A NUMERICAL INVESTIGATION OF THE CREEP (VISCOPLASTIC) BEHAVIOUR OF CIRCULAR OPENING AND PILLAR IN ROCKSALT

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ABSTRACT

The mechanical behaviour of rocksalt is mostly controlled by inelastic straining. Analyses of rock structures in rocksalt must then take into account these rate-dependant strains. In this paper, three constitutive models have been used to characterize rocksalt mechanical response, namely: the Norton Creep Law, a Modified Strain Hardening Creep Law (SH) and a unified model ($SUVIC_{sh}$). Each model was used to perform numerical analyses with an object-oriented finite element code, on pressurized thick wall cylinders and mine pillars. The calculation results are compared in terms of time-dependent deformation and stress for each constitutive approach, to highlight their main features. Results showed that the model used has a great influence on the numerical results obtained.

RÉSUMÉ

Le comportement mécanique du sel est principalement contrôlé par les déformations inélastiques. Des analyses de structures rocheuses dans une roche saline doivent donc considérer ces déformations différées. Dans cet article, trois modèles constitutifs ont été utilisés pour caractériser le comportement mécanique du sel, soit : la loi de fluage de Norton, un modèle de loi d'écrouissage modifiée (SH) et un modèle unifié ($SUVIC_{sh}$). Chaque modèle a été utilisé pour réaliser des analyses numériques à l'aide d'un code d'éléments finis sur des ouvertures cylindriques à paroi épaisse sous pression et sur des piliers de mine. Les résultats numériques ont été comparés en terme de contraintes et de déformations dans le temps. Les comparaisons montrent que le modèle utilisé a une grande influence sur les résultats numériques obtenus.

1. INTRODUCTION

The mechanical behaviour of rocksalt shows very distinctive features in comparison with other common rock types such as hard rocks found in the Canadian Shield. The behaviour of rocksalt is more ductile, and its increased deformability is accompanied by a strong time-dependency. This non-linear rate-dependent behaviour must be taken into account for the analysis of underground openings in rocksalt. However, modeling such response is a challenging task, especially when dealing with the different inelastic phases which typically include quasi-instantaneous (elastic and/or plastic), transient and steady-state responses.

The mathematical formulation (or constitutive model) used to describe rocksalt behaviour can have a great influence on the results calculated with analytical or numerical solutions. In this paper, the authors present the results of numerical analyses of underground structures in rocksalt performed using three different models. A modified version of the strain-hardening model is shown to describe well the behaviour of openings under conditions relevant for geomechanical applications.

2. MODELLING THE INELASTIC BEHAVIOUR OF ROCKSALT

To describe the behaviour of rocksalt, it is customary to identify up to four different straining phases on the strain-time curve. During the loading stage, there is the pseudo-instantaneous strain phase which includes the elastic ε^e and plastic ε^p strains. Then follow three phases of time-dependant (creep) ε^c strains: transient ε_t (or primary), steady-state ε_s (or stationary,) and accelerating ε_a (or tertiary), with the last one being frequently omitted in usual applications. The total strain rate can then be expressed with the following portioned equations:

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p + \dot{\varepsilon}^c \quad [1]$$

with

$$\dot{\varepsilon}^c = \dot{\varepsilon}_t + \dot{\varepsilon}_s + \dot{\varepsilon}_a \quad [2]$$

Each component in these equations can be described by distinct functions (or laws). Alternatively, unified formulations can also be used. In this case, all the inelastic components (i.e. $\dot{\varepsilon}^p$ and $\dot{\varepsilon}^c$) are represented by a single kinetic law for $\dot{\varepsilon}^i$ (e.g. Aubertin et al. 1991). Here, three constitutive models have been used to characterize rocksalt inelastic response, namely: the Norton Creep Law associated to a steady-state flow law, a modified Strain Hardening Creep Law (SH) that also includes steady-state flow, and the $SUVIC_{sh}$ model containing multiaxial internal state variables, which was developed over the last decade or so. The isotropic hardening SH model introduced here is in fact a simplified version of the $SUVIC_{sh}$ model, which ignores the tensorial components of hardening. These models are briefly described below.

2.1 The Norton Creep Law

This constitutive model is a classical power law used to describe stationary creep. It can be written as:

$$(\dot{\varepsilon}_e)_s = B\sigma_e^n \qquad [3]$$

where $(\dot{\varepsilon}_e)_s$ is the steady-state creep rate (expressed using the equivalent von Mises strain), σ_e is the deviatoric stress state, B and n are material parameters. It is important to mention that the Norton Creep Law is only an approximation of the actual creep behaviour of salt. It neglects the straining occurring in the transient phase, and it idealises the stress-strain rate relationship (which has been shown to better obey hyperbolic sine law; e.g. Julien 1999, Yahya et al. 2000). This model is nevertheless largely used because of its simplicity of application (e.g, Klein 2003). However, fundamental limitations may induce some significant deviation from the actual rocksalt behaviour, especially under the complex loading conditions encountered in a natural geomechanical setting (e.g. Aubertin et al. 1993, 1999a).

