

A 3D ANALYTICAL SOLUTION FOR EVALUATING EARTH PRESSURES IN VERTICAL BACKFILLED STOPEs

Li Li, École Polytechnique de Montréal
Michel Aubertin*, École Polytechnique de Montréal
Tikou Belem, Université du Québec en Abitibi-Témiscamingue
Richard Simon, École Polytechnique de Montréal
Michael James, École Polytechnique de Montréal
Bruno Bussière, Université du Québec en Abitibi-Témiscamingue

ABSTRACT

Backfilling is being increasingly used in underground mines for ground control and wastes disposal. The behavior of backfill is significantly influenced by its interaction with the surrounding rock mass and support structures such as barricades. Previous work conducted on backfilled stopes, including some recent studies by the authors, indicates that the theory of arching can be used to estimate the earth pressures in narrow, vertical backfilled openings. In this paper, a new 3D analytical solution is proposed to evaluate the state of stress within a backfilled vertical stope. The solution is validated using laboratory experimental results taken from the literature.

RÉSUMÉ

Le remblai est souvent utilisé dans les mines souterraines comme méthode de contrôle de terrain et pour disposer des rejets miniers. Le comportement du remblai et son interaction avec le massif rocheux adjacent et avec les structures de support (tel que les barricades) doivent alors être analysés. Les travaux antérieurs portant sur les chantiers remblayés, incluant des études récentes menées par les auteurs, indiquent que la théorie des arches peut être utilisée pour estimer la pression dans les chantiers étroits. Dans cet article, une nouvelle solution analytique tridimensionnelle est proposée pour évaluer l'état de contraintes dans le remblai. La solution est ensuite validée à l'aide de résultats expérimentaux de laboratoire tirés de la littérature.

1. INTRODUCTION

In recent decades, backfilling has become widely used for ground support and waste disposal in underground mining throughout Canada and around the world (e.g., Thomas et al. 1979; Udd 1989; Hassani and Archibald 1998; Landriault et al. 2000). With the progressive exhaustion of surficial ore bodies and more intensive environmental pressure, backfilling of stopes may become mandatory as it can help reduce the environmental impact of mine operations by minimising the quantity of waste to be disposed of in surface impoundments (e.g., Aubertin et al. 2002).

The backfill material is usually fairly similar to soil in its characteristics and behavior, so it is much more compressible than the surrounding rock mass. The interaction between backfill and rock mass is rather complex. Accordingly, soil and rock mechanics have to be considered simultaneously (Thomas et al. 1979) to evaluate the behavior of the backfill and the state of stress within the backfill and against the rock walls. Numerical modeling constitutes a powerful means to investigate the response of such system, as it allows for various factors (such as in situ stress conditions, mining sequence, rock discontinuities, wall convergence, and backfill consolidation) to be taken into account (e.g., Pariseau 1981; Brummer et al. 1996; Hassani and Fotoohi 1997;

Brechtel et al. 1999; Li et al. 2003). Nevertheless, analytical methods can provide rapid and low cost approximate solutions of the behavior of backfilled stopes (e.g., Mitchell 1983; Hassani and Archibald 1998; Aubertin et al. 2003). In that regard, previous work conducted on this issue, including some recent studies by the authors, indicates that the theory of arching may be used for estimating the earth pressures in narrow, (sub-) vertical stopes (e.g., Knutsson 1981; Hustrulid et al. 1989; Aubertin 1999; Aubertin et al. 2003; Li et al. 2003). However, most existing solutions are based on 2D limit equilibrium analysis in which the two long walls of backfilled stopes are considered identical. In many cases, these solutions are inappropriate, especially when the backfilled stope has a limited length or when the two long walls have different characteristics. In this paper, a more general 3D analytical solution is proposed and is validated using representative experimental results.

2. PROPOSED 3D SOLUTION

When a frictional particulate material, such as backfill, is placed in a rigid container, the material yields internally as it compresses due to gravity. However, the surrounding rigid walls will tend to resist the downward movement of the material by frictional forces along the interfaces. Part of the load due to the weight of the material is thus transferred to the walls, so the resulting vertical stresses

* Corresponding author : michel.aubertin@polymtl.ca; tel 514-340-4711 ext. 4046; fax: 514-340 4477.

in the material will be less than those calculated based on the unit weight of the material. This phenomenon is known as "arching" (e.g., Richmond and Gardner 1962).

Arching was observed long ago in the powder industry where silos and bins are used to store granular materials. A common prerequisite for handling these materials is the need to estimate the minimum span (of silos or bins) to avoid the formation of a stable arch that could obstruct the flow of the material in the containers (e.g., Richards 1966; Cowin 1977). The arching theory, initially proposed by Janssen (1895) for the analysis, was later introduced to geotechnical engineering by the pioneering work of Marston (1930) and Terzaghi (1936). Marston (1930) adapted arching theory to investigate the pressure on buried conduits in trenches (e.g., McCarthy 1988), while Terzaghi (1936, 1943) applied arching theory to evaluate the stress distribution above tunnels. Arching theory has been further applied to retaining walls (e.g., Frydman and Keissar 1987; Take and Valsangkar 2001) and to dams (e.g., Kutzner 1997).

structures. Three dimensional consideration has also been given in cases where one of the lateral walls was removed (e.g., Mitchell 1983).

