INFLUENTIAL FACTORS OF INTERMEDIATE SAND LAYERS FOR CONSOLIDATION OF CLAYEY SUB-SOIL WITH VERTICAL DRAINS

Udeni P.Nawagamuwa, Yokohama National University, Japan
Goro Imai, Yokohama National University, Japan

ABSTRACT
Design of vertical drains is usually based on Barron's theory considering the clay layer is always homogeneous. However it has often been recognized in several situations that many natural deposits have considerable in-homogeneities, such as laminations of coarser material within the clay layer. Field data from such clay layers improved by vertical drains have sometimes shown that the commonly used conventional equations should be modified. In this paper finite difference approach is used to predict the degree of consolidation for the combined case of both vertical and radial consolidation. Two kinds of new parameters are defined as $K=(k_s/k_c)(H_s/H_c)$ and $t_{98(2D)}/t_{98(radial)}$ considering different coefficients of permeability and sand/clay thickness. With these definitions, the importance of thin intermediate sand layers, which were previously overlooked in designs, are discussed in this paper.

RÉSUMÉ
La conception des verticaux drains est habituellement basée sur la théorie de Barron's vu que la couche d'argile est toujours homogène. Cependant on l'a souvent identifié dans plusieurs situations que beaucoup de dépôts normaux ont des inhomogénéités considérables, telles que des stratifications d'un matériel plus brut dans la couche d'argile. Les données de champ de telles couches d'argile se sont améliorées par les drains verticaux ont parfois prouvé que les équations conventionnelles généralement utilisées devraient être modifiées. Dans cette rapport l'approche de la différence finie est employée pour prévoir le degré de consolidation pour le cas combiné de la consolidation verticale et radiale. Deux genres de nouveaux paramètres sont définis en tant que $K=(k_s/k_c)(H_s/H_c)$ et $t_{98(2D)}/t_{98(radial)}$ vu différents coefficients de perméabilité et d'épaisseur de sand/clay. Avec ces définitions, l'importance des couches intermédiaires minces de sable, qui ont été précédemment négligées dans les conceptions, sont discutées en cet article.

1. INTRODUCTION

For over 50 years now, vertical drains have been installed in compressible soils to speed up consolidation. Improvement of soft ground, such as that encountered during construction of roads, railways and airports on near-shore or reclaimed shallow-water areas, is often achieved by the use of preload and vertical drains. The best-known study on this topic of vertical drains was carried out by Barron (1948). Design of vertical drains is usually based on Barron’s theory although there is evidence that the predicted results of settlements are sometimes very much different with the field observations, which cannot be explained by such a simplified approach proposed for a uniform soil.

It was shown by Rowe (1968) using several field observations, that the real drainage behavior of a deposit as a whole depends on the geological details of its formation. Quite small layers, veins of silt along fissures, or organic inclusions can transform the permeability of the mass compared with that of small samples.

Permeable sand or silt layers often are found within the poor soil. Unfortunately, these natural and highly effective drainage layers are often overlooked when they are thin, especially when continuously sample borings are not made. However, even if continuous sand or silt layers are found, they may be so thin or have such a low permeability that head losses in them become excessive if the drainage path is long. Where the effectiveness of intermediate sand or silt layers is in doubt or when such layers do not exist, positive means for accelerating drainage may be desirable.

As Johnson (1970) mentioned, vertical drains are an expense, they obviously should be installed only where subsoil studies show them to be required. For safe and cost effective design of embankment on clayey subsoil, it is very important to detect intermediate sand layers between the clayey subsoil. The existence of the intermediate sand layers will reduce the drainage path and increase the rate of consolidation thus reduce significantly the waiting time to achieve the required settlement, and in certain case even the use of vertical drains can be omitted (Gue and Tan, 2001). According to them, 1m center to center (*c*/*c*) spacing vertical drains will cost 300% more than 2m *c*/*c* spacing vertical drains. In view of the cost sensitive nature, it is very important to acquire sufficient information of the subsoil, so that a cost effective design can be carried out.