2.2 The $SUVIC_{sh}$ model

The SUVIC model (for Strain rate history-dependant Unified Viscoplastic model with Internal variables for Crystalline materials) is an unified model with internal state variables (ISV). Since its first version (Aubertin et al. 1991), several modifications have been performed (Aubertin et al. 1999a,b; Yahya et al. 2000a,b), leading to the $SUVIC_{sh}$ formulation (where subscript sh stands for hyperbolic sine law for the saturation of ISV).

Under uniaxial stress, the basic equation used for the flow law is given by (Aubertin et al. 1991):

$$\dot{\varepsilon}^i = A\left\langle \frac{\sigma - \sigma_i}{K} \right\rangle^N \qquad [4]$$

with $\sigma_i = B + R$ [5]

where $\dot{\varepsilon}^i$ is the inelastic strain rate, A and N are material parameters, B, R and K are internal state variables, and σ_i is an internal stress. The model can also be expressed in a tensorial form (Aubertin et al. 1999b):

$$\dot{\varepsilon}_{ij}^i = A\left\langle \frac{X_{ae} - R}{K} \right\rangle^N n_{ij}$$

with

$$X_{ae} = \left\| S_{ij} - B_{ij} \right\| = \left[\frac{3}{2}\left(S_{ij} - B_{ij}\right)\left(S_{ij} - B_{ij}\right) \right]^{1/2} \qquad [6]$$

$$n_{ij} = \frac{3}{2}\frac{\left(S_{ij} - B_{ij}\right)}{\left\| S_{ij} - B_{ij} \right\|}$$

where S_{ij} is the deviatoric stress tensor and B_{ij} is the kinetic creep tensor with $\langle x \rangle = \frac{1}{2}\left(x + |x|\right)$.

The three internal state variables (B, R, K) evolve during straining at a rate defined by their respective growth law. Upon steady-state, each one has reached its saturation values (B', R', K'). More details about the $SUVIC_{sh}$ model can be found in Aubertin et al. (1999), Julien (1999), Yahya et al. (2000a,b) and Boulianne (2003). This model can also include a damage component to describe the semi-brittle (mean-stress dependant) behaviour up to failure, but this aspect is not treated here.

2.3 The Modified Strain-Hardening Model (SH)

Although the $SUVIC_{sh}$ model is very powerful and efficient in its description of the inelastic behaviour of rocksalt, it can be somehow arduous to apply because of its numerous parameters. For that reason, a simplified version has been proposed, where the internal stresses (B and R) are neglected. This new formulation is then given by (Boulianne 2003):

$$\dot{\varepsilon}_{ij}^i = A^*\left\langle \frac{\sqrt{3/2}\left\| S_{ij} \right\|}{K} \right\rangle^{N^*} \frac{3}{2}\frac{S_{ij}}{\left\| S_{ij} \right\|} \qquad [7]$$

In this kinetic law, the isotropic strain-hardening (SH) is related to the following evolution law:

$$\dot{K} = A_5^*\left(1 - \frac{K}{K'}\right)\dot{\varepsilon}_e^i \qquad [8]$$

$$K' = \frac{\sigma_0^* a^* \sinh\left(\dot{\varepsilon}_e^i / \dot{\varepsilon}_0^*\right)^{1/n^*}}{\left(\dot{\varepsilon}_e^i / A^*\right)^{1/N^*}} \qquad [9]$$

where A^*, A_5^*, a^*, σ_0^*, $\dot{\varepsilon}_0^*$, N^* et n^* are material parameters and K is the isotropic stress. The value of K' represents the isotropic hardening variable K at saturation, which is reached when the flow is in a steady-state.

This SH model initially behaves as a classical strain-hardening law at small inelastic strain, but contrary to the latter, it ultimately reaches a stationary creep condition at larger strain. Figure 1 shows a simulation of creep tests on rocksalt at different stress levels, comparing the SH and the $SUVIC_{sh}$ models responses. As it can be seen, the SH model can approximate fairly well the creep behaviour obtained with the $SUVIC_{sh}$ model, which has been shown to provide a good description of rocksalt response. The details on these calculations are found in Boulianne (2003).

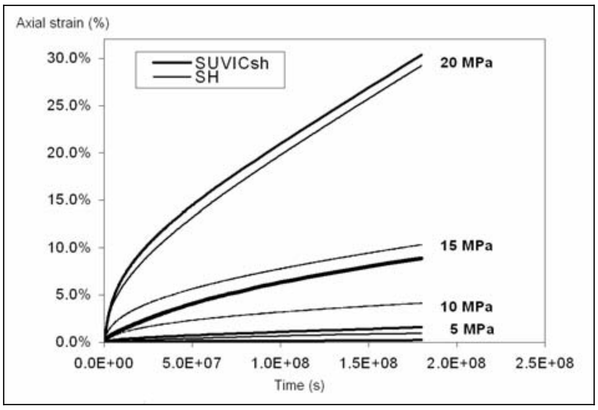

Figure 1. Comparison between the SH and $SUVIC_{sh}$ models for simulated creep tests on rocksalt at different stress levels (after Boulianne 2003).

