A UNIAXIAL LINEAR VISCOELASTIC- VISCOPLASTIC MODEL FOR HIGH DENSITY POLYETHYLENE

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ABSTRACT:
A rheological constitutive model, which considers both viscoelastic and viscoplastic components of high density polyethylene behaviour, is developed using data from axial compression tests. A linear viscoelastic model, developed previously, is modified and complemented with a viscoplastic model, and the stress-strain results of five compressive tests at different engineering strain rates are used to determine the model parameters. The ability of the model to predict the response to different loading conditions such as creep, stress relaxation, constant load rates, unloading and strain reversal is examined by comparing the model simulation with the available experimental data. It was found that the model can well predict the response to different loading conditions but is not as accurate in predicting the response to unloading and strain reversal conditions.

RÉSUMÉ :
Un modèle constitutif rhéologique, qui considère les aspects viscoélastique et viscoplastique du comportement de polyéthylène à haute densité, est développé en utilisant des données des essais de compression axiale. Un modèle viscoélastique linéaire, développé précédemment, est modifié et complété avec un modèle viscoplastique, et les résultats de contrainte-déformation de cinq essais de compression à différentes vitesses de déformation sont employés pour déterminer les paramètres du modèle. La capacité du modèle de prévoir la réponse à différentes conditions de chargement telles que le fluage, la relaxation, le chargement à vitesses constantes, le déchargement et l'inversion de déformation est examinée en comparant la simulation du modèle aux données expérimentales disponibles. Il s'avère que le modèle peut prévoir la réponse aux différentes conditions de chargement mais n'est pas précis en prévoyant la réponse aux conditions d'inversion de déformation et de déchargement.

1. INTRODUCTION:

High density polyethylene is now a widely used material in geotechnical engineering applications due to its low cost, light weight and durability. Examples include underground drainage pipes and geogrids and geotextiles used for soil reinforcement. New underground pipe installation technologies such as horizontal directional drilling and pipe bursting use high density polyethylene pipes due to their relative flexibility and light weight.

Even at ambient temperatures, the mechanical properties of high density polyethylene are highly time and rate dependent and its response can differ significantly when subjected to different loading conditions. The mechanical behaviour of HDPE consists of elastic, viscoelastic and viscoplastic components. The instantaneous response is elastic but viscoelastic and viscoplastic strains develop with time. Moreover, there is no well-defined yielding point beyond which plastic behaviour is experienced. Permanent viscoplastic deformation initiates at relatively low stresses and dominates at high stresses.

Different constitutive models have been developed to simulate the mechanical behaviour of HDPE. Chua (1986) carried out compression tests on polyethylene pipes and proposed the following time dependent relaxation modulus:

$$E(t) = \frac{\sigma}{\varepsilon(t)} = 52.6 + 460t^{-0.097786} \quad [1]$$

Where the modulus E is given in MPa and t is time in hours. Hashash (1991) performed tests on corrugated HDPE pipes and proposed the following time dependent modulus:

$$E(t) = \frac{\sigma}{\varepsilon(t)} = 329t^{-0.0859} \quad [2]$$

where again t is time in hours. Hashash's formula proposes that the relaxation modulus diminishes with time while Chua's equation suggests that the modulus reaches a minimum limit of 52.6 MPa after a long period of loading.

Moore and Hu (1996) developed a linear viscoelastic model that can be mechanically represented by a combination of a spring and nine Kelvin elements in series (figure 1). The constitutive relations for that model can be written as:

$$\sigma(t) = E_o\varepsilon_o = E_i\varepsilon_i + \eta_i\dot{\varepsilon}_i \quad [3.a]$$

$$\varepsilon(t) = \varepsilon_o(t) + \sum_{i=1}^{9} \varepsilon_i(t) \qquad [3.b]$$

Where i denotes the Kelvin element number. The elasticity and damping constants of the second to ninth Kelvin elements were chosen to be multiples of the elasticity and damping constants, respectively, of the first Kelvin element. The independent spring predicts the instantaneous response while each Kelvin element dominates the behaviour over one cycle of log time.

