

HYSTERESIS EFFECTS ON THE WATER RETENTION CURVE: A COMPARISON BETWEEN LABORATORY RESULTS AND PREDICTIVE MODELS

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ABSTRACT:

Description and prediction of water flow through unsaturated soils imply an understanding of their hydraulic properties. One of these unsaturated soil properties is the water retention curve (WRC), which relates the water content to suction, exhibits hysteresis effects. An experimental study of hysteresis phenomenon was undertaken to characterize the hydraulic behaviour of sandy soils used in engineered covers and to assess the performance of three predictive hysteresis models available in the literature. For the studied sands (fine, coarse and silty sand), hysteresis effect is clearly seen, and should not be ignored. The comparison between experimental and predicted results with the three different models indicates that Universal Mualem model did not predict adequately the WRC. However, the two versions of the Parlange model allow good predictions of the main drying curves. Moreover, because of their relatively simple formulation, these can easily be incorporated into numerical models for a better prediction of water flow and/or contaminant transport through unsaturated soils.

RÉSUMÉ :

La description et la prédiction du mouvement de l'eau à travers les sols non saturés impliquent une bonne connaissance de leurs propriétés hydriques. Une de ces propriétés de base est la courbe de rétention d'eau (CRE), qui relie la teneur en eau et la succion, montre des effets d'hystérésis. Une étude expérimentale de ce phénomène d'hystérésis a été entreprise avec l'objectif de caractériser le comportement hydrique de trois sols sableux typiquement utilisés dans la construction de recouvrements et d'évaluer la performance de trois modèles de prédiction d'hystérésis tirés de la littérature. Pour les sables étudiés (fin, grossier et sable silteux), l'effet d'hystérésis est bien présent et ne devrait pas être ignoré. Les résultats pour la prédiction des courbes principales de drainage permettent de conclure que le modèle Universel de Mualem n'est pas adapté pour obtenir de bonne prédiction de la CRE. Cependant, les deux versions du modèle de Parlange permettent d'obtenir de bonnes prédictions des courbes principales de drainage. De plus, leur formulation simple fait que les deux versions pourraient être facilement incorporées dans les modèles numériques pour une meilleure prédiction de l'écoulement d'eau et/ou du transport de contaminants à travers les sols non saturés.

1. INTRODUCTION

Description and prediction of water flow through unsaturated soils imply an understanding of unsaturated soil properties. The main unsaturated soil properties used in engineering calculations are the relationships between suction (or water pressure) h (cm of water or kPa) and volumetric water content θ (cm^3/cm^3) and between suction and hydraulic conductivity (k). These two relationships are known as the water retention curve (WRC) and permeability function, respectively. Due to the complex nature of the liquid-phase configuration in an unsaturated porous medium, the relationship between water pressure and water content is not unique and presents hysteresis effects (e.g. Haines 1930; Poulvassilis 1962; Topp and Miller 1966; Dane and Wierenga 1975). As shown in Figure 1, a soil typically shows a volumetric water content that is less for a wetting process (such as infiltration) than

for a drying process (such as evaporation or drainage) at a given water pressure.

The hysteresis effect can be attributed to 4 main causes (e.g. Hillel 1980; O'Kane et al. 2004): i) geometric nonuniformity of individual pores, resulting from the so-called "Ink Bottle" effect, ii) different spatial connectivity of pores during drying or wetting process, iii) variation in liquid-solid contact angle, and iv) air entrapment. Over the last 40 years, different models have been developed to describe the different hysteresis curves (main, primary and secondary curves) of the WRC. However, except for a few field studies (e.g., Si and Kachanoski 2000), these hysteresis phenomena continue to be neglected in most practical applications. Consequently, the results from laboratory and field tests can sometimes differ substantially and part of these differences can be attributed to hysteresis effects (Basile et al. 2003; Bussière et al. 2004).