3. NUMERICAL MODELLING

These three models presented above have been introduced into a numerical code to perform geomechanical calculations. The software used for the numerical modelling is a finite element code called ZéBuLoN that was developed at the École Nationale des Mines de Paris (Burlet et Cailletaud, 1991). The version 8 of the software offers a great versatility, allowing the user to implement its own constitutive models. A programming language called ZebFront (Foerch 1996) was developed so that users could implement models without having to modify the main code.

The software already included the Norton creep law. The $SUVIC_{sh}$ model was implemented in Zébulon by Julien (1999; see also Julien et al. 1996), and the SH model was implemented by Boulianne (2003).

3.1 Thick cylinder submitted to an internal pressure

3.1.1 Validation of the numerical model

The Norton creep law lends itself to some analytical solutions for simplified problems, such as thick walled cylinder submitted to internal and/or external pressure (e.g. Boyle and Spence 1983; Skrzypek et Hetnarski 1993). For a thick cylinder with an internal radius a and outside radius b, the long term radial σ_r and tangential σ_θ stresses at a radius r ($a \le r \le b$) are given by :

$$\sigma_r = -P_i \left(\frac{\left(b/r\right)^{\frac{2}{n}} - 1}{\left(b/a\right)^{\frac{2}{n}} - 1} \right) \quad [10]$$

$$\sigma_\theta = P_i \left(\frac{1 - \left(1 - 2/n\right)\left(b/r\right)^{\frac{2}{n}}}{\left(b/a\right)^{\frac{2}{n}} - 1} \right) \quad [11]$$

where P_i is the internal pressure. A 2D model of a thick cylinder was created with Zébulon with an internal radius of 2.5 m and an outside radius of 7.5 m and submitted to a constant internal pressure of 25 MPa. Figure 2 shows the comparison of the numerical results using the Norton creep law with the analytical solution given by Eq. 10 and 11. As can be seen, the numerical results are similar to the analytical solution when the steady-state is reached after about one day of loading.

Many other calculations were performed to validate the results from the numerical modeling (Boulianne 2003).

3.1.2 Comparison of constitutive models

The thick cylinder was analysed further using the Norton model, the $SUVIC_{sh}$ model and the SH model to illustrate the importance of the constitutive equation. The same dimensions were used but the loading conditions consisted of an internal pressure of 5 MPa applied for 120 Ms (3.8 years) followed by a pressure of 7.5 MPa applied from 120 to 180 Ms (5.7 years) as shown in Figure 3. Figure 4 to 6 shows the results obtained with the different models.

As it can be seen in Fig. 4, there is an almost perfect correlation between the numerical results and the analytical solution with the Norton equation. However, Fig. 5 shows that the results obtained with the $SUVIC_{sh}$ are different, particularly for the long term. As the $SUVIC_{sh}$ constitutive model is considered to be more representative of the real creep behaviour of rocksalt (compared to the Norton law), results shown in Fig. 5 indicate that the use of the steady-state power law can induce some significant deviations for the stress distribution.

As for the results obtained with the SH model (Fig. 6), they compare well with the $SUVIC_{sh}$ results, although the long term tangential stress is slightly overestimated near the inside radius (Fig. 6d).

The difference between the models is even more pronounced when one looks at the strains calculated at the internal radius of the cylinder (Fig. 7). Here the $SUVIC_{sh}$ and the SH models again give fairly similar results but the Norton model largely underestimates the strain because it only considers the steady-state creep phase.

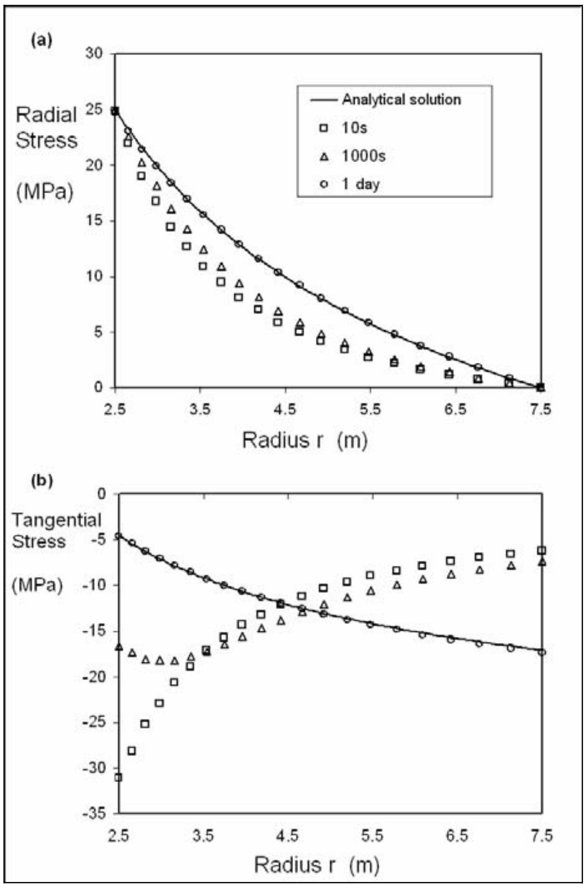

Figure 2. Variation of stresses inside along the cylinder radius and comparison with the analytical solution for a Norton creep law. a) Radial stress. b) Tangential stress (for a power law exponent n = 4, after Boulianne 2003).

