THE PERFORMANCE OF LONG LATERALLY LOADED PILE EMBEDDED IN SAND BELOW WATER TABLE SUBJECTED TO CYCLIC LOADING – SENSITIVITY INVESTIGATIONS

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ABSTRACT
The sensitivity analysis is concerned with the relationship between physical parameters that define the system and the system performance characterized by a response functional. In the paper the free head long pile is embedded in a soil located below water table subjected to cyclic horizontal force of variable values. The cyclicity of the loading is considered in implicit fashion. The stiffness of the pile and the parameters used for the description of p-y relationships of sand below water table are considered as the design variables of continuous type that are space dependent. The performance functional that describes the maximum generalized deformations of a pile-soil system is formulated with the aid of a non-linear primary system. The sensitivity results in the form of sensitivity integrands that affect the changes of the maximum generalized deformation of the pile-soil system caused by the changes of the variations of continuous design variables are discussed.

RÉSUMÉ
Cette analyse de sensibilité concerne le rapport entre paramètres physiques définissant le système, d'une part, et d'autre part la performance de système caractérisée par un fonctionnel de réaction. On décrit un pieu long à tête libre, construit dans un sol situé sous la nappe phréatique et sujet à une force horizontale cyclique ayant des valeurs variables. La cyclicité de cette charge est considérée de façon implicite. La raideur du pieu, et les paramètres utilisés pour décrire les rapports p-y du sable sous la nappe phréatique, sont considérés comme des variables continues de conception qui dépendent de l'espace. Le fonctionnel de performance qui décrit les déformations généralisées maximales d'un système pieu-sol est formulé à l'aide d'un système primaire non linéaire. On discute les résultats de sensibilité, sous forme d'intégrands de sensibilité qui affectent les changements de la déformation généralisée maximale du système pieu-sol causés par les changements des variations des variables continues de conception.

1. INTRODUCTION

The pile foundations are used to resist axial and lateral loads applied to the pile head. The pile-soil interaction can be simulated by a number of different approaches. One of the most popular approaches used in the geotechnical community is p-y method referred also as local-transfer method. In p-y method, p stands for soil reaction whereas y defines lateral displacement. The pile structure in the pile-soil interaction system is considered as an elastic beam element. The soil p-y model represents nonlinear springs distributed along the pile axis that deform locally which means that the p-y model itself does not transfer the deflection y to the soil neighborhood. A number of p-y curves were developed for sand (Murchison and O'Neil 1983, Reese et al. 1974).

The objective of this paper is focused on the following aims:

1. To develop the theoretical formulation of sensitivity analysis of distributed parameters of laterally loaded pile embedded in p-y sand located below water table subjected to cyclic load.
2. To conduct the numerical sensitivity investigations of maximum performance of laterally loaded pile subjected to horizontal force of discrete variability, affected by the variations of the design variables of the system.

2. THEORETICAL FORMULATION

2.1 Brief description of p-y model used in investigations

The pile structure together with the adjacent soil model and specified physical parameters of the pile-soil system subjected to investigations is shown in Fig. 1.

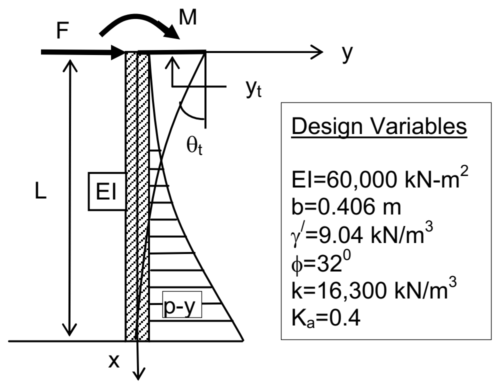

Figure 1. Schematic view of a laterally loaded pile.

The response of the soil is modeled according to p-y notion whereas the pile structure is simulated as a one dimensional beam element. The interactive pile-soil system satisfies the differential equation denoted as Eq. (1). The solution of it represents the performance (deflection y, angle of flexural rotation θ) of the system. Thus,

$$EIy^{IV} + p(y) = 0 \quad [1]$$

where EI stands for pile's bending stiffness, p(y) denotes the soil reaction being the function of lateral deflection y.

The soil p-y model for sand proposed by Reese et al. (1974) describes the soil behavior when adjacent laterally loaded pile is embedded in sand below water table subjected to cyclic loading. It employs the ultimate soil resistance p_c which depends on the depth x and the soil strength parameters such as an angle of internal friction ϕ, a submerged unit weight of soil γ', a coefficient of lateral earth pressure of Rankine type K_a, a modulus of subgrade reaction k and width b of a pile where the soil reaction can develop. The p_c is expressed by means of two equations that differentiate themselves by the fact, that one part of p_c denoted as p_{ct} can develop close to the soil surface whereas p_{cd} is generated at the deeper depth. The transition from p_{ct} to p_{cd} occurs at such depth x_r that provides the continuity of p_{ct} and p_{cd}. Thus, for $x \le x_r$

$$p_{ct} = \gamma' x \left[\frac{K_0 x \tan\phi \sin\beta}{\tan(\beta-\phi)\cos\alpha} + \frac{\tan\beta}{\tan(\beta-\phi)}(b + x\tan\beta\tan\alpha) + K_0 x \tan\beta(\tan\phi\sin\beta - \tan\alpha) - K_a b \right] \quad [2]$$

and for $x \ge x_r$, $p_{cd} = K_a b\gamma' x(\tan^8\beta - 1) + K_0 b\gamma' x \tan\phi \tan^4\beta$ [3]

where $\alpha = \phi/2$ and $\beta = (45^0 + \phi/2)$.