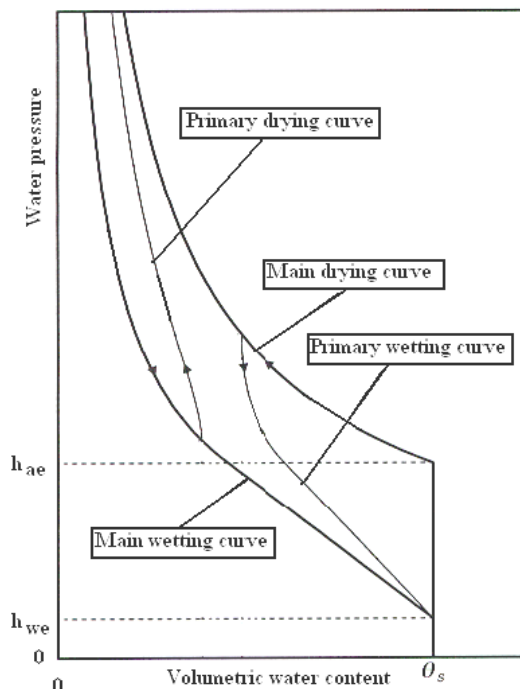


Figure 1. Schematic representation of water retention curves with hysteresis effects

This paper presents the main results from a laboratory experimental study of hysteresis phenomena in soils. This research was undertaken in a broader project aiming at better understanding of the hydraulic behaviour of engineered covers. After this brief introduction on the hysteresis phenomenon on the WRC, the main type of predictive models for hysteresis effects and the models retained in this study are presented. A description of the materials and methods follows. The main results obtained under different drying and wetting processes are discussed and then compared with the predictions obtained from three models. The study focuses on the prediction of the main drying curves from the main wetting curves and will not address the prediction of primary and secondary drying and wetting curves. The issue of primary and secondary curves will be addressed in future publications.

2. HYSTERESIS MODELS

The different approaches that have been used to describe hysteresis in WRC can be categorized into two main groups: conceptual and empirical models. The first group is based on the domain theory and the second group mainly relies on an analysis of WRC shape and properties. Another classification often found in the literature, which will not be discussed further in this paper, is based on the so-called *n*-curves used in model calibration and can be referred to as the *n*-branches classification.

2.1 Conceptual models

The first group of conceptual models is based on the independent domain theory developed by Néel (1942-1943) and used by different authors (Everett and Whitton 1952; Everett and Smith 1954; Everett 1954, 1955; Enderby 1955). This theory assigns soil water to domains and each domain wets at a characteristic water pressure (or suction) h_w and dries at a water pressure h_d , regardless of the neighbouring domains. The first application of this theory into hysteresis models was done by Poulvassilis (1962, 1970) and Topp and Miller (1966). These two models need 4 branches for model calibration. Mualem models I and II (1973, 1974), which are 2-branch models, and Mualem Universal model (1977), which presents a universal relation between the main and drying curves using only one branch, also use the independent domain theory. Parlange (1976) has proposed a one branch model based on the same theory (named Parlange model in the following).

Other authors have proposed modifications to the independent domain theory to take into account interactions between domains. Many models requiring more than two branches for calibration were developed (Poulvassilis and Childs 1971; Topp 1971; Mualem and Dagan 1975). The latest model developed by Mualem (1984) needs only two branches for calibration; this model (Mualem IV) uses a correction factor for water content variations calculated on the basis of the independent domain theory.

2.2 Empirical models

Empirical models are based on an analysis of WRC shape and properties. These models use close-formed empirical expressions to represent hysteresis curves. They are often developed for a specific soil and they do not claim general validity because their derivation is not based on a physical representation of hysteresis. Among these models, one can identify: i) the scaling-down model developed by Scott et al. (1983); ii) the linear model developed by Hanks et al. (1969); iii) the interpolation model developed by Pickens and Gilham (1980), Hoa et al. (1977) and Dane and Wierenga (1975); iv) the slope model developed by Jaynes (1984) derived from Dane and Wierenga (1975) work.