A unique fill-wall friction angle is included with most existing 3D solutions. However, in practice, it is not uncommon for the material of the two lateral walls (ore or cemented backfill) to be different from that of the hanging and/or foot walls (rock) of a stope. A set of 3D equations is developed here to evaluate the stress along the vertical walls of narrow backfilled stopes having different properties.

2.1 Equations development

Figure 1 is a schematic of a typical vertical, narrow backfilled stope, with various force components (based on uniformly distributed stresses on the isolated element); H is the backfill height, B is the stope width and L is the stope length. At position h the horizontal layer element is subjected to lateral compressive forces C_i ($i = 1$ to 4),

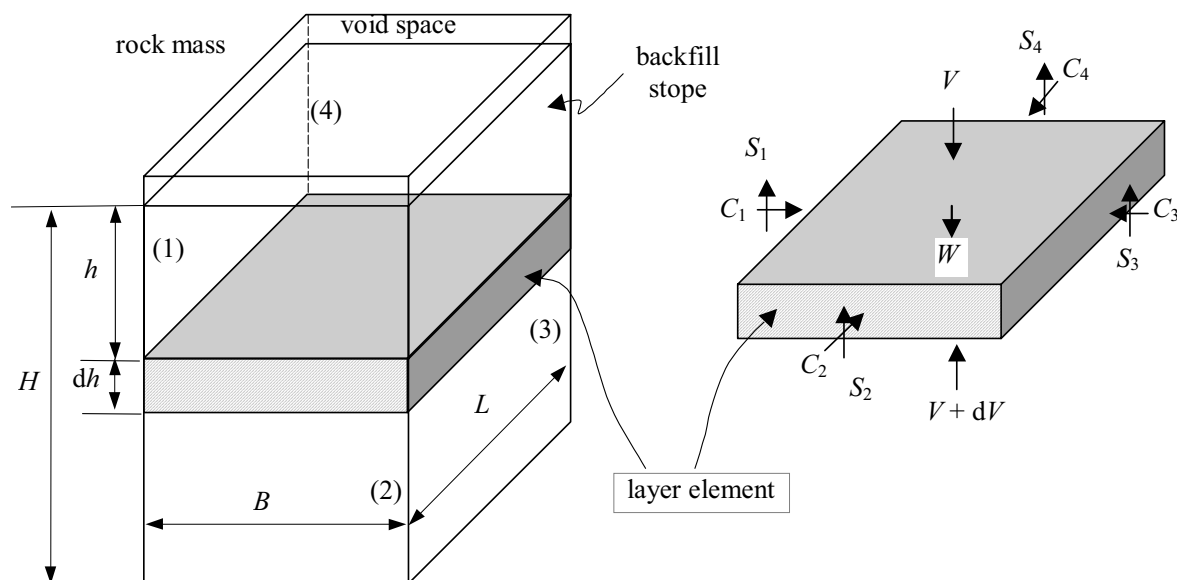


Figure 1. A vertical backfilled stope with the forces acting on an isolated layer element.

Containers used in the powder industry typically have finite dimensions on all sides, so three dimensional models had to be considered early (e.g., Richmond and Gardner 1962; Richards 1966; Cowin 1977; Blight 1986; Williams et al. 1987). In geotechnical engineering, many structures have a dimension that is much larger than the other two, so the problems are commonly treated with 2D (plane strain) models (e.g., Spangler 1948; Ladanyi and Hoyaux 1969; Atkinson et al. 1974; Hustrulid et al. 1989; Iglesia et al. 1999; Aubertin et al. 2003; Li et al. 2003). There are, nevertheless, notable exceptions, such as the solution given by Van Horn (1964) who used the Marston theory to obtain three dimensional loads on underground

structures. Three dimensional consideration has also been given in cases where one of the lateral walls was removed (e.g., Mitchell 1983).

$$W = \gamma B L dh \quad [1]$$

where γ is the unit weight of the backfill, and dh is the thickness of the layer element.

The vertical force V is obtained by assuming a uniform vertical stress distribution along the horizontal plane:

$$V = \sigma_{vh} B L \quad [2]$$

where σ_{vh} is the vertical stress at position h .

Using the relationship between the vertical stress σ_{vh} and horizontal stress σ_{hhi} in the backfill layer element, the lateral compressive force C_i ($i = 1$ to 4) on each wall can be expressed as:

$$C_m = dh L \sigma_{hhm} = dh L K_m \sigma_{vh} = dh K_m V/B, \quad m = 1, 3 \quad [3a]$$

$$C_n = dh B \sigma_{nhn} = dh B K_n \sigma_{vh} = dh K_n V/L, \quad n = 2, 4 \quad [3b]$$

where the reaction coefficient K_i ($i = 1$ to 4) is the ratio of the horizontal stress to vertical stress. In eq. [3] the horizontal stress could be considered isotropic, then K would be defined as:

$$K = K_i = \sigma_{hhi}/\sigma_{vh}, \quad i = 1 \text{ to } 4 \quad [4]$$

Depending on the wall movement conditions, three particular cases can be considered for the analytical reaction coefficient K based on the Rankine solution (Bowles 1988):

- When the walls do not move and there is no lateral strain, the backfill is in an "at rest" state. The reaction coefficient is then defined as (Jaky 1948):

$$K = K_0 = 1 - \sin\phi'_{bf} \quad [5a]$$

- When the walls converge inward to compress the backfill, the fill is in a passive state. The passive reaction coefficient is then defined as:

$$K = K_p = (1 + \sin\phi'_{bf})/(1 - \sin\phi'_{bf}) \quad [5b]$$

- When the walls diverge and move away from the backfill, the backfill is in an active state. The active reaction coefficient is expressed as:

$$K = K_a = (1 - \sin\phi'_{bf})/(1 + \sin\phi'_{bf}) \quad [5c]$$

In these eq. [5], ϕ'_{bf} is the friction angle of the backfill; the effect of cohesion is neglected in these representations, but it could easily be introduced (as will be shown in a companion paper to be published). In most cases, K_i is expected to be somewhere between K_0 and K_p .