The equity of Eq. (2) and (3) allows for determination of x_r which is given as:

$$x_r = \frac{b\tan\beta\left[K_a\tan^7\beta + K_0\tan\phi\tan^3\beta - \frac{1}{\tan(\beta-\phi)}\right]}{\frac{K_0\tan\phi\sin\beta + \tan^2\beta\sin\alpha}{\tan(\beta-\phi)\cos\alpha} + K_0\tan\beta(\tan\phi\sin\beta - \tan\alpha)} \quad [4]$$

At arbitrary depth x the soil lateral displacement is marked by three characteristic values. They are denoted as y_k, y_m and y_u. The y_k defines this interval (0-y) of lateral deflection y at an arbitrary point x where the soil reaction p demonstrates a linear behaviour. When the lateral displacement y is located in the interval contained between y_k and y_m=b/60 the soil reaction p is a parabolic function of y. The y_u=3b/80 marks the value of lateral displacement y where the soil reaction p passes from the bilinear state to the plastic flow. The corresponding values of soil reaction p associated with characteristic points y are denoted as p_k, p_m and p_u, respectively and are shown in Fig. 2.

The set of suitable physical relationships for p-y soil discussed is given as:

for $y \le y_k$, p = kxy [5]

$$\text{for } y_k \le y \le y_m,\ p = B_c p_c \left(\frac{60}{b}y\right)^{0.8\left(\frac{A_c}{B_c}-1\right)} \quad [6]$$

where A_c and B_c are experimentally determined functions of dimensionless variable (x/b) that take into account the effect of cyclic loading on development of soil reaction p, and

$$\text{for } y_m \le y \le y_u,\ p = p_c\left[B_c + \frac{48}{b}\left(y - \frac{b}{60}\right)(A_c - B_c)\right] \quad [7]$$

It is worth noting that the ultimate soil reaction p_c that appears in Eqs. (6) and (7) is defined by Eq. (2) for $x \le x_r$ and by Eq. (3) for $x \ge x_r$.

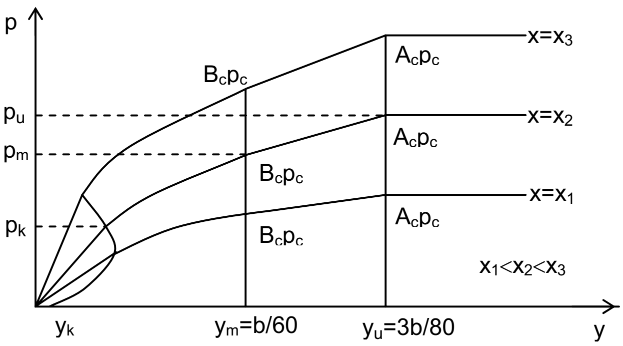

Figure 2. Graphical representation of p-y curves for sand below water table subjected to cyclic loading, the variability of y_k along the depth x is also indicated on the p-y curves.

Figure 2 shows that the lateral displacements y_m and y_u do not change with depth x. The same conclusion cannot be extended to y_k whose value changes with depth x. The locations of intersection points of Eq. (5) with parabolic portions of p-y curves given by Eq. (6) are shown in Fig. 2 for variable values of depth x. It is apparent that interval (0-y_k) within which the soil behaviour is of linear type changes with depth. The type of variability of (0-y_k) as a function of x is important in explanation of distributions of sensitivity integrands affecting the performance of maximum value of generalized deflection. Therefore the physical variability of

y_k, y_m and y_u in the vicinity of the laterally loaded pile subjected to variable in discrete fashion forces F_1, F_2, F_3 are shown in Fig. 3, which contains also the possible deflection lines y_1, y_2, y_3 generated by the applied forces F_1, F_2, F_3.

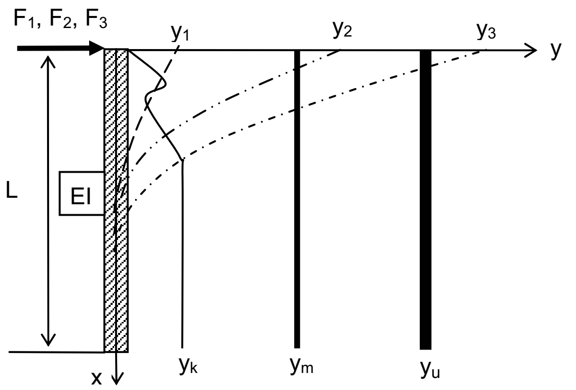

Figure 3. Typical distributions of y_k, y_m and y_u values together with deflection curves y_1, y_2, y_3 of a laterally loaded pile embedded in p-y sand located below water table subjected to variable forces F_1, F_2, F_3 of cyclic type.

The distributions of functions A_c and B_c of Eq. (6) and (7) that represent the cyclicity effect on the behaviour of laterally loaded pile-soil system embedded in sand below water table are shown in Fig. 4.