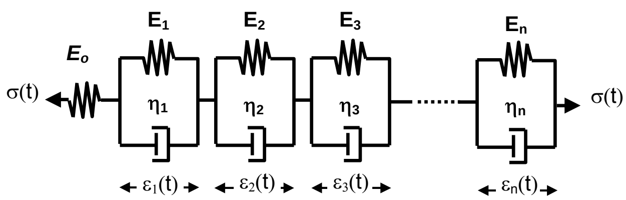

Figure 1: Multi-Kelvin linear viscoelastic model (after Moore and Hu, 1996)

Zhang and Moore (1997) carried out extensive testing on cylindrical polyethylene samples extracted from commercially available plain HDPE pipes. The results of creep tests under different engineering stress levels were used to develop a nonlinear viscoelastic model (figure 1) which consists of an independent spring and six Kelvin elements, and in which the model parameters E_o, E_1 and η_1 depend on the stress level. Zhang and Moore (1997) also developed a viscoplastic model using their constant engineering strain rate tests. The Viscoplastic model was developed using the framework proposed by Bodner (Bodner and Partom, 1972, 1975) to characterize the plastic behaviour of metals. It was assumed that inelastic deformation occurs at all stages of loading. To account for the dependence of the model's behaviour on the loading rate and plastic work level W_{vp}, a state variable X was introduced as:

$$X(W_{vp}, \dot{\varepsilon}_{vp}) = \frac{1}{\alpha(\dot{\varepsilon}_{vp}) + \sqrt{\beta/(\gamma + W_{vp})}} \qquad [4]$$

Where α, β and γ are model parameters. The viscoplastic strain rate is then related to the stress through the relation:

$$\dot{\varepsilon}_{vp} = C\left(\frac{\sigma}{X}\right)^n \qquad (n \geq 1) \qquad [5]$$

Where C is a scalar constant.

Suleiman and Coree (2003) used the constant strain rate test results provided by Zhang and Moore (1997) and developed a hyperbolic constitutive formula for the tangent modulus of HDPE, given by:

$$E_t = 2932.6\, \dot{\varepsilon}^{0.1207}\left(1 - \frac{\sigma}{39.396 + 1.526\ln(\dot{\varepsilon})}\right)^2 \qquad [6]$$

They also used a systematic "focus point" approach that yielded their preferred expression for the tangent modulus:

$$E_t = \frac{1}{10^{-3} + \frac{0.0523}{39.433 + 1.524\ln\dot{\varepsilon}}}\left(1 - \frac{\sigma}{39.433 + 1.524\ln\dot{\varepsilon}}\right)^2 \qquad [7]$$

In some geotechnical engineering applications, the installed product is subjected to cyclic loading. For example, in a horizontal directional drilling or pipe bursting process, the stresses induced in the pipe due to the installation process fluctuate significantly. While most of the models mentioned in the review have generally proven to be effective in calculating HDPE response, they perform poorly when the HDPE is subjected to strain reversal. In particular, the Non-linear Viscoelastic model overestimates strain recovery on unloading, while the Viscoplastic model underestimates the strain recovery.

Therefore, there is a need to develop a model that can effectively predict HDPE behaviour during unloading, strain reversal and cyclic loading. A pure linear viscoelastic (LVE) model does not consider permanent strain after load removal, while a pure viscoplastic (VP) model cannot predict time-dependent recovery.

2. UNIAXIAL VISCOELASTIC-VISCOPLASTIC MODEL

The mechanical behaviour of high density polyethylene include both viscoelastic and viscoplastic components. The model developed in this study combines, in series, a linear viscoelastic model (LVE), similar to the one proposed by Moore and Hu (1996), and a viscoplastic model similar to that developed by Zhang and Moore (1997). The results of the constant strain rate tests (Zhang and Moore, 1997) are used to develop the model. The linear viscoelastic model has the following parameters (modified after Moore and Hu, 1996):

$$E_o = 1400 \text{ MPa}$$
$$E_1 = 3615.5 \text{ MPa}$$
$$\eta_1 = 43459.2 \text{ MPa.s}$$
$$S = 0.845 \qquad D = 10$$

Where S and D are multipliers to determine the elasticity and damping for the second to ninth Kelvin elements, as shown in figure 2.