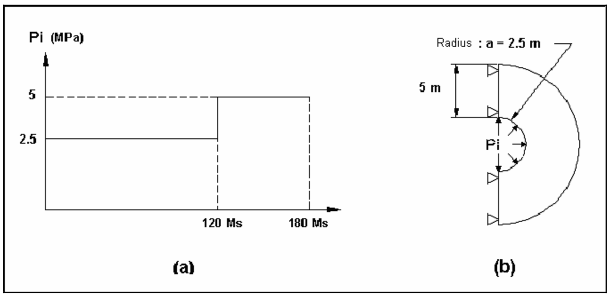

Figure 3. Model of the thick cylinder submitted to an internal pressure: (a) applied pressure as a function of time; (b) Schematic representation of the model.

These results serve to further highlight the inherent limitations of the Norton creep law, when it comes to describing (and predicting) the actual response of rocksalt, for a simple cylindrical opening submitted to a simple external loading state. Such limitation can be even more pronounced in the case of more complex geometry and/or loading conditions.

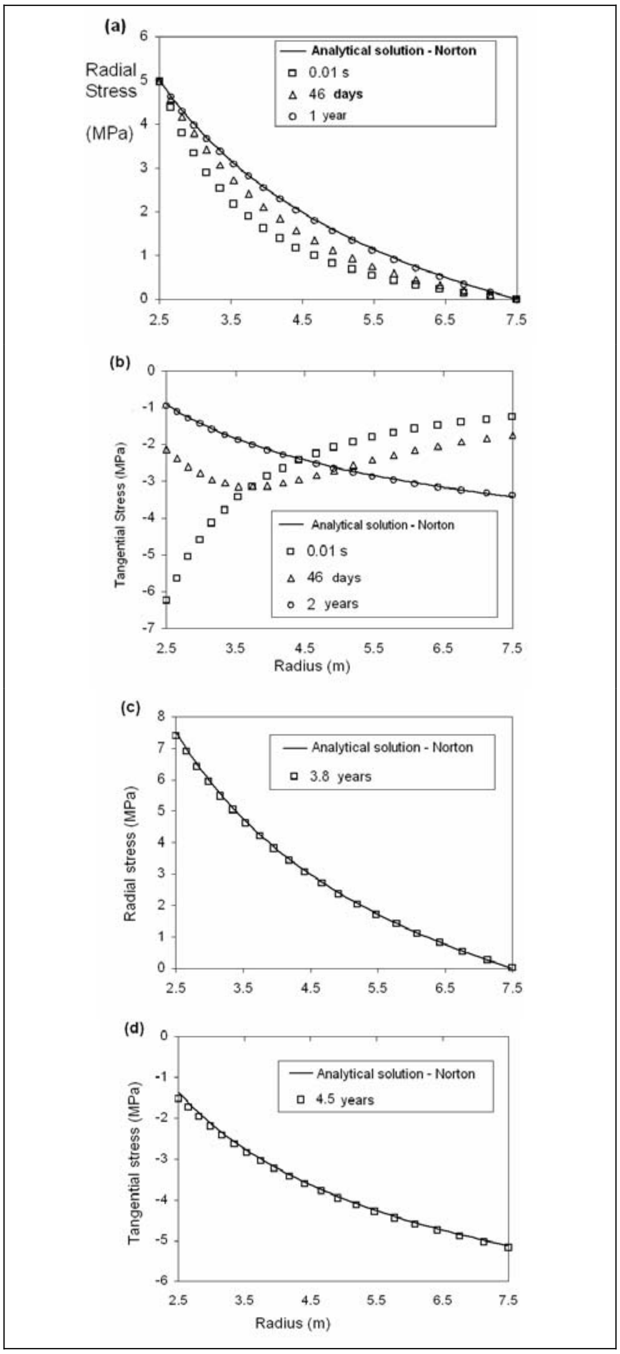

Figure 4. Evolution of the stress distribution inside a thick cylinder using the Norton model. a) Radial stress with P_i = 5 MPa. b) Tangential stress with P_i = 5 MPa. c) Radial stress with P_i = 7.5 MPa. d) Tangential stress with P_i = 7.5 MPa (after Boulianne 2003).