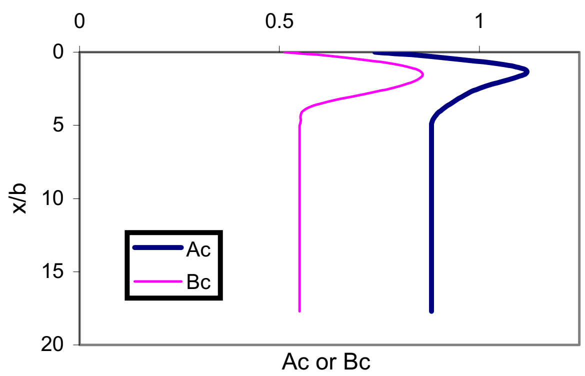

Figure 4. Distributions of non-dimensional functions A_c, B_c contributing to the effects of cyclic loading affecting the performance of laterally loaded pile embedded in sand.

2.2 Formulation of sensitivity performance of laterally loaded pile-soil system with distributed parameter

The performance of the pile in this analysis is defined by maximum lateral deflection and maximum angle of flexural rotation. In the investigated case both these components of maximum generalized deflection **u** are located at the pile head and are denoted as y_t and θ_t. The p-y pile-soil system explored in the framework of sensitivity theory by means of the adjoint structure method is shown in Fig. 5.

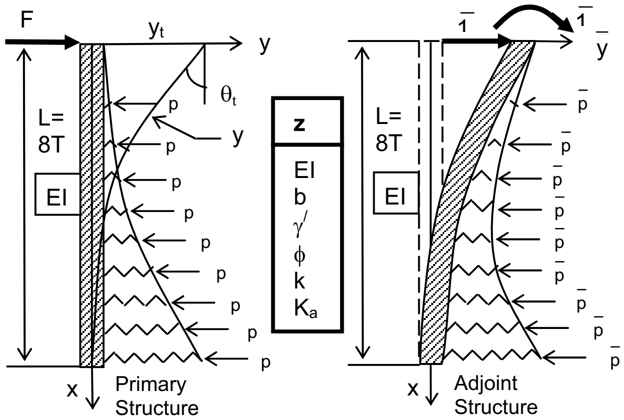

Figure 5. The pile-soil structure subjected to sensitivity analysis with distributed design variables.

The indicated physical and geometrical parameters affecting the performance of the pile-soil system are taken as the design variables of distributed type. They are considered as being functions of spatial variable x and are arranged in vector **z** defined as:

$$\mathbf{z} = [EI, k, \gamma', \phi, b, K_a]^T \qquad [8]$$

The generalized maximum deflection can be determined based on virtual work principle (Washizu 1976). The original structure is further called as a primary structure. It requires introduction of a temporary system called the adjoint structure (Fig. 5) that is subjected to suitable generalized unit load (Haug et al. 1986). It satisfies the same differential equation of the system as well as the physical equations as the primary structure does.

The changes of the design variables by $\delta\mathbf{z}$ in the presence of the unchanged load F enables one to write the virtual work principle in variational form (Kleiber et al. 1997) as:

$$\bar{1} \bullet \delta\mathbf{u} = -\int_0^L \bar{M}\delta y'' dx + \int_0^L \bar{p}\delta y dx \qquad [9]$$

where $\bar{M}, \bar{p}$ are internal forces of the adjoint structures, $\delta y''$ and δy are the first variations or increments of suitable generalized deflections such as increment of second derivative of deflection and increment of deflection itself, $\delta\mathbf{u}$ stands for the first variation of maximum generalized deflection caused by the change of the design variables $\delta\mathbf{z}$.

The unknown variations of $\delta y''$ and δy resulting from changes of the design variables can be determined considering the relationship between increment of internal forces and increments of state variables as well as changes in the design variables. Formally, the pile structure and the adjacent soil satisfy the following equations:

$$M = -EIy'' \quad [10]$$

$$p = y(\mathbf{z}) \quad [11]$$

where y(**z**) represents the lateral deformation of the soil being the function of the design variable vector **z** that does not contain bending stiffness EI of the pile, which is associated with pile structure of Eq. (10).

The increments of internal forces described by Eqs. (10) and (11) are due to the changes of state variables y, y'' as well as changes of the design variables vector **z**. Thus,

$$\delta M = \frac{\partial M}{\partial EI}\delta EI + \frac{\partial M}{\partial y''}\delta y'' \quad [12]$$

$$\delta p = \frac{\partial p}{\partial y}\delta y + \frac{\partial p}{\partial \mathbf{z}}\delta \mathbf{z} \quad [13]$$

Since the investigated system is subjected to the constant load therefore the increments of internal forces are equal to zero. This means that the following conditions are satisfied:

$$\delta M = 0 \quad [14]$$

$$\delta p = 0 \quad [15]$$

Consequently, Eq. (14) when combined with Eq. (12) allows determining the sought $\delta y''$. Similarly, implementation of Eq. (15) into Eq. (13) leads to determination of δy. Thus,

$$\delta y'' = -\frac{\partial M}{\partial \mathbf{z}}\frac{\partial y''}{\partial M}\delta \mathbf{z} \quad [16]$$

$$\delta y = -\frac{\partial p}{\partial \mathbf{z}}\frac{\partial y}{\partial p}\delta \mathbf{z} \quad [17]$$