The Coulomb criterion is now used to define the shearing forces S_i ($i = 1$ to 4) along the walls:

$$S_m = (\sigma_{hhm}\tan\delta_m + c)Ldh = (K\sigma_{vh}\tan\delta_m + c)Ldh, \quad m = 1, 3 \quad [6a]$$

$$S_n = (\sigma_{nhn}\tan\delta_n + c)Bdh = (K\sigma_{vh}\tan\delta_n + c)Bdh, \quad n = 2, 4 \quad [6b]$$

where δ_i ($i = 1$ to 4) is the effective friction angle between the fill and i^{th} wall, c is the cohesion along the interface.

The static equilibrium of the layer element in the vertical direction implies that:

$$W = dV + \sum_{i=1}^4 S_i \quad [7]$$

Introducing eqs. [1], [2] and [6] into eq. [7], the following is obtained:

$$\frac{d\sigma_v}{dh} + K\sigma_v \left(\frac{\tan\delta_1 + \tan\delta_3}{B} + \frac{\tan\delta_2 + \tan\delta_4}{L} \right) + 2c \left(\frac{1}{B} + \frac{1}{L} \right) - \gamma = 0 \quad [8]$$

From eq. [8], the vertical stress acting across the horizontal plane at position h is deduced as follow:

$$\sigma_{vh} = (\gamma - 2c(B^{-1} + L^{-1})) \times \frac{1 - \exp\left\{-Kh((\tan\delta_1 + \tan\delta_3)B^{-1} + (\tan\delta_2 + \tan\delta_4)L^{-1})\right\}}{K((\tan\delta_1 + \tan\delta_3)B^{-1} + (\tan\delta_2 + \tan\delta_4)L^{-1})} \quad [9]$$

with $\delta_i \leq \phi'_{bf}$ ($i = 1$ to 4). Note that for $\delta_i > \phi'_{bf}$ ($i = 1$ to 4) along the interface, yielding would take place in the fill (rather than directly along the fill-wall interface), so the value of ϕ'_{bf} should be taken for δ_i ($i = 1$ to 4). The horizontal stress σ_{hh} can be obtained from eqs. [4] and [9].

2.2 Special cases

In practice, the hanging and foot walls are often made of one material while the two side (lateral) walls are of another material (i.e., $\delta_1 = \delta_3$ and $\delta_2 = \delta_4$). In this particular case, eq. [9] becomes:

$$\sigma_{vh} = \frac{1 - \exp\left\{-2Kh(B^{-1}\tan\delta_1 + L^{-1}\tan\delta_2)\right\}}{2K(B^{-1}\tan\delta_1 + L^{-1}\tan\delta_2)} (\gamma - 2c(B^{-1} + L^{-1})) \quad [10]$$

When the four walls of the stope are composed of a unique material (i.e., $\delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta$), eq. [9] becomes identical to Van Horn's (1964) solution:

$$\sigma_{vh} = \frac{1 - \exp\left\{-2Kh(B^{-1} + L^{-1})\tan\delta\right\}}{2K\tan\delta} \left\{ \gamma(B^{-1} + L^{-1}) - 2c \right\} \quad [11]$$

If the stope length is significantly larger than the width (i.e. $L \gg B$), eq. [11] reduces to a 2D solution:

$$\sigma_{vh} = (\gamma B - 2c) \left(\frac{1 - \exp(-2KhB^{-1}\tan\delta)}{2K\tan\delta} \right) \quad [12]$$

Furthermore, when the friction angle of the fill is equal to the fill-wall friction angle (i.e. $\delta = \phi'_{bf}$), eq. [12] becomes:

$$\sigma_{vh} = (\gamma B - 2c) \left(\frac{1 - \exp(-2KhB^{-1}\tan\phi'_{bf})}{2K\tan\phi'_{bf}} \right) \quad [13]$$

For a cohesionless fill (i.e. $c = 0$), eq. [13] is identical to one of the Marston's solutions (McCarthy 1988; Aubertin et al. 2003):

$$\sigma_{vh} = \gamma B \left(\frac{1 - \exp(-2KhB^{-1}\tan\phi'_{bf})}{2K\tan\phi'_{bf}} \right) \quad [14]$$

2.3 Schematic representation

Figure 2 shows the average horizontal (Fig. 2a) and vertical (Fig. 2b) stresses versus elevation h in a backfilled stope of 5-m wide and 10-m long using the general solution given by eq. [9]. The fill friction angle, ϕ'_{bf} , is 35° and the fill-wall friction angles are $\delta_1 = 20^\circ$, $\delta_2 = 25^\circ$, $\delta_3 = 30^\circ$, and $\delta_4 = 35^\circ$. The horizontal and vertical stresses are significantly affected by fill-wall friction with lower friction angles resulting in higher stresses (less arching) and higher friction angles resulting in lower stresses (more arching).