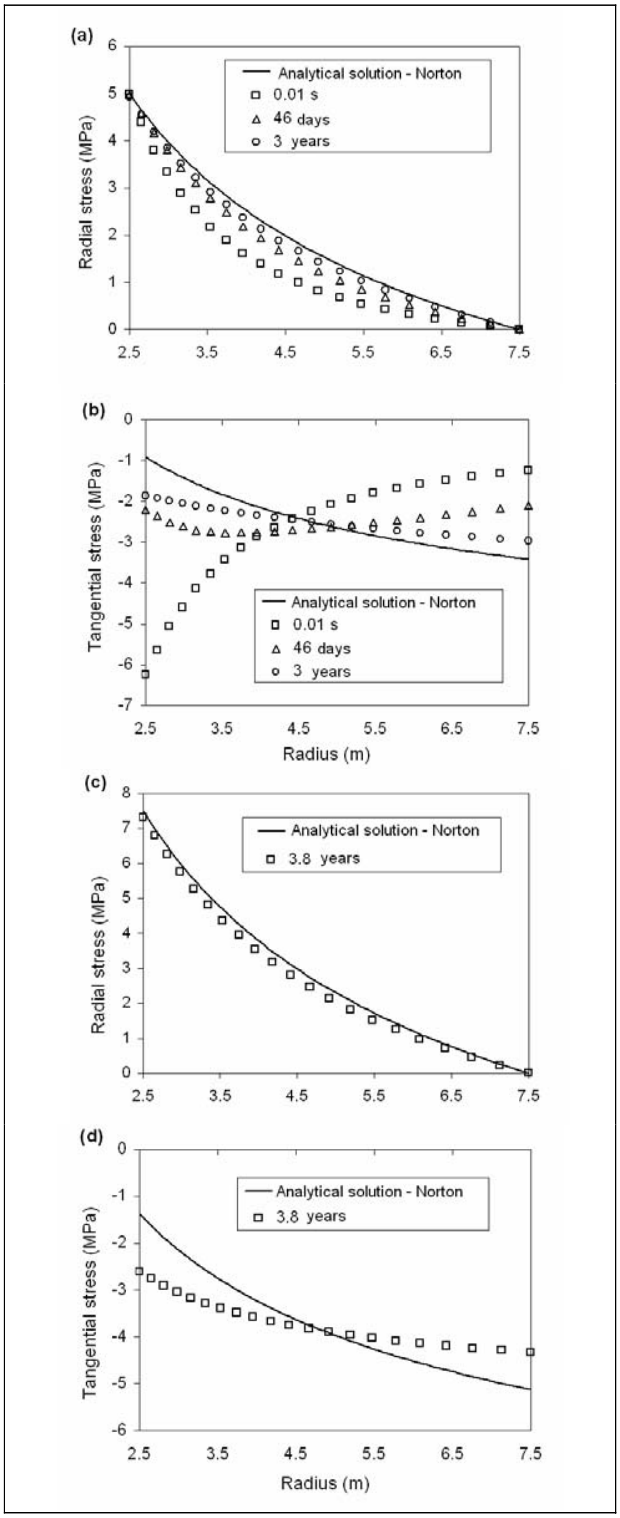

Figure 5. Evolution of the stress distribution inside a thick cylinder using the $SUVIC_{sh}$ model. a) Radial stress with P_i = 5 MPa. b) Tangential stress with P_i = 5 MPa. c) Radial stress with P_i = 7.5 MPa. d) Tangential stress with P_i = 7.5 MPa (after Boulianne 2003).