In Eq. (16), $\delta\mathbf{z}$ contains the change of pile material stiffness, i.e. δEI, whereas $\delta\mathbf{z}$ of Eq. (17) contains the changes of the physical parameters that affect the behavior of the soil. Thus, performing the required operations of differentiation demanded by Eq. (16) with aid of Eq. (10), it is arrived at:

$$\delta y'' = -(-y'')\left(-\frac{1}{EI}\right)\delta(EI) \quad [18]$$

Based on the definition of the adjoint structure the behavior of the adjoint system is governed by the following equation;

$$\bar{M} = -EI\bar{y}'' \quad [19]$$

Thus, combining Eq. (18) and (19) with the function under first integral of Eq. (9), the following relationship emerges:

$$\bar{M}\delta y'' = y''\bar{y}''\delta(EI) \quad [20]$$

The full form of Eq. (17) that takes explicitly into account all design variables associated with soil model is the following:

$$\delta y = -\frac{\partial p}{\partial b}\frac{\partial y}{\partial p}\delta b - \frac{\partial p}{\partial \gamma'}\frac{\partial y}{\partial p}\delta\gamma' - \frac{\partial p}{\partial \phi}\frac{\partial y}{\partial p}\delta\phi - \frac{\partial p}{\partial K_a}\frac{\partial y}{\partial p}\delta K_a - \frac{\partial p}{\partial k}\frac{\partial y}{\partial p}\delta k \quad [21]$$

The determination of partial derivative $(\partial y / \partial p)$ is based on Eq. (11). It is apparent that the soil adjacent to the adjoint structure is characterized by the equation similar to Eq. (11), which is now given as:

$$\bar{p} = \bar{y}(\mathbf{z}) \quad [22]$$

Thus, combining Eq. (21) with Eq. (22) the product of functions under second integral of Eq. (9) is given as:

$$\bar{p}\delta y = -\frac{\partial p}{\partial b}\frac{\partial y}{\partial p}\bar{y}(\mathbf{z})\delta b - \frac{\partial p}{\partial \gamma'}\frac{\partial y}{\partial p}\bar{y}(\mathbf{z})\delta\gamma' - \frac{\partial p}{\partial \phi}\frac{\partial y}{\partial \phi}\bar{y}(\mathbf{z})\delta\phi - \frac{\partial p}{\partial K_a}\frac{\partial y}{\partial p}\bar{y}(\mathbf{z})\delta K_a - \frac{\partial p}{\partial k}\frac{\partial y}{\partial p}\bar{y}(\mathbf{z})\delta k \quad [23]$$

Introducing relationships (20) and (23) into Eq. (9), the following relationship is attained:

$$\bar{1}\bullet\delta\mathbf{u} = -\int_0^L y''\bar{y}''\delta(EI)dx - \int_0^L \frac{\partial p}{\partial b}\frac{\partial y}{\partial p}\bar{y}(\mathbf{z})\delta b dx - \int_0^L \frac{\partial p}{\partial \gamma'}\frac{\partial y}{\partial p}\bar{y}(\mathbf{z})\delta\gamma' dx - \int_0^L \frac{\partial p}{\partial \phi}\frac{\partial y}{\partial p}\bar{y}(\mathbf{z})\delta\phi dx - \int_0^L \frac{\partial p}{\partial K_a}\frac{\partial y}{\partial p}\bar{y}(\mathbf{z})\delta K_a dx - \int_0^L \frac{\partial p}{\partial k}\frac{\partial y}{\partial p}\bar{y}(\mathbf{z})\delta k dx \quad [24]$$

The brief look at Eq. (24) enables one to notice that LHS contains changes (first variation of $\delta\mathbf{u}$) of generalized maximum deflection whereas integrals of RHS contain changes (first variations) of the design variables of the pile-soil system. Therefore, Eq. (24) represents the sensitivity of

$\delta\mathbf{u}$ due to the changes of the design variable vector $\delta\mathbf{z}$. It is worth noting that each of the design variables bears a different unit. Moreover, the results of integration of all integrals of RHS are of the same units as LHS of Eq. (24). These facts drive towards conclusion that normalization of the variations $\delta\mathbf{z}$ of the design variables with respect to their initial values $\mathbf{z}$ leaves the integrands having the same units. The described process of normalization of the design variables means that the remaining integrands bear units of a force, which after integration with respect to x gives units of work or energy.

Thus, Eq. (24) can be now written in more concise form as:

$$\bar{1}\bullet\delta\mathbf{u}=\int_0^L P_{EI}^{F\mathbf{u}}\delta(EI)_N dx+\int_0^L P_b^{F\mathbf{u}}\delta b_N dx+\int_0^L P_{\gamma'}^{F\mathbf{u}}\delta\gamma'_N dx+\int_0^L P_{\phi}^{F\mathbf{u}}\delta\phi_N dx$$
$$+\int_0^L P_{K_a}^{F\mathbf{u}}\delta(K_a)_N dx+\int_0^L P_k^{F\mathbf{u}}\delta k_N dx \qquad [25]$$

The mathematical form of sensitivity integrands $P_{(***)}^{F\mathbf{u}}$ can be obtained by comparison with suitable integrals of Eq. (24). It is worth noting that sensitivity analysis is conducted in the vicinity of the applied loads. For the linear systems, the sensitivity integrands $P_{(***)}^{F\mathbf{u}}$ can be normalized with respect to the applied load F. This means that for linear elastic system the numerical results of Eq. (25) for one single load F can be suitably employed for entire spectrum of load values applied to the system investigated (Budkowska 1997a, 1997b). However, for non-linear system the proportionality law does not apply. In the discussed case, the source of non-linearity, which is attributed to the p-y relationship, is then extended to the relationship between force and generalized displacement. Therefore, the sensitivity investigations of non-linear systems are conducted in a discrete fashion for identified values of external loads.

