-based safety factor have clearer physical meaning for designing the foundation engineering, and is easy to be applied to determine the bearing capacity for complicated boundary conditions. Allowable pressures such as q_{cr}, $q_{1/4}$ and $q_{1/3}$ are also studied using the energy-based factor of safety. Finally, discussion and examples are given to provide deeper insights into the meaning of the traditional safety factor from the viewpoint of energy stability.

RÉSUMÉ

Aucun résumé français fourni par l'auteur

1. INTRODUCTION

The concept of energy and energy analysis has been widely utilized in civil engineering, including the analysis of the stability of various structure systems. This approach can also be used for the study of bearing capacity of footings. Numerous researches have been carried out to determine the bearing capacity of footings under various conditions and resulted in many valuable results [1-4]. Conventionally, the factor of safety is defined in terms of the applied load or pressure with respect to the maximum resistance provided by soil masses. However, owing to the failure and the functionality of a structure is influenced by both the strength and deformation of soils, the factor of safety defined in the traditional way does not reflect the non-functionality related to large deformation of soil. A factor of safety defined based on energy may provide an alternate approach for this problem, and is a new try to explain the stability of the bearing capacity of soil mass, it provides a deeper understand about the bearing capacity of footing.

This paper attempts to develop a new method to determine the bearing capacity of strip footings, i.e. footing with large length to width ratios. The footing is assumed to be rigid while the interface between the soil and the footing is smooth. In addition, the soil is assumed to be an isotropic, homogeneous and elastic-perfectly plastic material. Mohr-Coulomb criterion is used to describe the failure condition while the plastic deformation is calculated by the associated flow rule. Based on the theory of limit analysis and the energy analysis, a factor of safety will be defined based on energy stability to determine the bearing capacity of strip footings under various conditions.

2. DEFINITION OF THE ENERGY SAFETY FACTOR AND ITS REASONABLE VALUES

2.1 Definition of The Energy Safety Factor

For the soil mass under footing, the energy safety factor FS is defined as a ratio between resistance work rate $\dot{W}_R$ and driving work rate $\dot{W}_D$ under some failure mechanisms, and is defined as

$$FS = \dot{W}_R / \dot{W}_D \qquad [1]$$

If c, φ, and γ are soil cohesion, internal friction angle, and unit weight respectively, q_0 is surcharge of the footing, the Prandtl mechanism consisting of five distinct zones, which is proven to be a reasonable mechanism in predicting bearing capacity of footing by theory and practice, is adopted here in Figure 1. The abc is translating vertically as a rigid body with the same downward velocity v_1 as the footing. The downward movement of the footing and wedge is accompanied by the lateral movement of the adjacent soil as indicated by the radial shear zone bcd and wedge bdef. According to the Figure 1, some relationships are given by Appendix 1.

As shown in Figure 1, velocities v_r, v_1, and v_3 are expressed as

$$v_r = \frac{\sin\zeta}{\cos(\zeta-\varphi)} v_o, \; v_1 = \frac{\cos\varphi}{\cos(\zeta-\varphi)} v_0,$$
$$v_3 = v_0 \exp(\Theta \tan\varphi) \qquad [2]$$

in which $\Theta = \pi + \beta - \zeta - \eta$.

Resistance work rate dissipated by cohesion c along bc is gotten as

$$\dot{W}_{R1} = c \cdot bc \cdot v_r \cos\varphi \quad [3]$$

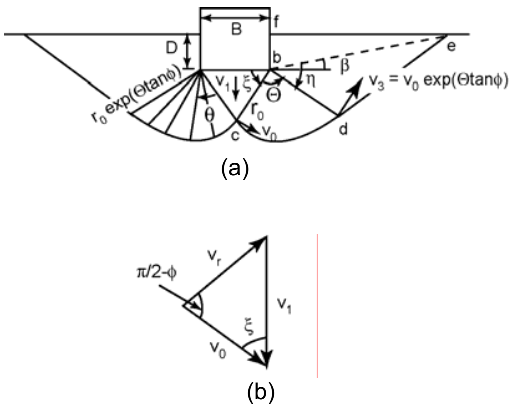


Figure 1. General Prandtl mechanism (from Chen 1975)