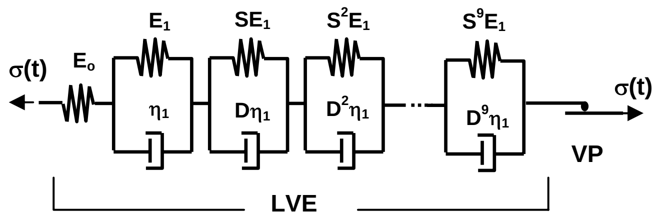

Figure 2: The LVE-VP model

Using this LVE model, the viscoelastic strains ε_{ve} were calculated from the known stress histories (in the constant strain rate tests). Since the total strain is considered, according to the incremental theory of plasticity, to be the summation of the viscoelastic and viscoplastic strains, the viscoplastic strain component ε_{vp} is calculated as:

$$\varepsilon_{vp}(t) = \varepsilon(t) - \varepsilon_{ve}(t) \qquad [8.a]$$

$$\dot{\varepsilon}_{vp}(t) = \dot{\varepsilon}(t) - \dot{\varepsilon}_{ve}(t) \qquad [8.b]$$

Where $\varepsilon(t)$ is the total strain obtained from the experimental results (Zhang and Moore, 1997). Figure 3 shows the viscoelastic and viscoplastic strain components.

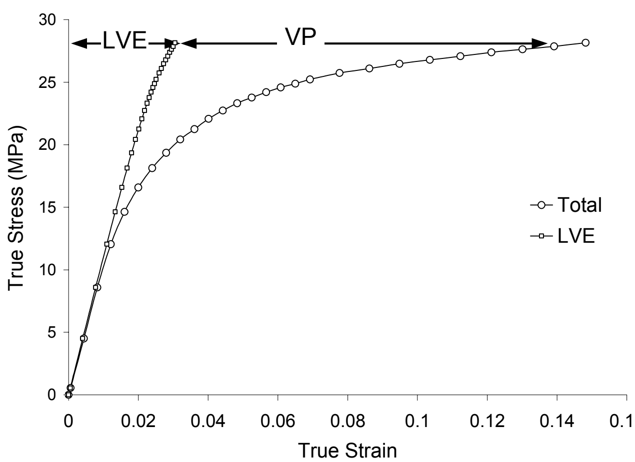

Figure 3: VIscoelastic and viscoplastic strains for the 0.01 s^{-1} strain rate test.

The viscoplastic strain rates were calculated from the viscoplastic strains using central divided difference numerical differentiation, and the viscoplastic work given by:

$$W_{vp} = \int \sigma \, d\varepsilon_{vp} \qquad [9]$$

was calculated using Simpson's numerical integration. For each set of strain rate test results, two sets of the state variable X were calculated using Equations 4 and 5, starting with selected values of the parameters *C*, n, α, β and γ. Nonlinear curve fitting was performed to match the two sets as shown in figure 4, and the parameters were determined to be:

$$C = 0.01 \qquad n = 8$$

$$\alpha = 0.0457(\dot{\varepsilon}_{vp})^{0.0755}$$

$$\beta = 8.225 \times 10^{-5}(\dot{\varepsilon}_{vp})^{0.07163}\,\text{MPa}$$

$$\gamma = 10^{-3}\,\text{MPa}$$

For negative strain rates, the absolute values are used to determine α and β. The new LVE-VP model predicts the stress-strain behaviour very well as shown in figure 5.

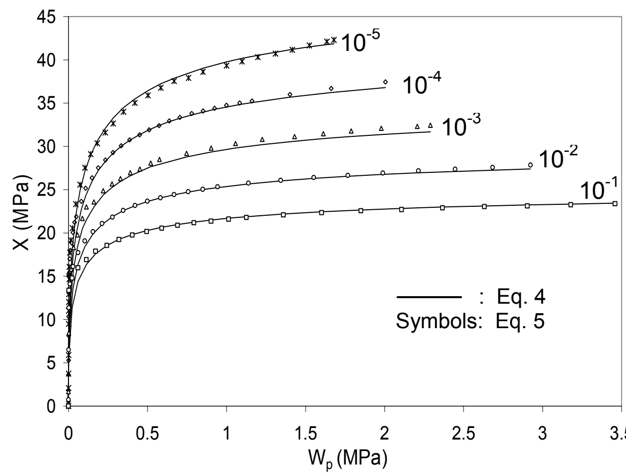

Figure 4: State variable X for different strain rates.