The sensitivity integrands are the functions of spatial variable x, which are subjected to integration with respect to x. They represent the effect of spatial nature of the design variables $(***)$ on the changes of the generalized deflections. In case when the pile-soil system is subjected to bending moment M, then the superscript F of sensitivity integrands will be changed to M. The Eq. (25) can be used for sensitivity investigations of maximum lateral deflection y_t as well as maximum angle of flexural rotation θ_t. Accordingly, the superscript **u** in Eq. (25) will be replaced by y or θ and consequently, the unit horizontal load $\bar{1}$ of Eq. (24) in latter case stands for unit bending moment applied to the adjoint pile-soil system.

The distribution of sensitivity integrands $P_{(***)}^{F\mathbf{u}}$ given by Eq. (25) provide an important information on the spatial ability of changes of the parameters of the system, which are of crucial importance to the performance of the system. They also play significant role in clear indication of critical locations of the parameters of the system, which are important for the behavior of deep foundations as well as superstructure. This aspect of sensitivity investigations is of special interest to users of the system as well as at the design stage of the system. The $P_{(***)}^{F\mathbf{u}}$ sensitivity integrands allow to predict the inevitable changes of the performance of the system subjected to constant load when soil profile is subjected to changes of properties of the soil model used. The knowledge resulting from sensitivity integrands $P_{(***)}^{F\mathbf{u}}$ provides the basis to develop suitable preventive measures to extend the life-service of the system. The sensitivity integrands formulated in the scope of the sensitivity theory of distributed parameters constitute the key notion that sustainable development philosophy is looking for.

3. NUMERICAL INVESTIGATIONS

The objective of this section is to implement the presented theoretical formulation to the numerical investigations of laterally loaded long piles. The initial input data for the pile-soil system that is based on the recommendations of COM624P (1993) and CISC (2001) are shown in Fig. 1. The geometry of the free head pile-soil structure of length L=8T(14.9m) subjected to force F of discrete variability is shown in Fig. 5. The relative stiffness factor T that is used in assessment of length of non-linear p-y pile-soil system is defined (Evans and Duncan 1982) as equal to $(y_t\,EI/A_y\,F)^{1/3}$ with A_y being the pile head restraint constant and y_t is the pile head lateral deflection.

The generated top lateral deflections y_t with corresponding values of F are transformed to the p-y curve constructed for $x \approx 0$, that is as close as possible to the soil surface. The results of this transformation are shown in Fig. 6. The points y_t and F when placed on p-y curve of $x \approx 0$ provide the information on the possible soil (p-y) physical phases that will develop within the soil adjacent to the pile.

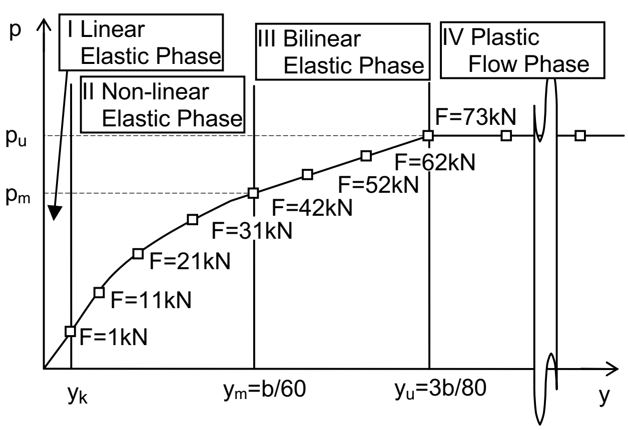

Figure 6. The p-y curve for x=0.01m with marked values of force F used in investigations.

The usefulness of the display of the results (y_t, F) on p-y curve of Fig. 6 becomes apparent in further discussion on sensitivity results in terms of $P^{F_y}_{(***)}$ and $P^{F_\theta}_{(***)}$ and their utilization for engineering applications. The non-linearity of the system besides results presented in Fig. 6 can also be assessed by means of the outcomes of the adjoint system when loaded by suitable unit load. For linear systems, all internal forces $\bar{M}, \bar{V}$ as well as deflection lines $\bar{y}$ are the same, independently of the magnitude of the applied load (Budkowska and Szymczak 1992). However, for non-linear system, this rule does not apply. The numerical investigations of sensitivity of laterally loaded p-y pile-soil system are conducted by means of the program COM624P (1993). The sensitivity integrands $P^{F_y}_{(***)}$ of Eq. (25) for forces F shown in Fig. 6 are presented in Figs. 7-11.