Resistance work rate dissipated by the radial shear zone bcd is written as

$$\dot{W}_{R2} = c v_0 r_0 \frac{\exp(2\Theta\tan\varphi) - 1}{\tan\varphi} \quad [4]$$

Resistance work rate dissipated by cohesion c along de is calculated by

$$\dot{W}_{R3} = c \cdot de \cdot v_3 \cos\varphi \quad [5]$$

Resistance work rate dissipated by the part weight of radial shear zone bcd is obtained as

$$\begin{aligned}\dot{W}_{R4} &= -\frac{\gamma}{2}\int_{\pi/2-\zeta}^{\pi+\beta-\zeta-\eta} r^2 v \cos(\theta+\zeta)\,d\theta \\ &= \frac{\gamma}{2} r_0^2 v_0 \{\frac{[3\tan\varphi\cos(\eta-\beta) - \sin(\eta-\beta)]\exp(3\Theta\tan\varphi)}{1+9\tan^2\varphi} + \\ &\quad \frac{\exp[3(\pi/2-\zeta)\tan\varphi]}{1+9\tan^2\varphi}\}\end{aligned} \quad [6]$$

Resistance work rate dissipated by weight of wedge bde is formulated as

$$\dot{W}_{R5} = W_{bde} v_3 \cos(\eta-\beta) \quad [7]$$

Resistance work rate dissipated by weight of wedge bef is expressed as

$$\dot{W}_{R6} = W_{bef} v_3 \cos(\eta-\beta) \quad [8]$$

Therefore total resistance work rate is obtained as

$$\dot{W}_R = \dot{W}_{R1} + \dot{W}_{R2} + \dot{W}_{R3} + \dot{W}_{R4} + \dot{W}_{R5} + \dot{W}_{R6} \quad [9]$$

At the same time, the driving work rate produced by the external pressure q is expressed as

$$\dot{W}_{D1} = qBv_1/2 \quad [10]$$

The driving work rate produced by weight of the wedge abc is written as

$$\dot{W}_{D2} = W_{abc} v_1/2 \quad [11]$$

The driving work rate produced by a part weight of the radial shear zone is gotten as

$$\begin{aligned}\dot{W}_{D3} &= \frac{\gamma}{2}\int_0^{\pi/2-\zeta} r^2 v \cos(\theta+\zeta)\,d\theta \\ &= \frac{\gamma}{2} r_0^2 v_0 \frac{\exp[3(\pi/2-\zeta)\tan\varphi] - \sin\zeta - 3\tan\varphi\cos\zeta}{1+9\tan^2\varphi}\end{aligned} \quad [12]$$

By adding up all terms associated with the driving work rate, one has

$$\dot{W}_D = \dot{W}_{D1} + \dot{W}_{D2} + \dot{W}_{D3} \quad [13]$$

For the Prandtl mechanism of Figure 1, corresponding energy safety factor is defined as

$$FS = \dot{W}_R / \dot{W}_D = FS(\zeta,\eta),\ FS_{\min} = FS(\zeta_{cr},\eta_{cr}) \quad [14]$$

The $FS_{\min}$ is furnished by the stationary conditions

$$\frac{\partial FS(\zeta,\eta)}{\partial\zeta} = 0,\ \frac{\partial FS(\zeta,\eta)}{\partial\eta} = 0 \quad [15]$$

Based on Eq. (14-1), $FS_{\min}$ is searched via an optimization strategy. The ζ, and η corresponding to the $FS_{\min}$ are called critical angles ζ_{cr}, and η_{cr} separately.

2.2 Reasonable Values of $FS_{\min}$

Reasonable values of $FS_{\min}$ can be evaluated by using of some known stability bearing capacity problems. Literature [4] points out that the soil mass under footing is stable when allowable pressures $q_{cr}, q_{1/4}$, and $q_{1/3}$ are met. Here, referring to Figure 2, the q_{cr} is a pressure that plasticity just begins at both points a and b, the $q_{1/4}$ is a pressure that corresponding maximum depth of plasticity zone developed under the footing is equal to $B/4$, and the $q_{1/3}$ is a pressure that its maximum depth of plasticity zone developed under the footing is equal to $B/3$. These allowable pressures are expressed as

$$q_{cr} = N_q \cdot \gamma D + N_c \cdot c \cdot \tan(\pi/2 - \varphi) \quad [16]$$