However in all probability, it can be implied that vertical drains would be especially useful in stratified deposits. Much of the uncertainty hitherto associated with the prediction of the consolidation rates of clay strata for the purpose of vertical drain designs would appear to be removed once attention is paid to the geological structure of the clay (i.e. especially proper consideration of intermediate sand layers into consolidation calculations) and the appropriate testing techniques. Even though there are lot of field observations on the effects of consolidation with vertical drains due to intermediate sand layers, so far little theoretical studies has been done considering the influence of intermediate sand layers. In this study, 2-dimensional numerical studies have been done considering clay layers having several intermediate sand layers, using the non-linear void ratio, permeability and effective stress relationships. A good relationship is proposed to predict the time needed for actual consolidation due to the influence of intermediate sand layers when the time needed for radial only case obtained by Barron's theory for a uniform clay layer is available.

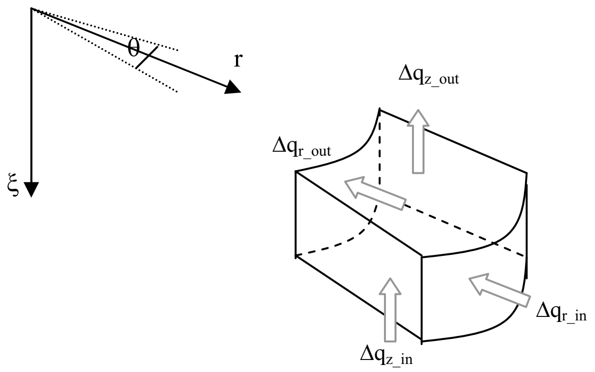

Figure 1. Movement of water in a small clay element

2. THEORETICAL BACKGROUND

In reference to Figure 1, the vertical inflow of water into the small element at time Δt

$$\Delta q_{z\_in} = [v_z + \frac{\partial v_z}{\partial \xi}\Delta\xi].r\theta.\Delta r.\Delta t \qquad [1]$$

The vertical outflow of water at that time is,

$$\Delta q_{z\_out} = v_z.r\theta.\Delta r.\Delta t \qquad [2]$$

At the same time horizontal inflow of water is,

$$\Delta q_{r\_in} = [v_r + \frac{\partial v_r}{\partial r}\Delta r].(r+\Delta r)\theta.\Delta\xi.\Delta t \qquad [3]$$

and the horizontal outflow of water is,

$$\Delta q_{r\_out} = v_r r\theta.\Delta\xi.\Delta t \qquad [4]$$

Therefore the change in volume of water from this small element is,

$$\Delta q = (\Delta q_{r\_in} - \Delta q_{r\_out}) + (\Delta q_{z\_in} - \Delta q_{z\_out})$$

$$\frac{\Delta(r\theta.\Delta r.\Delta\xi)}{\Delta t} = \left[\left(\frac{\partial v_r}{\partial r} + \frac{v_r}{r}\right) + \frac{\partial v_z}{\partial \xi}\right] r\theta.\Delta r.\Delta\xi$$

In reduced coordinate system, z, the law of mass conservation for the combined process of vertical and horizontal drainage of water from a soil element is therefore changed to,

$$\frac{\partial e}{\partial t} = (1+e)\left(\frac{\partial v_r}{\partial r} + \frac{v_r}{r}\right) + \frac{\partial v_z}{\partial z} \qquad [5]$$

where $\Delta\xi$ =(1+e)Δz and $\frac{\partial \Delta\xi}{\partial t} = \frac{\partial e}{\partial t}\Delta z$ (Imai,1995)

As the pore water moves through the soil skeleton in accordance with Darcy's law ($v = ki$) both in horizontal and vertical direction, then,

radial velocity, $v_r = \frac{k_r}{\gamma_w}\left(\frac{\partial u}{\partial r}\right)$ and

vertical velocity $v_z = \frac{k_z}{\gamma_w}\left(\frac{1}{1+e}\frac{\partial u}{\partial z} - \gamma_w\right)$