As discussed previously, the sensitivity integrands of maximum angle of flexural rotation θ_t require application of unit bending moment $\bar{1}$ to the adjoint structure (see Fig. 5). The deliberated sensitivity integrands defined as $P^{F_\theta}_{(***)}$ have mathematical structure analogous to $P^{F_y}_{(***)}$. The

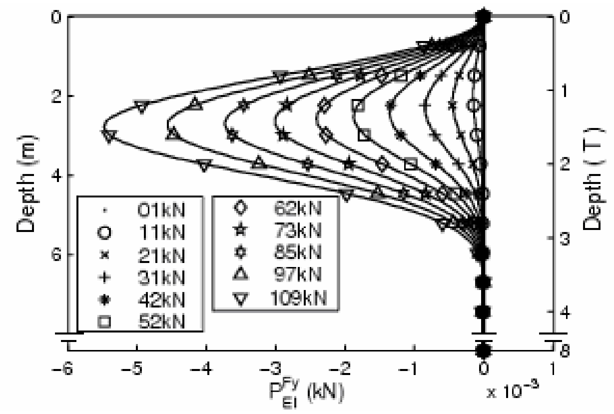

Figure 7. Distributions of sensitivity integrands, $P^{F_y}_{EI}$.

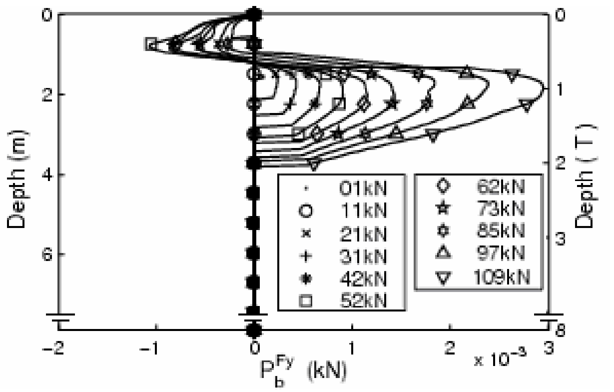

Figure 8. Distributions of sensitivity integrands, $P^{F_y}_{b}$.

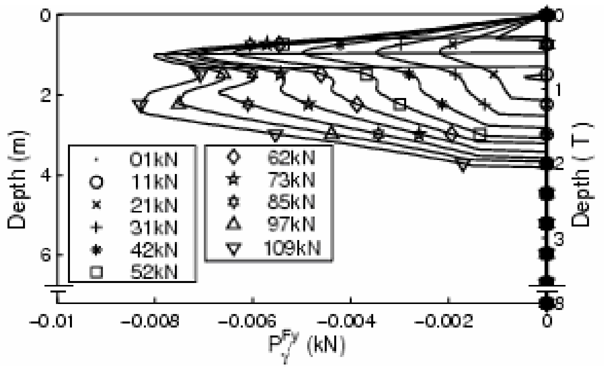

Figure 9. Distributions of sensitivity integrands, $P^{F_y}_{\gamma'}$.

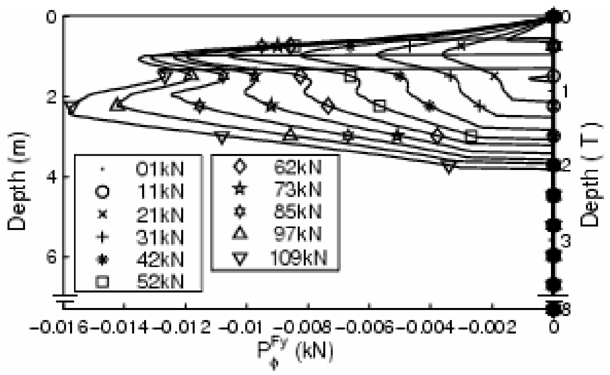

Figure 10. Distributions of sensitivity integrands, $P^{F_y}_{\phi}$.

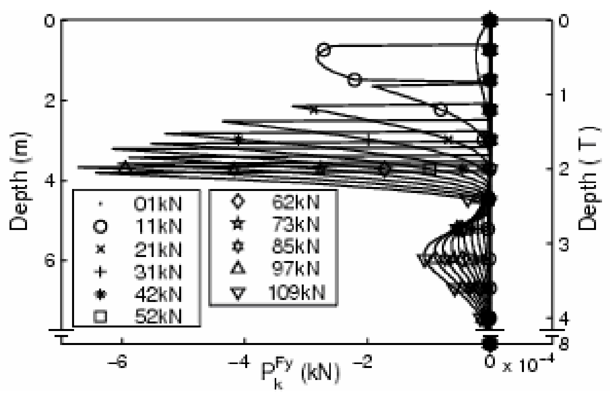

Figure 11. Distributions of sensitivity integrands, $P^{F_y}_{k}$.

differences between $P^{F_y}_{(***)}$ and $P^{F_\theta}_{(***)}$ are the consequence of the fact that the former requires application of the unit horizontal force $\bar{1}$ to the adjoint structure, whereas latter implies application of unit bending moment $\bar{1}$. Accordingly, the virtual load $\bar{M} = \bar{1}$ generates in the adjoint structure the generalized lateral deflections $\bar{y}_M, \bar{y}_M', \bar{y}_M''$. Consequently,

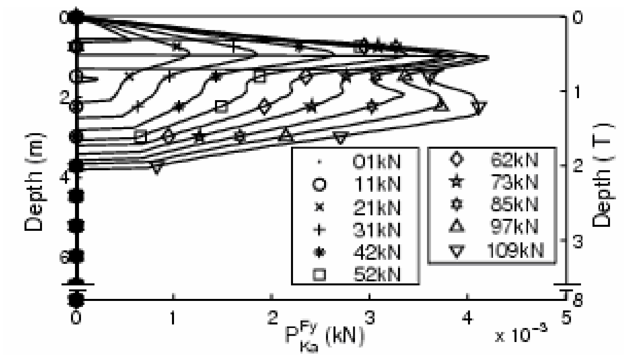

Figure 12. Distributions of sensitivity integrands, $P^{F_y}_{K_a}$.