$$q_{1/4} = N_q \cdot \gamma D + N_c (c \cdot \tan(\pi/2 - \varphi) + \gamma B/4) \quad [17]$$

$$q_{1/3} = N_q \cdot \gamma D + N_c (c \cdot \tan(\pi/2 - \varphi) + \gamma B/3) \quad [18]$$

in which

$$N_q = \frac{\tan(\pi/2 - \varphi) + (\pi/2 + \varphi)}{\tan(\pi/2 - \varphi) - (\pi/2 - \varphi)},$$
$$N_c = \frac{\pi}{\tan(\pi/2 - \varphi) - (\pi/2 - \varphi)} \quad [19]$$

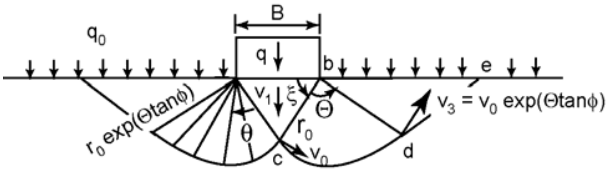


Figure 2. Idealized Prandtl mechanism

Using the idealized Prandtl mechanism showed in Figure 2, to above stable states for $q_{cr}, q_{1/4}$, and $q_{1/3}$, the reasonable $FS_{\min}$ values can be calculated by using of the Eq. (14), in which only $\dot{W}_{R6}$ is replaced by

$$\dot{W}_{R6} = q_0 \cdot be \cdot v_3 \cos(\eta - \beta)\cos\beta \quad [20]$$

in which $\beta = 0$.

Example 1: As shown by Figure 2, if $c = 20kN/m^2, \varphi = 30^\circ, B = 3m, \gamma = 18kN/m^3$ and $q_0 = 45kN/m^2$, different $FS_{\min}$ under different external pressure is searched based on Eq. (14). The results are listed as follows

(1) $q_{cr} = 410.33kN/m^2$, $FS_{\min} = 5.06$, $\zeta_{cr} = 54^\circ$, $\eta_{cr} = 30^\circ$, $\beta = 0^\circ$, $k = q_u/q_{cr} = 5.36$

(2) $q_{1/4} = 472.26kN/m^2$, $FS_{\min} = 4.44$, $\zeta_{cr} = 54^\circ$, $\eta_{cr} = 30^\circ$, $\beta = 0^\circ$, $k = q_u/q_{1/4} = 4.66$

(3) $q_{1/3} = 492.90kN/m^2$, $FS_{\min} = 4.26$, $\zeta_{cr} = 54^\circ$, $\eta_{cr} = 30^\circ$, $\beta = 0^\circ$, $k = q_u/q_{1/3} = 4.46$

(4) $q = 1084.70kN/m^2$, $FS_{\min} = 2.00$, $\zeta_{cr} = 53^\circ$, $\eta_{cr} = 30^\circ$, $\beta = 0^\circ$, $k = q_u/q = 2.03$

(5) $q_u = 2199.50kN/m^2$, $FS_{\min} = 1.00$, $\zeta_{cr} = 53^\circ$, $\eta_{cr} = 30^\circ$, $\beta = 0^\circ$, $k = q_u/q_u = 1.00$

Example 2: As shown by Figure 2, if $\varphi = 30^\circ$, $B = 3m, \gamma = 18kN/m^3$ and $q_0 = 45kN/m^2$, different $FS_{\min}$ under different c values can be calculated based on the Eq. (14). The results can be listed as follows

(1) $c = 10$, $q_{cr} = 330.88$, $FS_{\min} = 5.33$, $\zeta_{cr} = 53^\circ$, $\eta_{cr} = 30^\circ$, $\beta = 0^\circ$, $k = q_u/q_{cr} = 5.72$

(2) $c = 20$, $q_{cr} = 410.33$, $FS_{\min} = 5.06$, $\zeta_{cr} = 54^\circ$, $\eta_{cr} = 30^\circ$, $\beta = 0^\circ$, $k = q_u/q_{cr} = 5.36$

(3) $c = 30$, $q_{cr} = 489.73$, $FS_{\min} = 4.88$, $\zeta_{cr} = 55^\circ$, $\eta_{cr} = 30^\circ$, $\beta = 0^\circ$, $k = q_u/q_{cr} = 5.12$