3. NUMERICAL SIMULATION AND MODEL EVALUATION:

3.1 Introduction

The ability of the new LVE-VP model to predict well the HDPE behaviour under constant engineering strain rate does not ensure its ability to predict the behaviour under other different loading conditions such as creep, relaxation and cyclic loading. Therefore, the model should be tested for such conditions. Extensive experimental results for different loading conditions and combinations are available from Zhang and Moore (1997). These results were used to test the model. For displacement control tests, the engineering strain history is known and the stress history was predicted using the model. For load control tests, the engineering stress is known and the strain history was calculated. Considering compression to be positive, the true stress and strain are related to the engineering stress and strain by (Zhang, 1996):

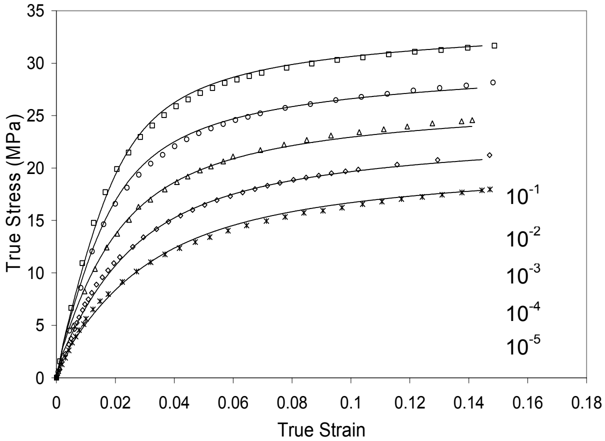

Figure 5: Experimental data and model simulation for constant strain rate tests.

$$\sigma = \sigma_{eng}\exp(-\varepsilon) \qquad [10.a]$$

$$\varepsilon = -\ln(1-\varepsilon_{eng}) \qquad [10.b]$$

3.2 Load Control Tests:

Figure 6 shows the true strain histories for creep tests under different engineering stress values. It can be seen that the model slightly underpredicts the creep strains at low stresses but overpredicts these at high stresses. There is less than 5% error and the general performance is considered good.

Figure 7 shows the experimental results and model calculation for a combination of different load rates with creep periods. The model works very well at low stresses, but slightly overestimates the creep strains at high stresses, which agrees with the observations made regarding Figure 6.

3.3 Displacement Control Tests:

The stress relaxation predicted by the LVE-VP model under different specified strain levels is compared with the experimental results as shown in Figure 8. For test 1, a strain of 0.1 was reached in 100 seconds before being fixed. In tests 2 and 3, the strain was increased at a rate of 0.1 s^{-1} and then fixed at 0.042 and 0.018, respectively. It can be seen that the model calculation is good for tests 1 and 3. However, the stress relaxation is overestimated for test 2. This may suggest that the model does not perform very well at very high strain rates such as 0.1 s^{-1}, which is not typical in geotechnical engineering applications.

The ability of the LVE-VP model to capture an abrupt change in the strain rate is examined in figure 9. The LVE-VP model calculates the behaviour within 6%. The change in stress is overestimated but the stress after the rate jump IS underestimated. This could be because the calculated viscoplastic work is less than that of the 0.01 strain rate test, which leads to underestimated work hardening.

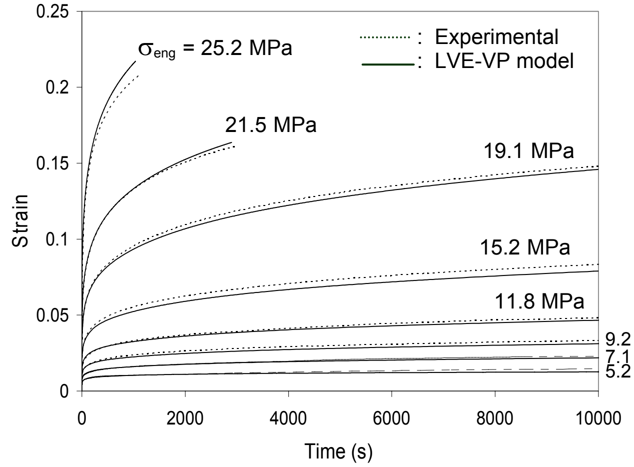

Figure 6: Experimental results and model simulation for creep tests.