Figure 3 illustrates the influence of the third dimension (stope length) on the horizontal stress σ_{hh} (Fig. 3a) and vertical stress σ_{vh} (Fig. 3b) in a stope of limited length ($L:B = 5:3$). The values were calculated using the three dimensional and two dimensional solutions, eq. [11] and eq. [13] respectively. Both the vertical and horizontal stresses are significantly overestimated by the two dimensional solution, and the degree of overestimation increases with depth. The reason for the difference between the two solutions is that the arching effect of two of the four walls is neglected in the two-dimensional solution.

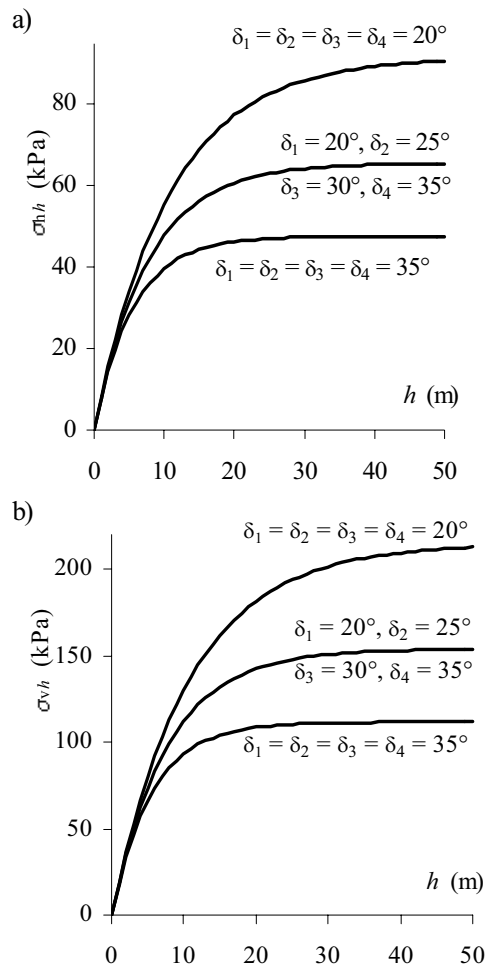


Figure 2. Horizontal (a) and vertical (b) stresses with $B = 5$ m, $L = 10$ m, $\phi'_{bf} = 35^\circ$, and $\gamma = 20$ kN/m³.

The overestimation of the 2D solution decreases as the stope becomes longer due to the reduced influence of the neglected walls. Figure 4 indicates that when the ratio of length to width of the stope, $L:B$, is larger than about 5 to 6, the two dimensional solution could be considered acceptable for most engineering applications, with a typical error of less than 10 to 15%.

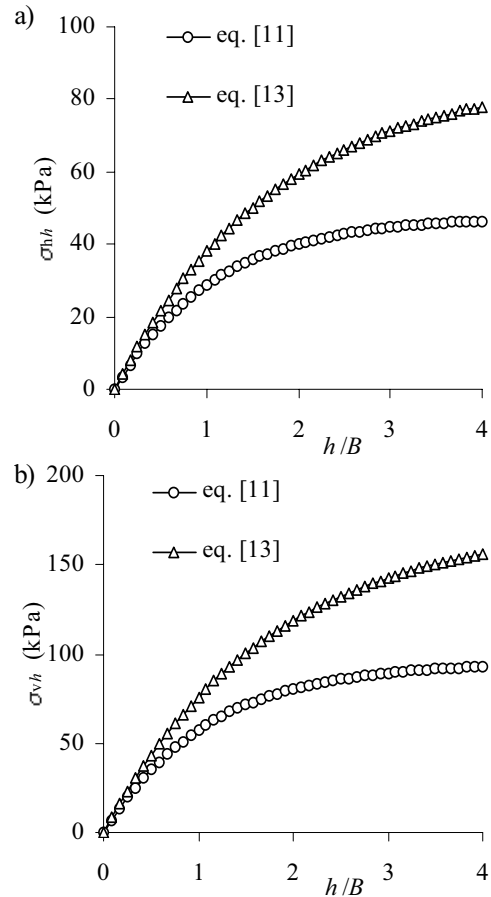


Figure 3. Horizontal (a) and vertical (b) stresses calculated with the 2D (eq. [13]) and 3D (eq. [11]) solutions, with $B = 6$ m, $L = 10$ m, $c = 10$ kPa, $\delta = \phi'_{bf} = 30^\circ$, $K = K_0 = 0.5$, $\gamma = 20$ kN/m³.

3. SAMPLE APPLICATIONS TO EXPERIMENTAL DATA

Some interesting centrifuge test results were reported by Take and Valsangkar (2001) (see also Take 1998). In their experiments, earth pressure cells were mounted inside an aluminum model of backfilled stope. To investigate the effect of boundaries with dissimilar frictional characteristics one wall of the aluminum model was covered with a sheet of 120A-grit sandpaper. The backfill used in these tests is classified as uniformly graded sand with little or no fines. The maximum and minimum dry densities were 1.62 and 1.34 g/cm³ respectively. The angle of internal friction of the sand and the interface friction angles between the sand and the aluminum and sandpaper are given in Table 1. The experiments were performed at an acceleration of 35.7g to simulate a 5 m high stope. With this acceleration, the unit weight of the dense backfill (with a 79% relative density) is equivalent to 0.554 MN/m³, while that of the loose backfill (34% relative density) becomes equivalent to 0.508 MN/m³. Testing was conducted with the sand in

either a loose state (34% relative density) or dense state (79% relative density).