Example 3: As shown by Figure 2, if $c = 20kN/m^2, B = 3m, \gamma = 18kN/m^3$, and $q_0 = 45kN/m^2$, different $FS_{\min}$ under different φ values can be calculated based on the Eq. (14). The results can be listed as follows

(1) $\varphi = 10^\circ$, $q_{cr} = 161.42$, $FS_{\min} = 1.84$, $\zeta_{cr} = 47^\circ$, $\eta_{cr} = 40^\circ$, $\beta = 0^\circ$, $k = q_u/q_{cr} = 1.95$

(2) $\varphi = 20^\circ$, $q_{cr} = 250.80$, $FS_{\min} = 2.85$, $\zeta_{cr} = 49^\circ$, $\eta_{cr} = 35^\circ$, $\beta = 0^\circ$, $k = q_u/q_{cr} = 3.03$

(3) $\varphi = 30^\circ$, $q_{cr} = 410.33$, $FS_{\min} = 5.06$, $\zeta_{cr} = 54^\circ$, $\eta_{cr} = 30^\circ$, $\beta = 0^\circ$, $k = q_u/q_{cr} = 5.36$

Based on calculating results of Examples 1~ 3, it is seen that:

(1) The reasonable energy safety factor value, which is not a constant, is influenced obviously by soil internal friction angle and is changed slightly by change of soil cohesion. For economic and safety purpose, the reasonable values of $FS_{\min}$ should be in the range $2.0 \le FS_{\min} \le 3.0$.

(2) For soil mass with higher φ value, the traditional allowable pressures such as $q_{cr}, q_{1/4}$, and $q_{1/3}$ give much more conservative designing results, therefore are not reasonable. Whereas for soil with smaller φ value, the allowable pressures $q_{cr}, q_{1/4}$, and $q_{1/3}$ are relative reasonable and economical.

2.3 General Prandtl Mechanism

For generalized Prandtl mechanism shown in Figure 1, its q value equivalent to q value corresponding to same $FS_{\min}$ of Figure 2 can be calculated.

Example 4: As shown by Figure 1, if $c = 20kN/m^2, \varphi = 30^\circ, B = 3m, \gamma = 18kN/m^3$ and $D = 2.5m$, different $FS_{\min}$ under different external pressure is searched based on Eq. (14). The results are listed as follows

(1) $q = 461.00$, $FS_{\min} = 5.06$, $\zeta_{cr} = 57^\circ$, $\eta_{cr} = 36^\circ$, $\beta = 8.77^\circ$, $k = q_u / q = 5.34$

(2) $q = 531.00$, $FS_{\min} = 4.44$, $\zeta_{cr} = 57^\circ$, $\eta_{cr} = 36^\circ$, $\beta = 8.77^\circ$, $k = q_u / q = 4.63$

(3) $q = 553.50$, $FS_{\min} = 4.26$, $\zeta_{cr} = 57^\circ$, $\eta_{cr} = 36^\circ$, $\beta = 8.77^\circ$, $k = q_u / q = 4.45$

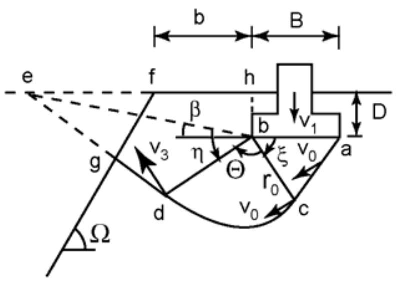
(4) $q = 1214.00$, $FS_{\min} = 2.00$, $\zeta_{cr} = 56^\circ$, $\eta_{cr} = 36^\circ$, $\beta = 8.91^\circ$, $k = q_u / q = 2.03$

(5) $q_u = 2460.00$, $FS_{\min} = 1.00$, $\zeta_{cr} = 56^\circ$, $\eta_{cr} = 36^\circ$, $\beta = 8.91^\circ$, $k = q_u / q_u = 1.00$

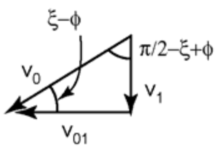
Based on above calculating results, it is seen that the embedding depth D of the footing can give a significant contribution to the bearing capacity of footing, we should consider the beneficial influence factor in our design for foundation engineering.