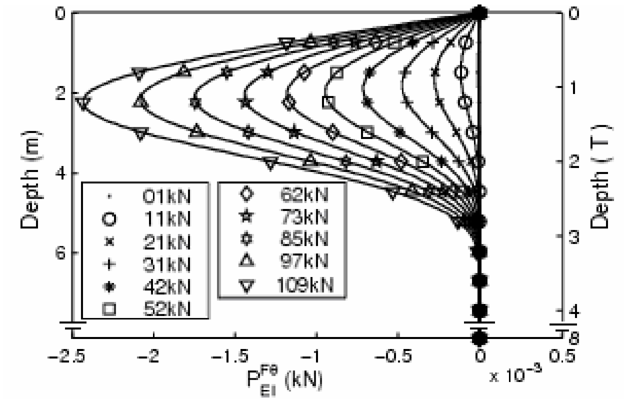

Figure 13. Distributions of sensitivity integrands, $P^{F_\theta}_{EI}$.

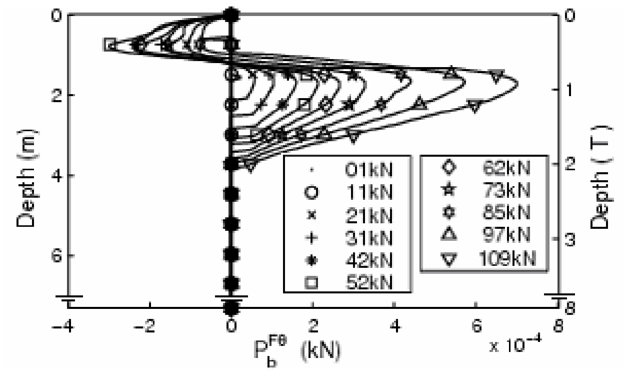

Figure 14. Distributions of sensitivity integrands, $P^{F_\theta}_{b}$.

the formulae for sensitivity integrands, $P^{F_\theta}_{(***)}$, affecting the maximum angle of flexural rotation, θ_t, due to the changes of the design variables, $(***)$, are obtained by substitution to Eqs. (19) and (20) as well as Eqs. (22), (23) and (24), $\bar{y}_M$ and $\bar{y}_M''$ instead of $\bar{y}$ and $\bar{y}''$. In this way determined sensitivity integrands, $P^{F_\theta}_{(***)}$ are presented in Figs. 13-18.

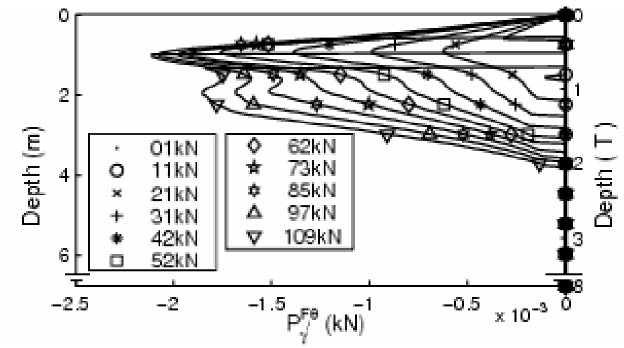

Figure 15. Distributions of sensitivity integrands, $P^{F_\theta}_{\gamma'}$.

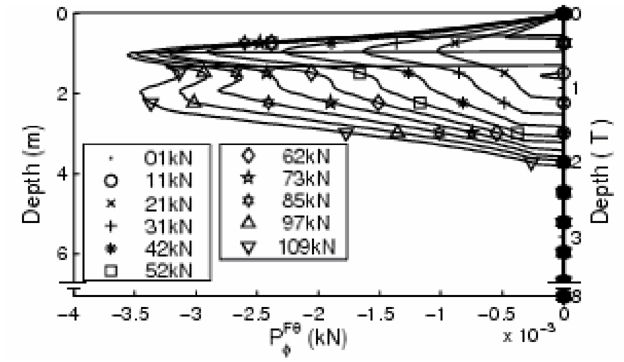

Figure 16. Distributions of sensitivity integrands, $P^{F_\theta}_{\phi}$.