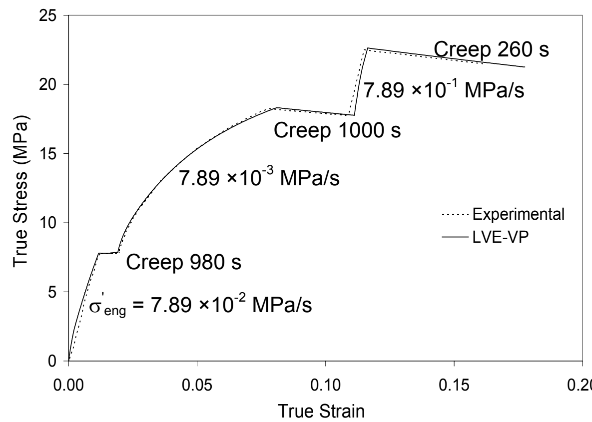

Figure 7: Experimental results and model simulation for constant load rates periods and creep periods

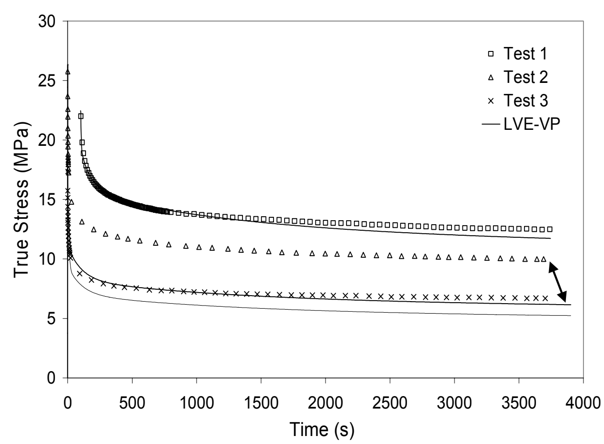

Figure 8: Experimental results and model simulation for stress relaxation tests.

Figure 10 shows the experimental results and model calculation for a combination of different engineering strain rates with relaxation periods. It can be seen that the model calculation is good although it slightly overpredicts the drop in stress due to the sudden change in the strain rate. This observation confirms the observation made earlier regarding the strain rate jump test (figure 9).

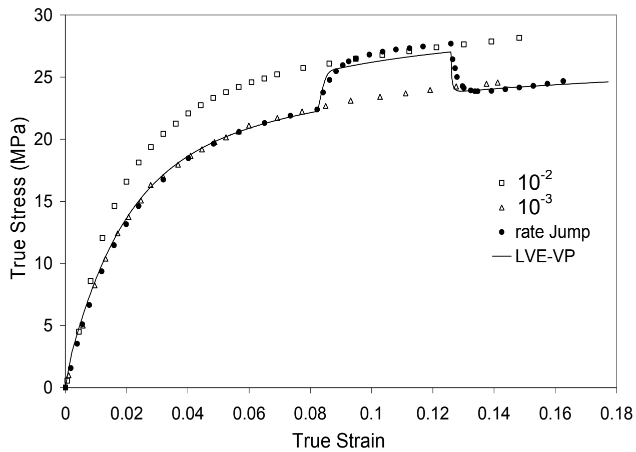

Figure 9: Strain rate jump test results and model simulation.