3 FURTHER APPLICATION OF ENERGY SAFETY FACTOR

3.1 Bearing Capacity Near Slope



(a)



(b)

Figure 3. Bearing capacity near slope

As shown in Figure 3, $ac, r_0, bd, be, de, W_{abc}$, and W_{bde} have been expressed in Appendix 1, other relationships are given in Appendix 2.

v_1 and v_3 are expressed as

$$v_1 = v_0 \sin(\zeta - \varphi),\ v_3 = v_0 \exp(\Theta \tan\varphi) \quad [21]$$

Then total resistance work rate is formulated as

$$\dot{W}_R = \dot{W}_{R1} + \dot{W}_{R2} + \dot{W}_{R3} + \dot{W}_{R4} + \dot{W}_{R5} + \dot{W}_{R6} - \dot{W}_{R7} \quad [22]$$

in which $\dot{W}_{R2}, \dot{W}_{R4}, \dot{W}_{R5}, \dot{W}_{R6}$ are expressed by Eqs. (4), (6), (7) and (8) respectively. The other items are expressed as

$$\dot{W}_{R1} = ac \cdot v_0 \cdot c\cos\varphi \quad [23]$$

$$\dot{W}_{R3} = dg \cdot v_3 \cdot c\cos\varphi \quad [24]$$

$$\dot{W}_{R7} = W_{efg} \cdot v_3 \cos(\eta - \beta) \quad [25]$$

And the total driving work rate is produced by

$$\dot{W}_D = \dot{W}_{D1} + \dot{W}_{D2} + \dot{W}_{D3} \quad [26]$$

in which

$$\dot{W}_{D1} = qBv_1 \quad [27]$$

$$\dot{W}_{D2} = W_{abc} v_0 \sin(\zeta - \varphi) \quad [28]$$

The term $\dot{W}_{D3}$ has been given in Eq. (12). Finally, EFS and $EFS_{\min}$ for the foundation shown in Figure 3 are expressed as

$$FS = \dot{W}_R / \dot{W}_D = FS(\zeta, \eta),\ FS_{\min} = FS(\zeta_{cr}, \eta_{cr}) \quad [29]$$

Example 5: For the foundation given in Figure 3, if $c = 20kN/m^2, \varphi = 30^\circ, B = 3m, b = 3m, \gamma = 18kN/m^3$, $D = 2.5m$, and $\Omega = 60^\circ$, different $FS_{\min}$ under different external pressure is searched based on the Eq. (29). The results are listed as follows

(1) $q = 218.90$, $FS_{\min} = 3.00$, $\zeta_{cr} = 71^\circ$, $\eta_{cr} = 51^\circ$, $\beta = 2.49^\circ$, $k = q_u / q = 3.42$

(2) $q = 351.19$, $FS_{\min} = 2.00$, $\zeta_{cr} = 71^\circ$, $\eta_{cr} = 51^\circ$, $\beta = 2.49^\circ$, $k = q_u / q = 2.13$

(3) $q_u = 747.70$, $FS_{\min} = 1.00$, $\zeta_{cr} = 70^\circ$, $\eta_{cr} = 51^\circ$, $\beta = 2.60^\circ$, $k = q_u / q_u = 1.00$

3.2 Bearing Capacity on Slope

Bearing capacity on a slope is a special case of the bearing capacity near a slope when $b = 0$. The energy safety factor for the foundation in Figure 4 can be obtained the Eq. (29) by assuming $b = 0$.

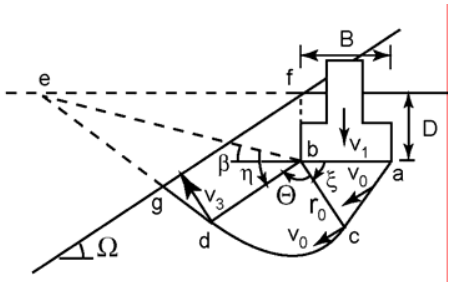


Figure 4. Bearing capacity on a slope

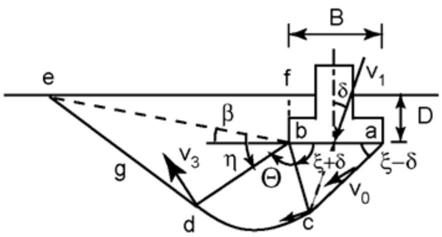
Example 6: As shown in Figure 4, if $c = 20kN/m^2, \varphi = 30^\circ, B = 3m, b = 0, \gamma = 18kN/m^3$, $D = 2.5m$, and $\Omega = 60^\circ$, different $FS_{\min}$ under different external pressure is searched by using corresponding Fortran optimization program. The results are listed as follows