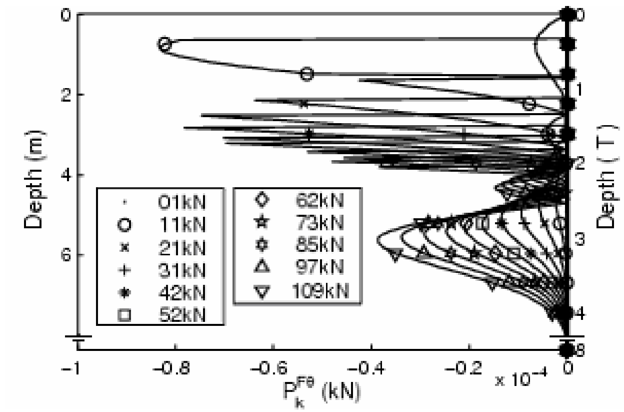

Figure 17. Distributions of sensitivity integrands, $P^{F_\theta}_{k}$.

4. CONCLUDING NOTES

The presented results of sensitivity integrands lead to the following conclusions:

1. the distributions of $P^{F_y}_{EI}$ associated with bending stiffness of the pile material (shown in Fig. 7) demonstrated the high degree of regularity for each value of load F,

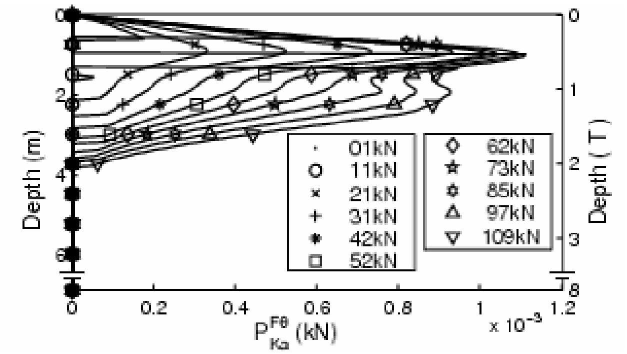

Figure 18. Distributions of sensitivity integrands, $P^{F_\theta}_{K_a}$.

2. the $P^{F_y}_{\gamma'}$, $P^{F_y}_{\phi}$ and $P^{F_y}_{K_a}$ (shown in Figs. 9,10 and 12) exhibit high degree of similarity regarding shape of their graphical representations for each value of F,
3. the discontinuity in diagrams of sensitivity integrands $P^{F_y}_{\gamma'}$, $P^{F_y}_{\phi}$ and $P^{F_y}_{K_a}$ observed are associated with entrances of their deflection lines y into y_k zone that is shown in Fig. 3 (for small F) or with development of plastic soil flow close to the soil surface for large F (85kN, 97kN, and 109kN),
4. a sort of irregularities observed at x ≈0.5T in sensitivity diagrams $P^{F_y}_{\gamma'}$, $P^{F_y}_{\phi}$ and $P^{F_y}_{K_a}$ are associated with cyclic effects that are taken into account by the correction functions A_c and B_c. They allow to consider a cyclic loading in quasi-static fashion,
5. the distributions of sensitivity integrands $P^{F_y}_{b}$ (shown in Fig. 8) differentiate themselves from sensitivity integrands $P^{F_y}_{\gamma'}$, $P^{F_y}_{\phi}$ and $P^{F_y}_{K_a}$. In contrast to $P^{F_y}_{EI}$, $P^{F_y}_{\gamma'}$, $P^{F_y}_{\phi}$ and $P^{F_y}_{K_a}$, the $P^{F_y}_{b}$ change sign. This fact is associated with the change of sense of the soil reaction p that is develop along the pile axis. This means, that $P^{F_y}_{b}$ is negative when the soil reaction p acts against force F, whereas $P^{F_y}_{b}$ is positive when sense of soil reaction p is in accord with force F,
6. the distributions of sensitivity integrands $P^{F_y}_{k}$ (shown in Fig. 11) connected with linear elastic soil phase are developed from the soil surface to the depth x ≈4T only for small values of load F applied. As the external load F increases, the deflection line y intersects the y_k envelope of Fig. 3 at progressively larger depth x that is demonstrated by rapid increase in values of $P^{F_y}_{k}$,
7. the scope of numerical variability of $P^{F_y}_{(***)}$ that affect the changes δy_t due to the changes of the design variables can be classified in ascending order with respect to the importance of the design variables as: k, b, EI, K_a, γ' and ϕ.

The review of sensitivity integrands $P^{F_\theta}_{(***)}$ affecting the maximum angle of flexural rotation of the pile head due to the changes of the design variables when subjected to force F of cyclic type and discrete variability presented in Figs. 13-18 leads to the similar observations that have been noticed for $P^{F_y}_{(***)}$. Regarding the numerical values of $P^{F_y}_{(***)}$ and $P^{F_\theta}_{(***)}$ for the same values of the forces F applied, the values of $P^{F_y}_{(***)}$ are substantially larger than corresponding values of $P^{F_\theta}_{(***)}$. More specifically, it is regarded that numerical values of the discussed sensitivity operators can be assessed as:

$P^{F_y}_{EI} \approx 2P^{F_\theta}_{EI}$; $P^{F_y}_{b} \approx 4.3P^{F_\theta}_{b}$; $P^{F_y}_{\gamma'} \approx 4P^{F_\theta}_{\gamma'}$; $P^{F_y}_{\phi} \approx 4.6P^{F_\theta}_{\phi}$; $P^{F_y}_{k} \approx 7.5P^{F_\theta}_{k}$; $P^{F_y}_{K_a} \approx 4P^{F_\theta}_{K_a}$.

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