Substituting the value of v_r and v_z in Eq.5, a combined governing equation of consolidation is formulated as,

$$\frac{\partial e}{\partial t} = [(C_1\frac{\partial^2 u}{\partial r^2} + C_2\frac{\partial u}{\partial r}) + (C_3\frac{\partial^2 u}{\partial z^2} + C_4\frac{\partial u}{\partial z} + C_5) \qquad [6]$$

where,

$$C_1 = \frac{(1+e)k_r}{\gamma_w}, C_2 = \frac{(1+e)}{\gamma_w}(\frac{k_r}{r} + \frac{\partial k_r}{\partial r}), C_3 = \frac{1}{\gamma_w}\frac{k_z}{(1+e)}$$

$$C_4 = \frac{1}{\gamma_w}(-\frac{k_z}{(1+e)^2}\frac{\partial e}{\partial z} + \frac{1}{1+e}\frac{\partial k_z}{\partial z}), C_4 = \frac{\partial k_z}{\partial z}$$

The following non-linear relationships are considered in the calculations to find out the change in the permeability (k) and the void ratio (e) due to an increase of effective stress σ'.

$$e = N_k + C_k \log k \qquad [7]$$

$$e = \Gamma - C_c \log \sigma' \qquad [8]$$

3. DEFINITION OF NEW PARAMETER *K*

It was shown by Gray (1945), that the rate of consolidation in a layered soil which consists of clay and sand is controlled by the parameter *R=(k'h)/(kh')*, a dimensionless factor that involved the permeability and thickness ratio of the two layers where *k* and *h* are the permeability and thickness of the clay, respectively; and *k'* and *h'* are the permeability and thickness of sand, respectively. However, Gray's analysis was one dimensional, which omits the effect of lateral drainage. Considering the predominant flow direction is vertical for the clay layers into the sand layers, and horizontal for the sand layers towards the connecting dykes, Gibson and Shefford (1968) evaluated filter efficiency. However, the effects of sand and clay thickness and length of flow path on the overall performance of the drainage layer were not elaborated. A characteristic factor called λ of a sand layer in the layered clay-sand scheme was proposed by Tan et.al. (1992), based on the average degree of consolidation of the system. $\lambda= (k_s/k_c)(H/L)(H_s/L)$, here k_s and k_c are permeability of sand and clay respectively and H and H_s are the thickness of clay and sand layers respectively, while L is the distance from the centerline to the sand dyke (horizontal flow length).

Since the above-mentioned parameter is defined for layered clay-sand scheme of land reclamation which should have a permeability of sand to clay ratio with an order of at least 10^6 for fully effective drainage, this λ parameter cannot be directly applied for vertical drain applications where they meet natural sand layers which cannot be always considered as pure sand with a very high permeability. In the present study, a new parameter K is proposed as $K=(k_s/k_c)(H_s/H_c)$ for vertical drain installation in the clayey soils which have intermediate sand layers. In this definition, k_s and k_c are permeability of sand and clay respectively and H_s and H_c are the thickness of sand and clay layers respectively.

4. PROOF OF *K* VALUE FOR FIELD CONDITIONS

Due to the possible difficulties encountered in obtaining k_s and H_s values in practical situations, the following simplification is proposed.

In general, natural soil deposits are stratified. If the stratification is continuous, the effective coefficients of permeability for flow in the horizontal and vertical directions can be calculated.