(1) $q = 123.80$, $FS_{\min} = 3.00$, $\zeta_{cr} = 69^\circ$, $\eta_{cr} = 53^\circ$, $\beta = 2.06^\circ$, $k = q_u / q = 3.63$

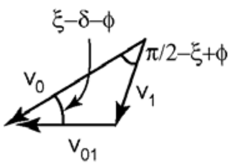
(2) $q = 206.30$, $FS_{\min} = 2.00$, $\zeta_{cr} = 68^\circ$, $\eta_{cr} = 53^\circ$, $\beta = 2.15^\circ$, $k = q_u / q = 2.18$

(3) $q_u = 449.50$, $FS_{\min} = 1.00$, $\zeta_{cr} = 63^\circ$, $\eta_{cr} = 52^\circ$, $\beta = 2.95^\circ$, $k = q_u / q_u = 1.00$

3.3 Bearing Capacity With Inclined Loads



(a)



(b)

Figure 5. Bearing capacity with inclined loads

Supposed corresponding failure mechanism is shown by Figure 5, v_1 and v_3 are expressed as

$$v_1 = v_0 \sin(\zeta - \delta - \varphi) / \cos\delta, \; v_3 = v_0 \exp(\Theta \tan\varphi) \quad [30]$$

in which $\Theta = \pi + \beta - \eta - \zeta - \delta$.

Then total resistance work rate is formulated as

$$\dot{W}_R = \dot{W}_{R1} + \dot{W}_{R2} + \dot{W}_{R3} + \dot{W}_{R4} + \dot{W}_{R5} + \dot{W}_{R6} \quad [31]$$

in which $\dot{W}_{R1}, \dot{W}_{R2}, \dot{W}_{R3}, \dot{W}_{R4}, \dot{W}_{R5}, \dot{W}_{R6}$ are expressed by Eq.s(23),(4),(5),(6),(7),(8) respectively, their corresponding $\Theta = \pi + \beta - \eta - \zeta - \delta$.

And the total driving work rate is produced by

$$\dot{W}_D = \dot{W}_{D1} + \dot{W}_{D2} + \dot{W}_{D3} \quad [32]$$

in which $\dot{W}_{D1}, \dot{W}_{D3}$ have been expressed by Eq.s (27) and (12) respectively, the items $\dot{W}_{D2}$ are written as

$$\dot{W}_{D2} = W_{abc} v_0 \sin(\zeta - \delta - \varphi) \quad [33]$$

FS and $FS_{\min}$ of Figure 5 are expressed by

$$FS = \dot{W}_R / \dot{W}_D = FS(\zeta, \eta), \; FS_{\min} = FS(\zeta_{cr}, \eta_{cr}) \quad [34]$$

Example 7: As shown by Figure 5, if $c = 20kN/m^2, \varphi = 30^\circ, B = 3m, \gamma = 18kN/m^3$ and $\delta = 10^\circ, D = 2.5m$, different $FS_{\min}$ under different external pressure is searched based on Eq. (34). The results are listed as follows

(1) $q = 401.00$, $FS_{\min} = 5.06$, $\zeta_{cr} = 70^\circ$, $\eta_{cr} = 36^\circ$, $\beta = 8.39^\circ$, $k = q_u / q = 5.51$

(2) $q = 462.00$, $FS_{\min} = 4.44$, $\zeta_{cr} = 70^\circ$, $\eta_{cr} = 36^\circ$, $\beta = 8.39^\circ$, $k = q_u / q = 4.78$

(3) $q = 482.50$, $FS_{\min} = 4.26$, $\zeta_{cr} = 70^\circ$, $\eta_{cr} = 36^\circ$, $\beta = 8.39^\circ$, $k = q_u / q = 4.58$

(4) $q = 1087.60$, $FS_{\min} = 2.00$, $\zeta_{cr} = 68^\circ$, $\eta_{cr} = 36^\circ$, $\beta = 8.16^\circ$, $k = q_u / q = 2.03$

(5) $q_u = 2208.00$, $FS_{\min} = 1.00$, $\zeta_{cr} = 68^\circ$, $\eta_{cr} = 36^\circ$, $\beta = 8.16^\circ$, $k = q_u / q_u = 1.00$

4 DESIGN PROCEDURES FOR THE BEARING CAPACITY OF FOOTING

Based on above parts, designing procedures can be concluded as follows :

Step 1: Given γ, c, φ values of soil mass; B, D values of footing and other geometrical boundary conditions.