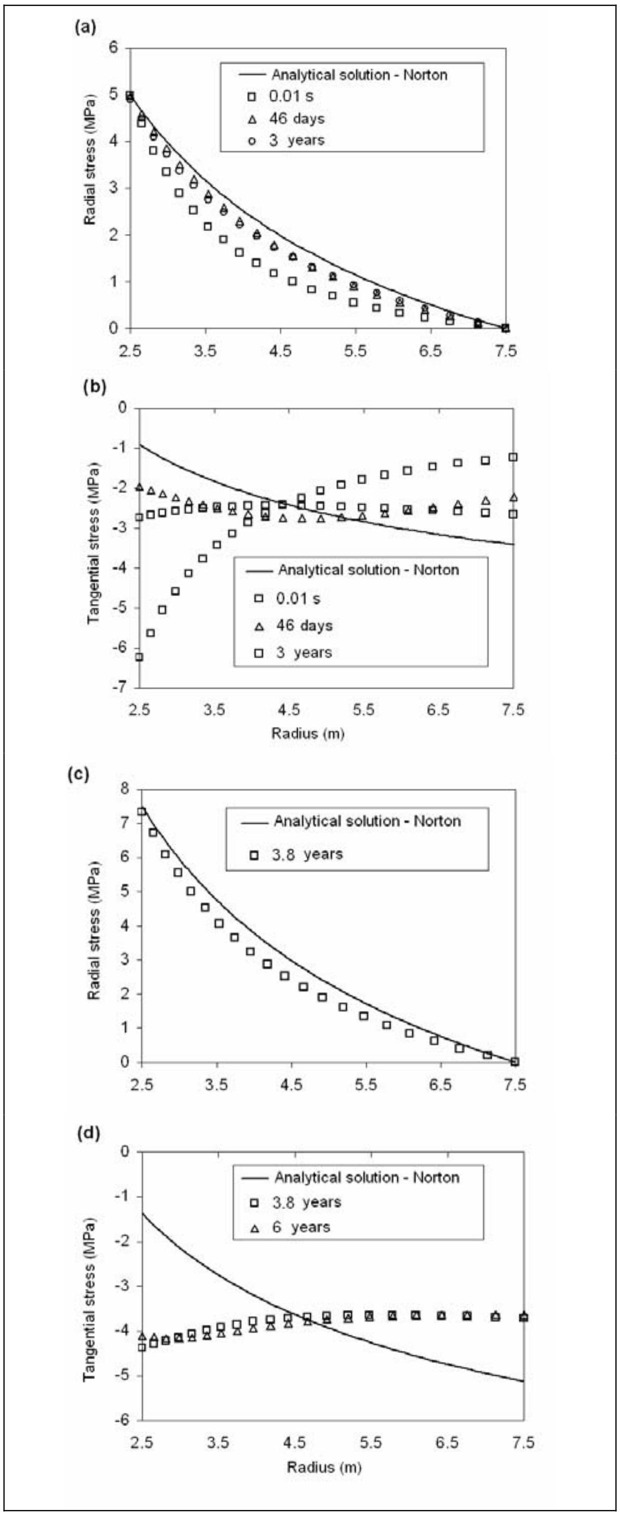

Figure 6. Evolution of the stress distribution inside a thick cylinder using the SH model. a) Radial stress with P_i = 5 MPa. b) Tangential stress with P_i = 5 MPa. c) Radial stress with P_i = 7.5 MPa. d) Tangential stress with P_i = 7.5 MPa (after Boulianne 2003).

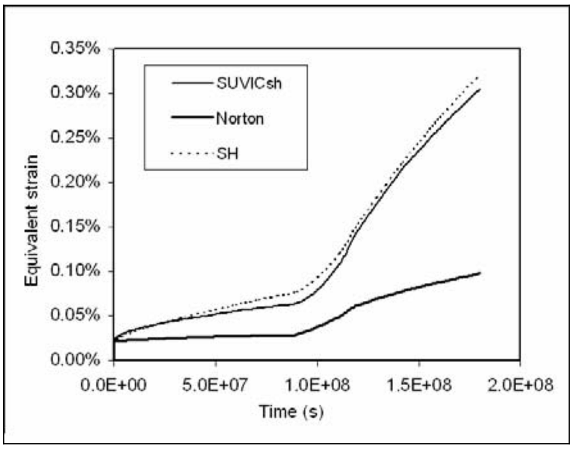

Figure 7. Comparison of the equivalent strain ε_e calculated at the internal boundary for the thick cylinder with the different models (after Boulianne 2003).

3.2 Numerical modelling of a pillar in rocksalt

A 2D model was built in Zébulon to evaluate the creep behaviour of rocksalt in a mine pillar, typical of a room-and-pillar mining method (Fig. 8). The pillar is located in a rock mass under a hydrostatic stress field of 5 MPa. To take into account the effect of the stripping ratio on the horizontal stress acting in the sill pillar in this small scale model, the tributary theory is applied; it yields a horizontal stress of 6.67 MPa. This situation was analysed using the three constitutive models.

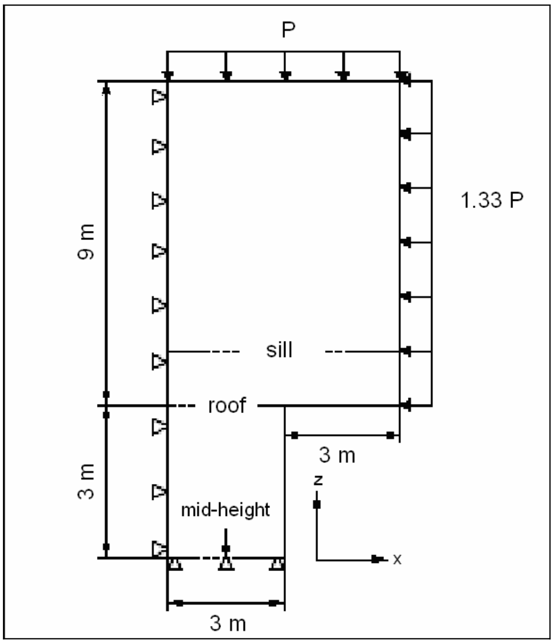

Figure 8. 2D model representing a ¼ of a long pillar symmetrical in the x and z axis with a Height/Width ratio of 1.0 in a hydrostatic stress field of 5 MPa.

Figure 9 and 10 show the results obtained with the different models at three locations in the rock mass after 1 year and 5.7 years respectively.

As can be seen, although the external stress levels are similar, the difference between the results is significant and tends to increases as time goes by. The stresses obtained with the Norton model are smaller in the pillar but larger in the roof of the excavation. The differences are seen both in the magnitude and distribution of the stresses. The difference between the models is even more pronounced when looking at the straining results shown in Figure 11. The strains obtained with the Norton creep law are less than half the ones obtained with the $SUVIC_{sh}$ model. This further indicates that the Norton model may not be appropriate for the modeling of openings in rocksalt, as it can underestimate strongly inelastic strains.