Flow in the horizontal direction can be expressed as follows.

$$\mathrm{k}_{\mathrm{e(h)}} = \frac{1}{\mathrm{H}}(k_{h1}H_1 + k_{h2}H_2 + k_{h3}H_3 + ...) \qquad [9]$$

Similarly the flow in the vertical direction can be expressed as follows.

$$\mathrm{k}_{\mathrm{e(v)}} = \frac{\mathrm{H}}{H_1/k_{v1} + H_2/k_{v2} + H_3/k_{v3} + ...} \qquad [10]$$

where k_{hi}- horizontal permeability
k_{vi}- vertical permeability
H_i – layer thickness
H – total thickness of the soil

These expressions can be used in the clay/sand layers as shown in Figure 3 and are used in this research as follows. For horizontal flow,

$$\mathrm{k}_{\mathrm{e(h)}} = \frac{1}{\mathrm{H}}(k_s H_s + k_c H_c) \qquad [11]$$

and

$$\mathrm{k}_{\mathrm{e(v)}} = \frac{\mathrm{H}}{H_c/k_c + H_s/k_s} \qquad [12]$$

Then the equivalent coefficient of permeability can be expressed as;

$$\mathrm{k}_{\mathrm{eq}} = \sqrt{\mathrm{k}_{\mathrm{e(h)}}.\mathrm{k}_{\mathrm{e(v)}}} \qquad [13]$$

By substituting Equations 11 and 12 in Eq. 13, the following expression can be found.

$$\mathrm{k}_{\mathrm{eq}} = \sqrt{\frac{1}{\mathrm{H}}(k_s H_s + k_c H_c).\frac{\mathrm{H}}{H_c/k_c + H_s/k_s}}$$

$$\mathrm{k}_{\mathrm{eq}} = \sqrt{(k_s H_s + k_c H_c).\frac{1}{\frac{H_c}{k_c}(1+\frac{H_s/k_s}{H_c/k_c})}}$$

when $H_s << H_c$ and $k_s >> k_c$, then $\frac{H_s/k_s}{H_c/k_c} \Rightarrow 0$

therefore,

$$\mathrm{k}_{\mathrm{eq}} = \sqrt{(k_s H_s + k_c H_c).\frac{k_c}{H_c}} = k_c\sqrt{\frac{k_s}{k_c}\frac{H_s}{H_c}+1}$$

$$(\frac{\mathrm{k}_{\mathrm{eq}}}{k_c})^2 = \frac{k_s}{k_c}\frac{H_s}{H_c}+1$$

Since $\mathrm{K} = \frac{k_s}{k_c}\frac{H_s}{H_c}$, it can be simplified that,

$$K = (\frac{\mathrm{k}_{\mathrm{eq}}}{k_c})^2 - 1 \qquad [14]$$

Here, k_c value can be found using the laboratory tests. As given in Eq. [13], to find out k_{eq}, it is needed to carry out pumping tests to obtain $k_{e(h)}$ value and $k_{e(v)}$ value can be calculated as follows.

Using the assumption of $\frac{H_s/k_s}{H_c/k_c} \Rightarrow 0$, the equation [11] can be modified as

$$\mathrm{k}_{\mathrm{e(v)}} = \mathrm{H}\frac{k_c}{H_c} \qquad [15]$$

Then k_{eq} value of Eq.13 can be obtained by using $k_{e(h)}$ from pumping tests and $k_{e(v)}$ from Eq.15, and K-value can be determined from Eq.14.

As shown in Figure 2, it can be seen a good linear

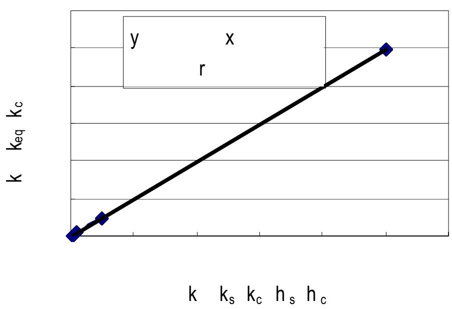

Figure 2. Variation of $K=(k_s/k_c)(H_s/H_c)$ and $K=(k_{eq}/k_c)^2-1$

relationship between two K values defined by the two methods, i.e., $K=(k_s/k_c)(H_s/H_c)$ and $K=(k_{eq}/k_c)^2-1$ with the square of the correlation coefficient of unity ($R^2=1$).