Step 2: Based on the Figure 2, calculating different $FS_{\min}$ values under different pressures of q_{cr} and $q_{1/4}$, and $q_{1/3}$ et al. by using of the Eq. (14).

Step 3: Combined the above $FS_{\min}$ values with the importance of footing engineering, the reasonable $FS_{\min}$ value is chosen as designing value in the end.

Step 4: Based on the designing $FS_{\min}$ value, corresponding designing pressure q is calculated by trial and error by using of the Eqs. (14), or (29), or (34).

5 CONCLUSIONS

(1) Energy safety factor has a more clear physical meaning to judge the stability of bearing capacity of footing, it provides a new and direct approach to design the bearing pressure in the reasonable value range $2.0 \le FS_{\min} \le 3.0$, especially for some complicated footing bearing capacity topics such as bearing capacity near a slope and bearing capacity with inclined loads.

(2) Reasonable value of $FS_{\min}$ is not only influenced obviously by soil property such as c and φ parameters, but also related to the importance of the structure system. The $FS_{\min}$ should be a bigger value for important engineering, and should be a smaller value for a normal or less important foundation engineering.

(3) We must pay more attention to the bearing capacity for footing near or on a slope, because the bearing capacity under this case is greatly discounted, the designing $FS_{\min}$ should be given a bigger value.

(4) Bearing capacity with inclined load is also deduced obviously as the inclined angle δ is increased.

(5) The traditional safety factor is defined as $k = q_u / q$, from Examples 1~7, one observes that k value is almost equal to $FS_{\min}$, especially when the external pressure q approaches the ultimate bearing pressure.

APPENDIX 1. SOME RELATIONSHIPS OF FIGURE 1

Some geometrical dimensions are given as follows:

$$bc = r_0 = B/(2\cos\zeta),\ bd = r_0 \exp(\Theta \tan\varphi),$$
$$be = \frac{\cos\varphi}{\cos(\eta+\varphi)} bd,\ de = \frac{\sin\eta}{\cos(\eta+\varphi)} bd \qquad [35]$$

Some weights of triangle wedges are expressed as

$$W_{abc} = \frac{\gamma}{2} B \cdot bc \sin\zeta,\ W_{bde} = \frac{\gamma}{2} bd \cdot be \sin\eta,$$
$$W_{bef} = \frac{\gamma}{2} be^2 \sin\beta \cos\beta \qquad [36]$$

APPENDIX 2. SOME RELATIONSHIPS OF FIGURE 3

Some geometrical dimensions are given as follows

$$ef = be\cos\beta - b,\ fg = \frac{\cos(\eta+\varphi-\beta)}{\cos(\beta+\Omega-\eta-\varphi)} ef,$$
$$eg = \frac{\sin\Omega}{\cos(\beta+\Omega-\eta-\varphi)} ef,\ dg = de - eg \qquad [37]$$

Some weights of triangle wedges are expressed as

$$W_{beh} = \frac{\gamma}{2} be^2 \sin\beta\cos\beta,\ W_{efg} = \frac{\gamma}{2} ef \cdot fg \sin\Omega \qquad [38]$$

6 AKNOWLEDGEMENT

Funding provided by the China Scholarship Council is gratefully acknowledged.

REFERENCES

Chen,W.F.1975. Limit analysis and soil plasticity. Elsevier Scientific Publishing Company,

Chen,W.F.1990. Limit analysis in soil mechanics. Elsevier Scientific Publishing Company,

Das,B.M.1987. Theoretical foundational engineering. Elsevier Scientific Publishing Company,

Jumikis,A.R.1969. Theoretical soil mechanics---with practical applications to soil mechanics and foundation engineering. Van Nostrand Reinhold Company,