A series of tests were completed by Take and Valsangkar (2001) with the dense backfill. One wall of the model was covered with sandpaper to simulate a rough surface; in this case, $\phi'_{bf} = 36^\circ$, $\delta_1 = \delta_3 = \delta_4 = 25^\circ$, $\delta_2 = 36^\circ$. No existing solutions were available for comparison with these tests, which included two different fill-wall friction angles. Two dimensional calculations with upper and lower bounds were made (Take and Valsangkar 2001) to compare with the experimental results. The proposed general equation (eq. [9]) can be used to more specifically evaluate such a situation. Figure 5 shows a comparison between the experimental results and the proposed solution (eq. [9]), plotted along with the overburden stress value. The proposed solution (eq. [9]) is seen to describe the experimental results quite well.

Take and Valsangkar (2001) conducted another series of tests with the loose backfill. One wall of the model was again covered with sandpaper to simulate a rough surface. The friction angles were $\phi'_{bf} = 30^\circ$, $\delta_1 = \delta_3 = \delta_4 = 23^\circ$, $\delta_2 = 32^\circ$. As indicated previously when the fill-wall friction angle is larger than the fill friction angle ϕ'_{bf} , the latter should be used for the fill-wall interface, so, $\delta_2 = \phi'_{bf} = 30^\circ$ is adopted here (see Table 2). Again, the proposed general solution (eq. [9]) gives a fairly good description to the experimental results (Fig. 6).

4. DISCUSSION

A general 3D analytical solution (eq. [9]) has been developed based on arching theory. This solution can be used to estimate the average earth pressures in a backfilled stope with four walls having different interface properties. Its capability has been shown in the previous section. However, the potential user should keep in mind that the proposed solution is based on some simplifying assumptions. These include:

- The walls around the backfilled stope are assumed to be rigid. The movement of the walls can only be qualitatively considered through the parameter K (isotropic)
- The authors (Aubertin et al. 2003; Li et al. 2003) have previously shown that the backfill placement sequence can significantly influence the stress distribution in backfilled stopes. This aspect can not directly be taken into consideration by the approach presented here.
- The shear stress along the interfaces between the rock and fill is deduced from the Coulomb criterion. Its value corresponds to the maximum stress sustained by the fill material as postulated in the limit equilibrium analysis approach (e.g., Chen and Liu 1990). According to numerical modeling results (Li et al. 2003), this assumption is not fully justified and the arching effect may sometimes be overestimated.
- In the proposed solution, both horizontal and vertical stresses are assumed to be uniformly distributed

across the full width or length of stope. According to numerical modeling results, this assumption has been shown to be acceptable for the 2D case (Li et al. 2003), but it remains to be verified for the 3D case.

- The reaction coefficient K was assumed here to be dependent exclusively on the fill property (isotropic). Li et al. (2003) showed that this is not too far from the numerical modeling results for the 2D case, but remains to be verified for the 3D case. The actual σ_h/σ_v ratio is expected to depend on the magnitude of wall movement and the condition of the backfill (normally consolidated, homogeneous and isotropic are assumed).

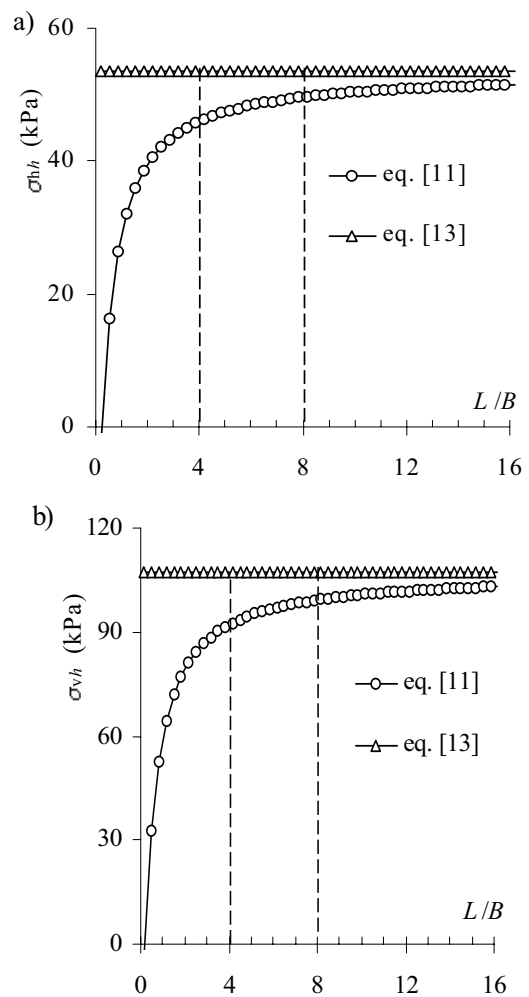


Figure 4. Horizontal (a) and vertical (b) stresses calculated with the 2D (eq. [13]) and 3D (eq. [11]) solutions for various stope length to width ratios, with $B = 6$ m, $h = 10$ m, $c = 10$ kPa, $\delta = \phi'_{bf} = 30^\circ$, $K = K_0 = 0.5$, $\gamma = 20$ kN/m³.