5. IDEALIZATION OF CLAY/SAND SOIL FOR NUMERICAL ANALYSIS

As shown in Figure 3, due to the symmetrical situation of clay and intermediate sand layers, half of the effective area is considered for the analysis. H_s and H_c are the thickness of sand and clay layers respectively. A well radius (r_w) of 20cm and different effective radii (r_e) are used. It is considered that the permeability of the vertical drain as infinite and the permeability in sand and clay as k_s and k_c respectively.

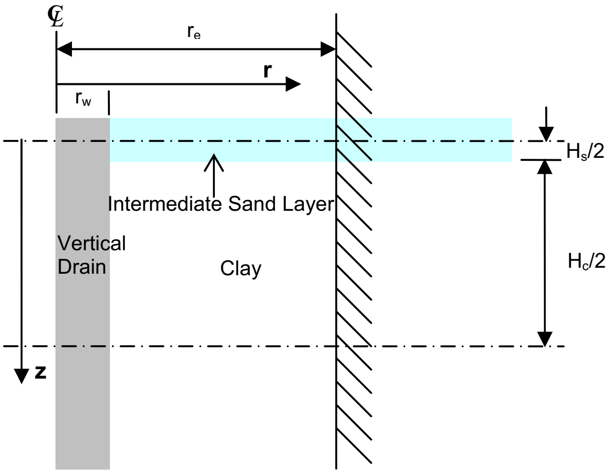

Figure 3. Schematic Diagram of Clay with Vertical Drains and Intermediate Sand layers

Since the overall stability and deformation depend on effective stress or pore pressure of the soil elements, the average degree of consolidation at time t can be considered as a measure of overall improvement of clay layer. It is calculated by,

$$U_t = 1 - \frac{\int_A u(r,z,t)dA}{\int_A u_0 dA} \qquad [16]$$

where u_0 and u are the initial and current pore water pressures respectively, and dA is the area of the small element corresponding to a grid point. Finite difference method is used for the analysis.

Initial and Boundary Conditions

Initial condition

$t = 0$ $\qquad u = u_0$

in the vertical drain ,

$0 \le r \le r_w$ $\qquad u = 0$

at the maximum radial distance,

$r = r_e$ $\qquad \frac{\partial u}{\partial r} = 0$

at the sand/ clay boundary,

$z = H_s/2$ $\qquad u_{sand}=u_{clay}$

$$k_{sand}\frac{\partial u}{\partial z} = k_{clay}\frac{\partial u}{\partial z}$$

at the mid-depth of the clay layer considering the symmetry, $z = H_s/2 + H_c/2$ $\qquad \frac{\partial u}{\partial z} = 0$

To consider the effect of both radial and vertical combined consolidation, another dimensionless ratio is defined as, $\frac{t_{98(2D)}}{t_{98(radial)}}$,

where $t_{98(2D)}$ and $t_{98(radial)}$ are required times for 98% consolidation for 2D and radial only cases respectively. Using this ratio, it can be evaluated how far, the actual time needed for consolidation deviates from the conventional Barron's solution, when intermediate sand layers are present in the clay. When this time ratio is unity, there is no effect of intermediate sand layers, while its effect is larger for a smaller time ratio.

6. NUMERICAL ANALYSIS

Numerical simulations have been done for a clay with fixed parameters of N_k=8.64, C_k=0.72, Γ=3.0, C_c=0.70; these are typical values of Japanese alluvial clays with w_L=100%, and r_w has been fixed to 20cm of sand drains. As shown in Figure 4, for different K values, the ratio of $t_{98(2D)}/t_{98(radial)}$ with clay thickness normalized by effective radius (H_c/r_e) is discussed for r_e=140cm. Here $K=\infty$ represents the situation where the intermediate sand layers have infinite permeability. When the clay layer is very thin, then $t_{98(2D)}/t_{98(radial)}$ is almost zero for $K=\infty$ case. For thin clay layers, it can be clearly seen that the effect of K value is significant especially when H_c/r_e ratio is less than 1.0. This indicates that the effect of K value is considerable when clay layer thickness is less than the effective radius of the vertical drain. In actual clayey soil, there may be several intermediate sand layers where the thickness of the clay layer (H_c) between two intermediate sand layers is very small. Using this relationship, it can be observed that lower K values have higher ratio of $t_{98(2D)}/t_{98(radial)}$ for smaller thickness of clay layers. However, for thick clay layers, this relationship is not observed. When the clay layer thickness is small, $t_{98(2D)}/t_{98(radial)}$ has smaller values. It means whenever there are closely spaced thin sand layers, where clay layer thickness between two intermediate sand layers is small, only consideration of radial consolidation may overestimate the time required for full consolidation.