4. DISCUSSION

The numerical modelling results have helped to illustrate some of the main differences between three constitutive models: the Norton creep law, the modified strain-hardening model SH and the $SUVIC_{sh}$ model. The first two models represent respectively the stationary creep and the transient/stationary creep. The $SUVIC_{sh}$ model is a unified model that can describe the entire inelastic behaviour of rocksalt.

From a user's point of view, the application of a model like $SUVIC_{sh}$ may be complex because of its numerous equations (not all shown in this paper) and the amount of information needed to find the several material parameters. But once these equations are implemented into a numerical code, they are expected to give a better description and representation of the creep behaviour of rocksalt and other similar soft rocks.

The results shown in this paper indicate that for the time frame considered, the creep behaviour is mostly controlled by the transient phase. Since the Norton creep law does not take into account the transient phase, the results obtained with that approach significantly underestimate inelastic strains. The results showed that the differences can reach more than 200% compared to the results obtained with the $SUVIC_{sh}$ model (Figs. 7 and 11a) and sometimes more than 500% (Fig. 11b).

From an engineering point of view, the use of a constitutive model like $SUVIC_{sh}$ may be required for applications where detailed predictions are needed. This would be the case in situations like the planning and design of large openings in salt and potash mines at great depth, where stresses and temperature induce significant stress distribution and large inelastic strains. In these situations, it would be advisable to use constitutive equations that can properly describe the inelastic behaviour of the material at hand.

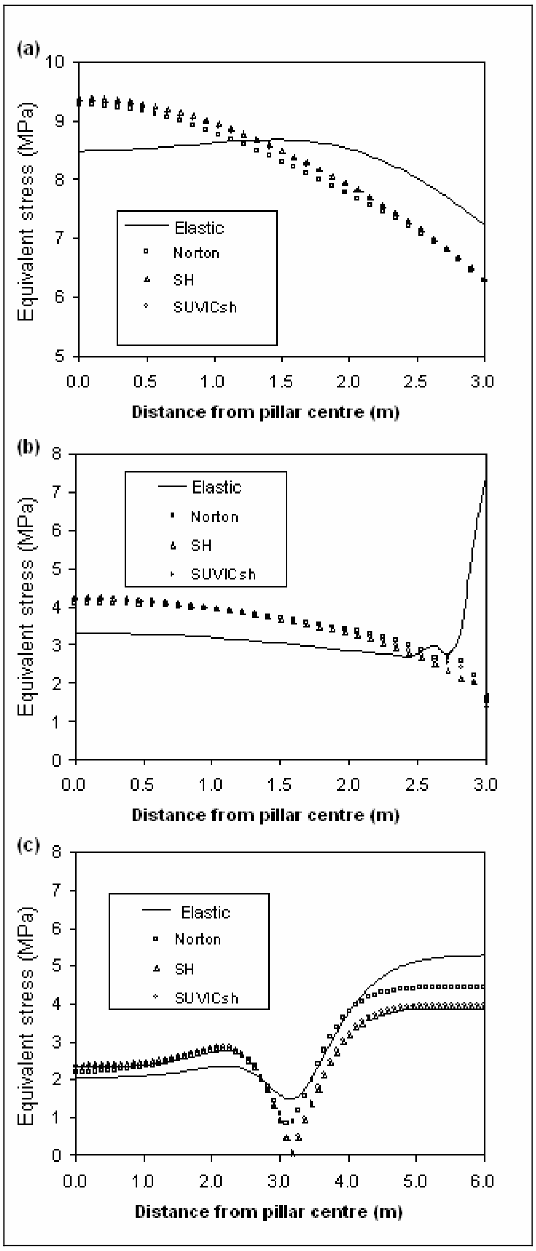

Figure 9. Comparison of the equivalent stress distribution obtained after 30 Ms (1 year): (a) pillar mid-height; (b) roof; (c) sill (after Boulianne 2003).