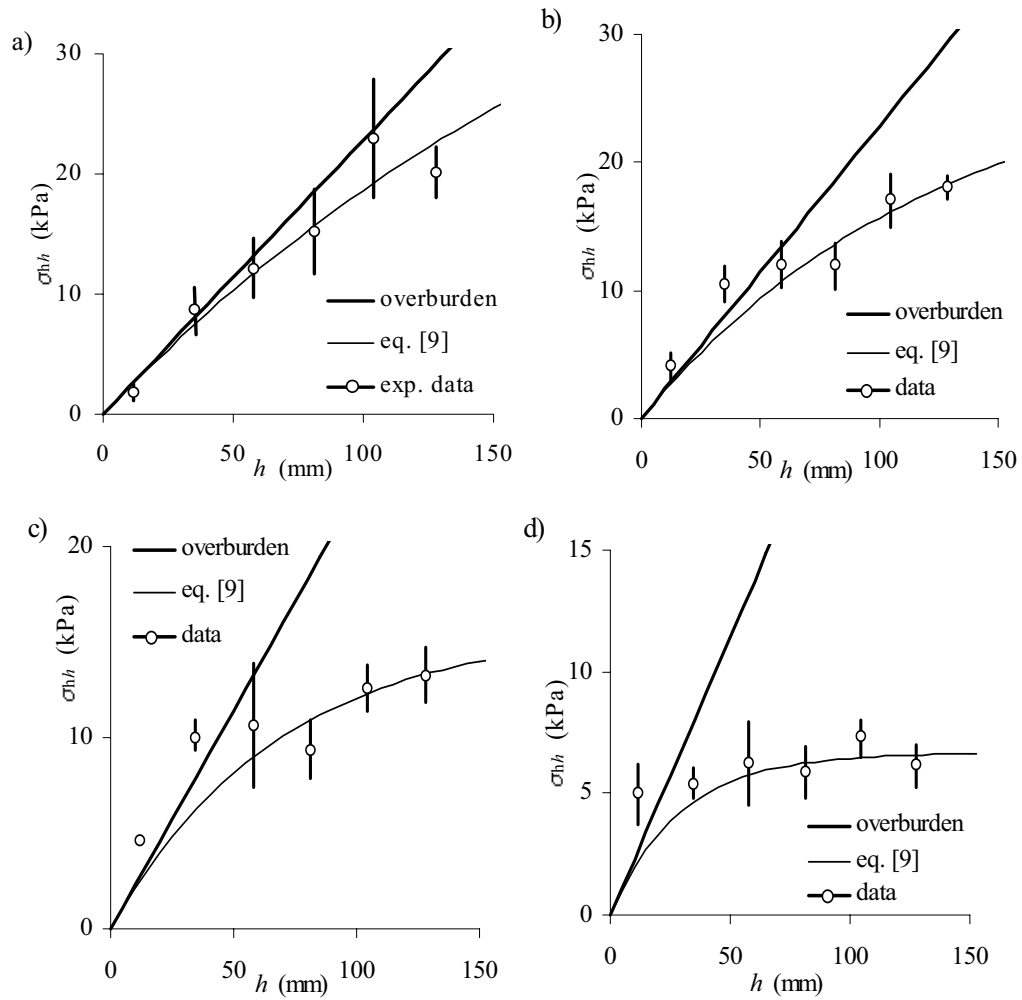


Figure 5. Comparison between the proposed general solution and the experimental results obtained on model backfilled with dense sand (data taken from Take and Valsangkar 2001; see text for more details).

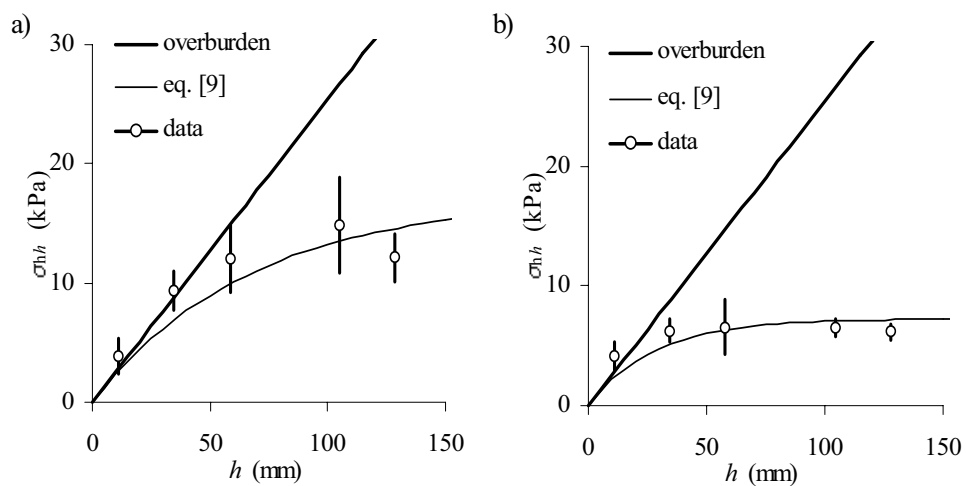


Figure 6. Comparison between the proposed solution and experimental results obtained on model backfilled with loose sand (data taken from Take and Valsangkar 2001; see text for more details).