On the other hand, when there are no closely spaced intermediate sand layers, i.e. clay layer thickness between two intermediate sand layers is high, consideration of radial only consolidation is reasonable and there is less effect on consolidation due to the presence of sand layers since $t_{98(2D)}/t_{98(radial)}$ value becomes unity.

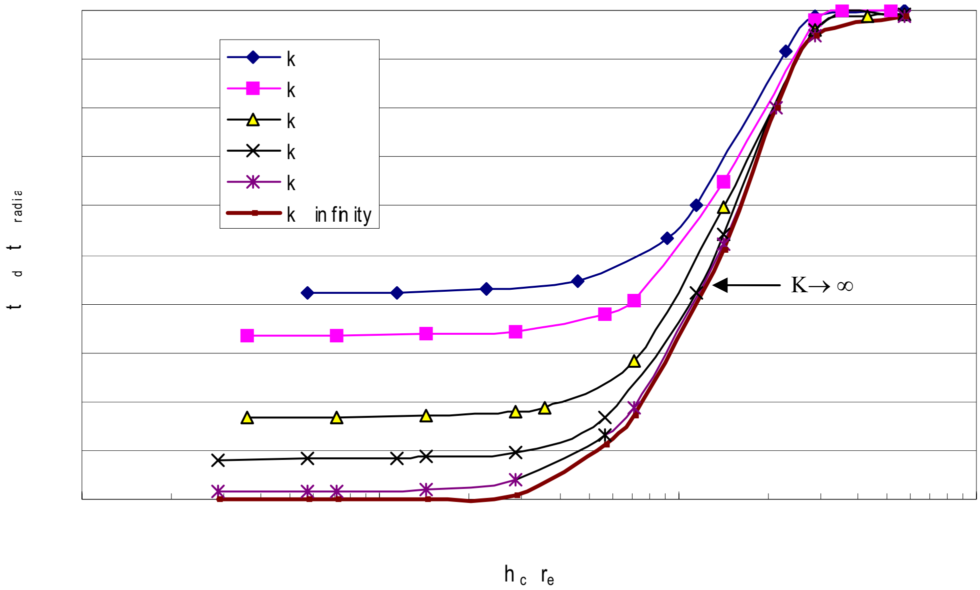

Figure 4. Variation of K value for r_e=140cm and r_w=20cm

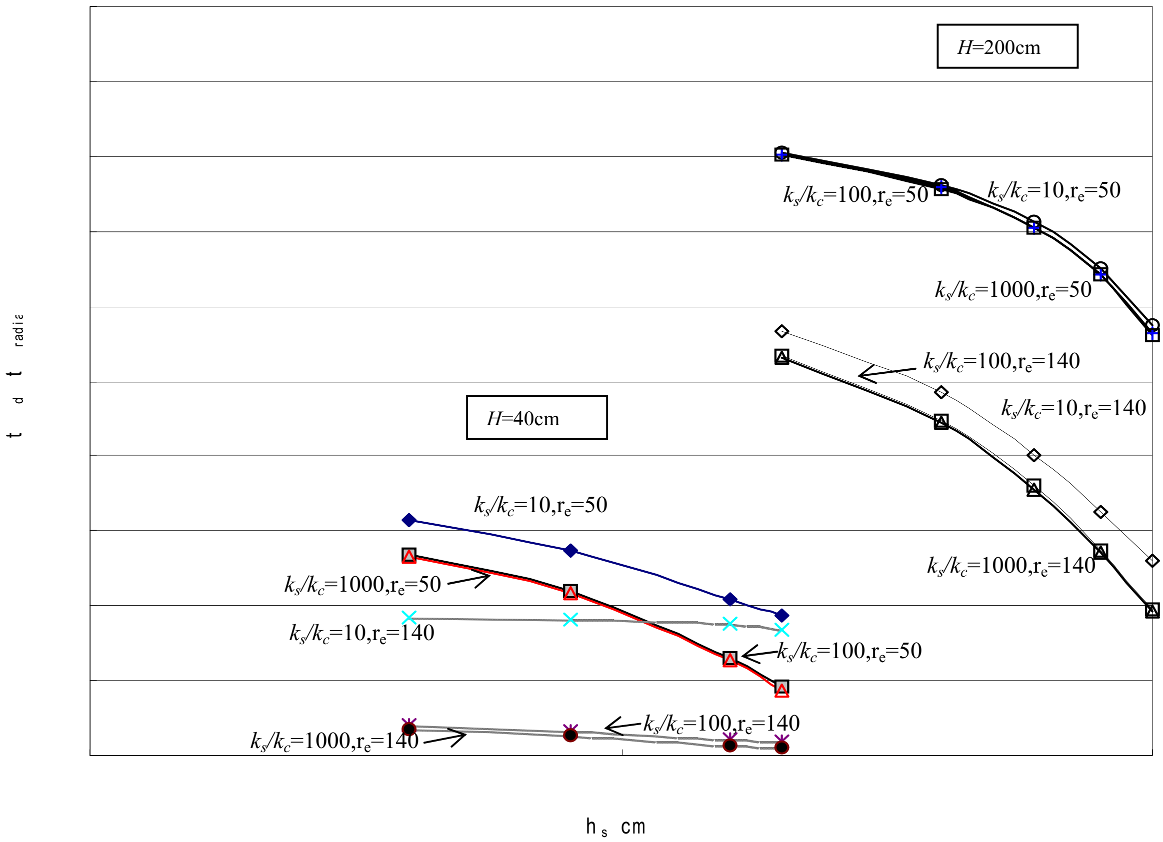

Figure 5. Variation of $t_{98(2D)}/t_{98(radial)}$ value with H_s

The variation of $t_{98(2D)}/t_{98(radial)}$ for two different total thickness (H) is shown in Figure 5. Here, r_e=50 and r_e=140cm cases are considered and it was found that lower r_e case has higher $t_{98(2D)}/t_{98(radial)}$ value compared to higher r_e case when the other conditions are same. So it can be concluded that the effect of intermediate sand layers on 98% consolidation is higher in larger r_e than smaller r_e, when compared with respective radial only consolidation.

Figure 6 shows the variation of $t_{98(2D)}/t_{98(radial)}$ value for different n (= r_e/r_w) values for the case of K=2.0. Figures 4 and 6 clearly show the importance of the spacing when H_c/r_e is less than 1.0. Therefore in order to evaluate the importance, another comparison is done as shown in Figure 7, between n values and $t_{98(2D)}/t_{98(radial)}$ for different K and n values for the region H_c/r_e<1.0 .

According to Figure 7, the effects of spacing of vertical drains become less important for higher K values. This clearly confirms the observation of Rowe (1968) on Derwent Reservoir, " it would seem therefore that the sand