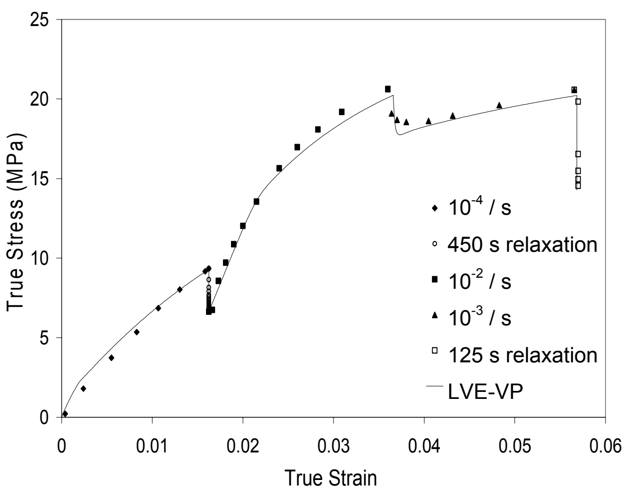

Figure 10: Experimental results and model prediction for constant strain rates and relaxation periods

3.4 Unloading, Strain Reversal and Cyclic Tests:

Modelling the cyclic behaviour is a challenge and none of the models mentioned in the literature review was efficient in calculating HDPE response to cyclic loading. For example, the nonlinear viscoelastic and the viscoplastic models developed by Zhang (1996) worked very well for most of the loading conditions but poorly for unloading or cyclic conditions. The LVE-VP model developed in this study was tested for such loading conditions and the results are shown in the following.

Figure 11 shows the model calculation for two unloading tests in which the unloading rate is equal to the loading rate. The model well predicts the initial unloading response but deviates as unloading continues. It tends to underestimate the recovery and overestimate the permanent strains.

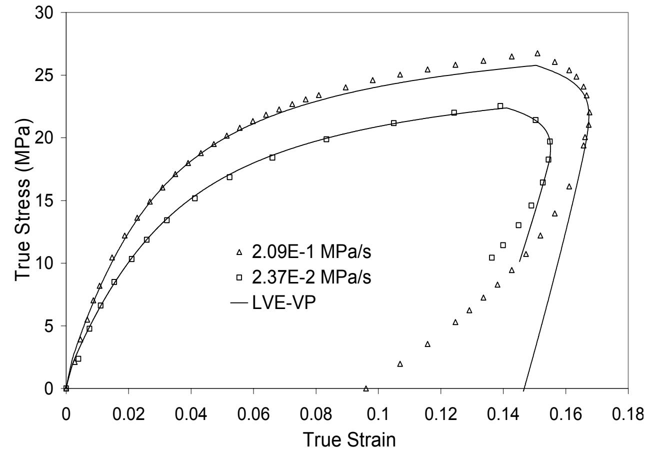

Figure 11: Experimental results and model simulation for constant loading rates followed by unloading with same rate.

Similar observations were made on strain reversal tests shown in figure 12. The recovery is underestimated and the permanent strain is overestimated. It can be noticed from the figure that the unloading slopes of the experimental curves decrease as unloading commences. At low stresses, this slopes become comparable to the "equivalent" stiffness of the LVE component E_∞. This may indicate that the LVE sub-model is overdamped i.e. it does not allow the release of the strains as fast as it should do.

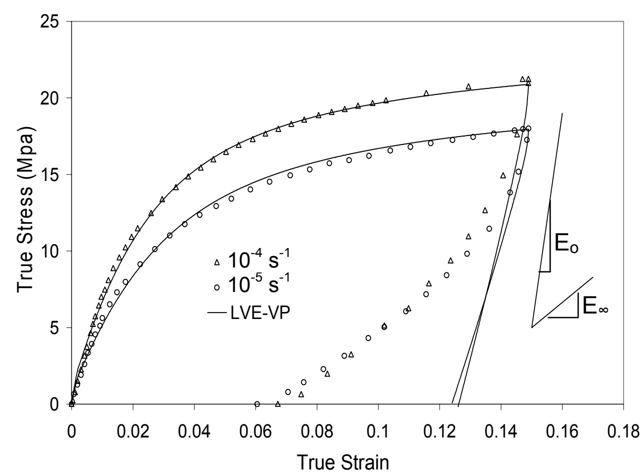

Figure 12: Model simulation for strain reversal. The reversal strain rates are equal to the loading ones.

The experimental results and model calculation for two cyclic loading conditions are shown in figure 13. Similarly, the recovery is underestimated although hysteresis can be clearly observed.