Table 1. Internal (ϕ') and interface (δ) friction angles of model backfill material (from Take and Valsangkar 2001).

	Loose backfill (34% relative density, unit weight 1.42 g/cm ³)	Dense backfill (79% relative density, unit weight 1.55 g/cm ³)
ϕ' backfill	30°	36°
δ backfill and aluminum	23°	25°
δ backfill and sandpaper	32°	36°

Table 2. Parameters used in Figs. 5 and 6.

Figure	Backfill	Geometry B (cm) \times L (cm)	Fill parameters		Wall-fill friction angle [‡] (°)			
			c	ϕ'_{bf} (°)	δ_1	δ_2	δ_3	δ_4
5a	dense	18.4 \times 25.4	0	36	25	36	25	25
5b	dense	7.5 \times 25.4	0	36	25	36	25	25
5c	dense	3.8 \times 25.4	0	36	25	36	25	25
5d	dense	1.5 \times 25.4	0	36	25	36	25	25
6a	loose	3.8 \times 25.4	0	30	23	32 [‡]	23	23
6b	loose	1.5 \times 25.4	0	30	23	32 [‡]	23	23

[‡] The backfill friction angle, ϕ'_{bf} , is used when the wall-fill friction angle δ is higher than ϕ'_{bf} .

- The backfill is assumed to obey a Coulomb failure criterion. Some more representative models that can take into account the mean pressure dependence behavior (e.g., Aubertin et al. 2000) could provide more realistic solutions.
- For the case where $\delta_1 \neq \delta_3$ or $\delta_2 \neq \delta_4$, the layer element in Fig. 1 may be distorted and warped. In this case, the procedure for deducing the general solution is not rigorous. Eq. [9] should be taken as an approximate solution.

The solution is applicable only for vertical stopes. Extension to inclined stope geometry is needed since underground mine stopes are commonly inclined to some degree.

5. CONCLUSION

A 3D analytical solution based on the arching theory has been developed to evaluate the earth pressures in narrow, vertical backfilled stopes. The solution takes into account different shear properties along the four walls. It is shown that the stope length has a significant influence on the earth pressure in the backfill, especially when the length to width ratio is smaller than about five to six. The versatility and descriptive capability of the proposed solution compare favorably with centrifuge test results taken from the literature. The proposed analytical solution can thus be used to estimate the earth pressure in narrow, vertical backfilled stope at the preliminary phase of project.

6. ACKNOWLEDGEMENT

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7. REFERENCES

- Atkinson, J.H., Cairncross, A.M., James, R.G. 1974. Model tests on shallow tunnels in sand and clay. *Tunnels and Tunnelling*, **6**(4): 28-32.
- Aubertin, M. 1999. Application de la mécanique des sols pour l'analyse du comportement des remblais miniers souterrains. Short Course (unpublished notes), 14^e Colloque en Contrôle de Terrain, Val-d'Or, 23-24 mars 1999. Association Minière du Québec.
- Aubertin, M., Bussière, B., Bernier, L. 2002. Environnement et gestion des rejets miniers. Manual on CD-ROM, Presses Internationales Polytechniques.
- Aubertin, M., Li, L., Simon, R. 2000. A multiaxial stress criterion for short term and long term strength of isotropic rock media. *International Journal of Rock Mechanics and Mining Sciences*, **37**:1169-1193.
- Aubertin, M., Li, L., Arnoldi, S., Belem, T., Bussière, B., Benzaazoua, M., Simon, R. 2003. Interaction between backfill and rock mass in narrow stopes. In P.J. Culligan, H.H. Einstein, A.J. Whittle (eds), *Soil and Rock America 2003*, vol. 1, pp. 1157-1164. Essen: Verlag Glückauf Essen (VGE).
- Blight, G.E. 1986. Pressure exerted by materials stored in silos. Part I: coarse materials. *Géotechnique*, **36**(1): 33-46.
- Bowles, J.E. 1988. *Foundation Analysis and Design*. McGraw-Hill.