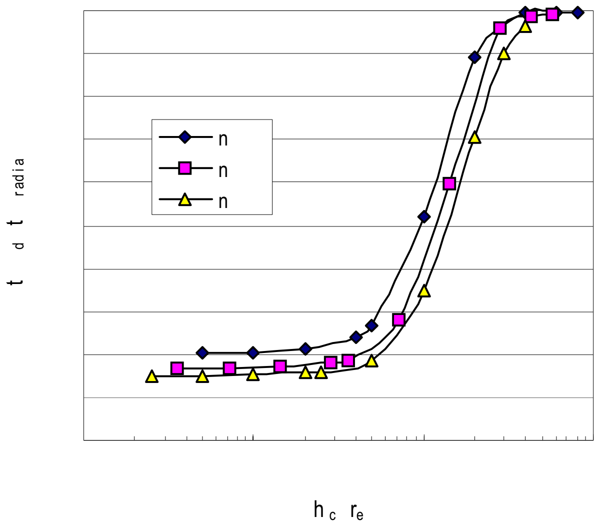

Figure 6. Variation of $t_{98(2D)}/t_{98(radial)}$ value with n [K=0.2]

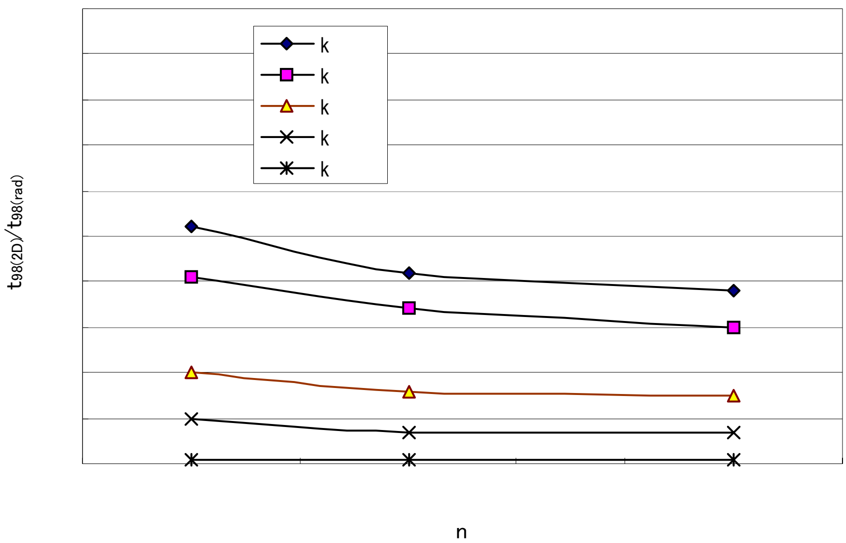

Figure 7. Variation of K value for different n when H_c/r_e<1

drains made an important contribution to construction but, solely on present considerations, fewer drains could have been used.” Both field observations and these numerical results confirm the need for treating the soil considering its geological features, especially the presence of intermediate sand layers to obtain the optimum solution with reference to time and cost.

7. CONCLUSIONS

When $H_c/r_e<1.0$ the effect of intermediate sand layers is very high and when $H_c/r_e>1.0$ effect of intermediate sand layers is negligible and Barron’s solution may serve the need. This means that the drainage path in vertical direction is effective when the effective radial distance is higher than the thickness of clay layers in between two sand layers. Whenever several intermediate sand layers are present in a particular area to be improved, the spacings will play a key role. (spacing = H_s+H_c ; for an optimum solution $H_c < r_e$)

When *n* is greater, the effects of intermediate sand layers are high. This does not mean that it is faster than low *n* cases, however compared with Barron’s solution, intermediate sand layers give a greater contribution to make it faster than the results obtained from Barron’s solution.

It is clear from this study that by reducing the number of vertical drains with appropriate consideration of the effects of intermediate sand layers, the cost for the vertical drains can be optimized. In some cases, even omission of vertical drains may successfully serve the need.

One major drawback of the above conclusions is, practically the intermediate sand layers should be continuous or peripheral drains should be considered. Another important requirement is, intermediate sand layers should not be in random directions in a horizontal plane. Since it will not give the optimum solution.

Real drainage behavior of any deposit as a whole depends on the geological details of its formation. Performance will entirely depend on the continuity and the spacing of the intermediate sand layers.

Simple laboratory test will not represent the actual drainage or permeability condition of the field, so it is required to couple both laboratory tests and field pumping tests to achieve the actual drainage conditions.

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