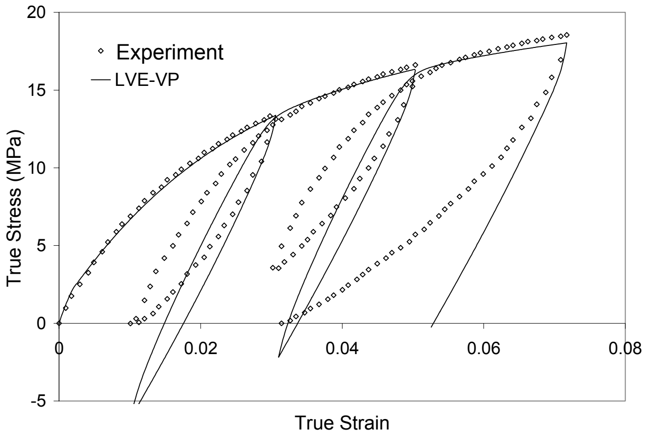

Figure 13: Model simulation for cyclic loading. Loading and unloading are at a strain rate of 10^{-4} s^{-1}.

4. CONCLUSION

A constitutive model that combines linear viscoelastic and viscoplastic behaviours was developed in this study. An existing linear viscoelastic model was used with modified independent spring constant. The parameters of the viscoplastic component were determined by curve fitting of a state variable using the results of five constant engineering strain rate tests.

The mechanical response of high density polyethylene is significantly affected by the loading rate, stress level and loading process, and finding a model that can simulate all these effects is challenging. The LVE-VP model developed in this study simulates most loading conditions very well and therefore it could be used for geotechnical applications where no significant strain reversal or unloading is expected such as geosynthetic applications. For unloading and cyclic conditions, the model works better than pure viscoelastic or viscoplastic models but it does not produce sufficiently accurate solutions. It underpredicts the recovery and overpredicts permanent strains. This may indicate that the plastic strains are exaggerated and viscoplastic component of the model is more dominant that it is supposed to be.

Further study is being undertaken to modify the LVE-VP model so that it can better capture the response to cyclic loading, which is the typical loading process in some recently introduced techniques for underground pipe installation i.e. horizontal directional drilling and pipe bursting. The LVE sub-model will be developed based on the available recovery data and then will be complemented by the VP sub-model.

The model developed in this study has the advantages that it can be readily generalized to a 3-D formulation, and easily implemented in computer algorithms

5. AKNOWLEDGEMENTS

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6. REFERENCE

Bodner, S. R. and Partom, Y. 1972. A large deformation elastic-viscoplastic analysis of a thick-walled spherical shell. Journal of applied mechanics, Vol. 39, No.3. pp. 751-757

Bodner, S. R. and Partom, Y. 1975. Constitutive equations for elastic-viscoplastic strain-hardening materials. Journal of applied mechanics, Vol. 42, No. 2, pp. 385-389

Chua, K. M. 1986. Time-dependent interaction of soil and flexible pipe. Ph.D. thesis, Texas A & M University, College station, Tex.

Hashash, N. M. A. 1991. Design and analysis of deeply buried polyethylene drainage pipes. Ph.D. thesis, Department of Civil Engineering, University of Massachusetts at Amherst, Amherst, Mass.

Moore, I. D. and Hu, F. 1996. Linear viscoelastic modelling of profiled high density polyethylene pipe. Canadian Journal of Civil Engineering, Vol. 23, pp. 395-407

Suleiman, M.T. and Coree, B. Constitutive Model for HDPE Material : A Systematic Approach. IN PRESS, ASCE Journal of Materials in Civil Engineering, Submitted July, 2003.

Zhang, C. 1996. Nonlinear mechanical response of high density polyethylene in gravity flow pipes. Ph.D. thesis, Department of Civil and Environmental Engineering, The University of Western Ontario, London, Ontario, Canada.

Zhang, C. and Moore, I. D. 1997. Nonlinear mechanical response of high density polyethylene. Part I: Experimental investigation and model evaluation. Polymer Engineering and Science, Vol. 37, No. 2, pp. 404-413.

Zhang, C. and Moore, I. D., 1997. Nonlinear mechanical response of high density polyethylene. Part II: Uniaxial constitutive modeling. Polymer Engineering and Science, Vol. 37, No. 2, pp. 414-420.