- Brechtel, C.E., Struble, G.R., Guenther, B. 1999. The evaluation of cemented rockfill spans at the Murray mine. *In* B. Amadei, Kranz, Scott, Smeallie (eds.), *Rock Mechanics for Industry*, vol. 1, pp. 481-487. Rotterdam: Balkema.
- Brummer, R.K., Gustas, R., Landriault, D.A., Steed, C.M. 1996. Mining under backfill - Field measurements and numerical modelling. *In* M. Aubertin, F. Hassani, H. Mitri (eds.), *Rock Mechanics: Tools and Techniques*, pp. 269-276. Rotterdam: A.A. Balkema.
- Chen, W.F., Liu, X.L. 1990. Limit analysis in soil mechanics. Amsterdam: Elsevier.
- Cowin, S.C. 1977. The theory of static loads in bins. *Journal of Applied Mechanics*, **44**: 409-412.
- Frydman, S.F., Keissar, I. 1987. Earth pressure on retaining walls near rock mass faces. *Journal of geotechnical Engineering, ASCE*, **113**(6): 586-599.
- Hassani, F., Archibald, J.H. 1998. Mine Backfill. CIM, CD-ROM.
- Hassani, F., Fotoohi, K. 1997. Quantitative evaluation of pastefill performance to alleviate rock burst. Séminaire sur l'Emploi du Remblai en Pâte dans les Mines Souterraines, Bibliothèque Nationale du Québec, vol. 1, pp. 65-77.
- Hustrulid, W., Qianyan, Y., Krauland, N. 1989. Modeling of cut-and-fill mining systems – Näsleden revisited. *In* F.P. Hassani, M.J. Scoble, T.R. Yu (eds), *Innovation in Mining Backfill Technology*, pp. 147-164. Rotterdam: Balkema.
- Iglesia, G.R., Einstein, H.H., Whitman, R.V. 1999. Determination of vertical loading on underground structures based on an arching evolution concept. *In* C. Fernandez, R.A. Bauer (eds.), *Geo-Engineering for Underground Facilities*, pp. 495-506. Geo-Institute of ASCE.
- Jaky, J. 1948. Pressure in silos. *Proceedings of the 2nd International Conference on Soil Mechanics and Foundation Engineering*, vol. 1, pp. 103-107. Rotterdam: Balkema.
- Janssen, H.A. 1895. Versuche über Getreidedruck in Silozellen. *Zeitschrift Verein Ingenieure*, **39**: 1045-1049.
- Knutsson, S. 1981. Stresses in the hydraulic backfill from analytical calculations and in-situ measurements. *In* O. Stephansson, M.J. Jones (eds), *Proceedings of the Conference on the Application of Rock Mechanics to Cut and Fill Mining*: 261-268. Institution of Mining and Metallurgy.
- Kutzner, C. 1997. Earth and rockfill dams: Principles of design and construction. Rotterdam: Balkema.
- Ladanyi, B., Hoyaux, B. 1969. A study of the trap-door problem in a granular mass. *Canadian Geotechnical Journal*, **6**(1): 1-14.
- Landriault, D.A., Brown, R.E., Counter, D.B. 2000. Paste backfill study for deep mining at Kidd Creek. *CIM Bulletin*, **93**(1036): 156-161.
- Li, L., Aubertin, M., Simon, R., Bussière, B., Belem, T. 2003. Modeling arching effects in narrow backfilled stopes with FLA. *In* R. Brummer, P. Andrieux, C. Detournay, R. Hart (eds.), *FLAC and Numerical Modeling in Geomechanics – 2003*, pp. 211-219. Lisse: A.A. Balkema.
- Marston, A. 1930. The theory of external loads on closed conduits in the light of latest experiments. Bulletin No. 96, Iowa Engineering Experiment Station, Ames, Iowa.
- McCarthy, D.F. 1988. *Essentials of Soil Mechanics and Foundations: Basic Geotechnics*. Prentice Hall.
- Mitchell, R.J. 1983. *Earth structures engineering*. Boston: Allen & Unwin.
- Pariseau, W.G. 1981. Finite-element method applied to cut and fill mining. *In* O. Stephansson, M.J. Jones (eds.), *Application of Rock Mechanics to Cut and Fill Mining: Proceedings of the Conference on the Application of Rock Mechanics to Cut and Fill Mining*, June 1-3, 1980, Lulea, pp. 284-292. Institution of Mining and Metallurgy.
- Richards, J.C. 1966. *The Storage and Recovery of Particulate Solids*. Institution of Chemical Engineers, London.
- Richmond, O., Gardner, G.C. 1962. Limiting spans for arching of bulk materials in vertical channels. *Chemical Engineering Science*, vol. 17, pp. 1071-1078.
- Spangler, M.G. 1948. Underground conduits – An appraisal of modern research. *Transactions, American Society of Civil Engineers*, **113**: 316-374.
- Take, W.A. 1998. Lateral earth pressures behind rigid fascia retaining walls. MSE thesis, University of New Brunswick.
- Take, W.A., Valsangkar, A.J. 2001. Earth pressures on unyielding retaining walls of narrow backfill width. *Canadian Geotechnical Journal*, **38**: 1220-1230.
- Terzaghi, K. 1936. Stress distribution in dry and in saturated sand above a yielding trap-door. *Proceedings of 1st International Conference on Soil Mechanics and Foundation Engineering*, Cambridge, Mass., vol. 1, pp. 307-311.
- Terzaghi, K. 1943. *Theoretical Soil Mechanics*. John Wiley & Sons.
- Thomas, E.G., Nantel, J.H., et Notley, K.R. 1979. *Fill Technology in Underground Metalliferous Mines*. International Academic Services, Kingston, Canada.
- Udd, J.E. 1989. Backfill research in Canadian mines. *In* F.P. Hassani, M.J. Scoble, T.R. Yu (eds), *Innovation in Mining Backfill Technology*, pp. 3-13. Rotterdam: Balkema.
- Van Horn, D.A. 1964. A study of loads on underground structures. *Proceedings of the Symposium on Soil-Structure Interaction*, 8 - 11 June 1964. Tucson, Arizona: University of Arizona, pp. 256-282.
- Williams, J.C., Al-Salman, D., Birks, A.H. 1987. Measurement of static stresses on the wall of a cylindrical container for particulate solids. *Power Technology*, **50**: 163-175.