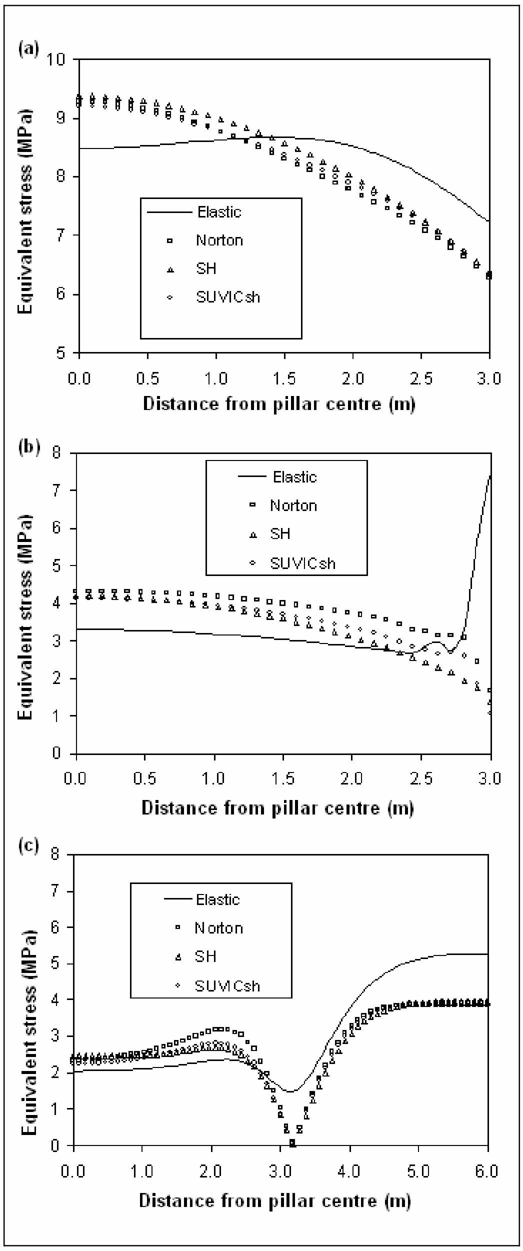

Figure 10. Comparison of the equivalent stress distribution obtained after 180 Ms (5.7 years): (a) pillar mid-height; (b) roof; (c) sill (after Boulianne 2003).

In cases where the complexity of a constitutive model like $SUVIC_{sh}$ may induce some problems (difficulty of implementation, long calculation times etc.), the SH model can provide an interesting alternative solution. The results above showed that the SH model can yield fair approximations of the inelastic strains that compare well with the results obtained with $SUVIC_{sh}$. The main advantage is that, contrary to the Norton approach, the flow is a function of stress and inelastic strain, thus taking the transient flow component into account. By neglecting internal stresses in the SH model, the number of material parameters is greatly reduced, so is the amount of laboratory tests needed to define these parameters.

5. CONCLUSION

Three different constitutive models were used to represent the mechanical behaviour of rocksalt. Numerical analyses simulated a thick cylinder submitted to an internal pressure and a single pillar in a room and pillar pattern. Results showed significant differences in the stresses and strains obtained. This indicates that simplified models such as the Norton creep law may not be adequate for the modelling of underground structures in rocksalt.

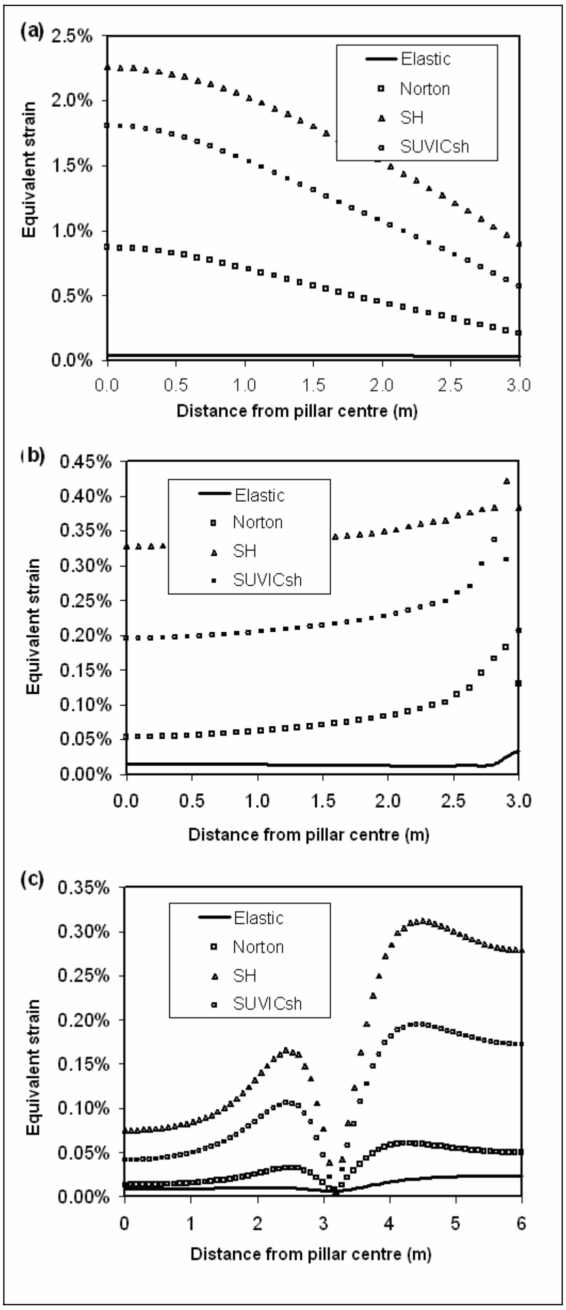

Figure 11. Comparison of the equivalent strain distribution obtained after 180 Ms (5.7 years): (a) mid-height; (b) roof; (c) sill (after Boulianne 2003).

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