Other models were recently developed using different concepts. For example, the mathematical model developed by Preisach (1938) was applied to describe the hysteresis effects in the water retention curve by O'Kane et al. (2004), using the concept of a continuous analog of a finite parallel connection of relays. The Haverkamp et al. (2002) model, based on geometric scaling, was recently modified and simplified (Gandola et al. 2004). Another hysteresis empirical model was developed for sandy soil using the basic concept of shape similarity between the WRC and the cumulative particle-size distribution function (Haverkamp and Parlange 1986). In this case, the

hysteresis is predicted from the basic properties of the soil, not from a WRC.

2.3 Model's comparison

Some authors compared different models. Viaene et al. (1994), following a statistical analysis of hysteresis models, concluded that the best 2 branch models were conceptual models (Mualem II and IV), while the Parlange model was selected as the best choice for hysteresis prediction using one branch. The same conclusion was reached by Si and Kachanoski (2000) about one branch models. However, Jaynes (1984, 1992) comparison led to the conclusion that none of the methods were consistently better than the others, even for the more complex models with more than two branches. Jaynes also concluded that the linear model (empirical type of model) appears to be the best approach to predict hysteresis. More recently, different studies (Braddock et al. 2001; Haverkamp et al. 2002) suggested that the Parlange model, that uses the concept of rational extrapolation, was the best equation to predict hysteresis of the WRC. Braddock et al. (2001) proposed a new formulation of the Parlange model using the van Genuchten (1980) equation instead of the Brooks and Corey (1964).

From the literature review, the authors have selected the Parlange model modified by Hogarth et al. (1988) and its new formulation proposed by Braddock et al. (2001), to test their ability to predict the hysteresis phenomenon for three soils that can be used in the construction of engineered covers. The Universal Mualem model proposed by Mualem (1977) was also retained because of its simplicity. The present study focuses (due to space limitation) on the prediction of the main drying curves using only one curve (main wetting curve); it should nevertheless be kept in mind that most of these models can also predict primary and secondary wetting and drying curves (but this is not addressed here).

3. DESCRIPTION OF SELECTED MODELS

In the following section, a brief description of the selected models is presented; only the equations used to predict the main drying curve are given.

3.1 Parlange model, modified by Hogarth et al. (1988)

The initial model presented by Parlange (1976) was mathematically complex (Mualem and Morel-Seytoux, 1978). By using a modified Brooks and Corey (1964) equation, the model was reformulated by Hogarth et al. (1988). Initially, the experimental data are fitted by using the modified Brooks and Corey equation which expresses the main wetting curves (MWC) as

$$\theta_w(h) = \theta_{ae} \left(\frac{h_{ae}}{h} \right)^\lambda \quad \text{for } h \geq h_{ae} \quad [1]$$

$$\theta_w(h) = \theta_{ae} \left(1 + \lambda - \lambda \frac{h}{h_{ae}} \right) \quad \text{for } h_{we} \leq h \leq h_{ae} \quad [2]$$

$$\theta_w(h) = \theta_s \quad \text{for } h \leq h_{we} \quad [3]$$

where h_{we} is the water entry pressure (see Figure 1), h_{ae} is the air entry pressure (see Figure 1), λ is a dimensionless shape factor, θ_s is the volumetric water content at saturation, θ_{ae} is the volumetric water content on the main wetting curve at air entry pressure, which is calculated as:

$$\theta_{ae} = \frac{\theta_s}{\left(1 + \lambda - \lambda \frac{h_{we}}{h_{ae}} \right)} \quad [4]$$

The parameters determined by fitting the main wetting curve are used to predict the main drying curve. This curve can be calculated using the expression:

$$\theta_d(h) = \theta_{ae} \left(\frac{h_{ae}}{h} \right)^\lambda \left(1 + \lambda - \lambda \frac{h_{we}}{h} \right) \quad \text{for } h \leq h_{ae} \quad [5]$$

$$\theta_d(h) = \theta_s \quad \text{for } |h| \leq |h_{we}| \quad [6]$$

where θ_d is the volumetric water content predicted, for a given water pressure h , during the drying process.

3.2 Parlange model, modified by Braddock et al. (2001)

The Parlange hysteresis model, which was initially formulated using Brooks and Corey equation (1964), was modified to use the well known van Genuchten (1980) equation to describe the WRC (Braddock et al. 2001). This model uses a simplified expression of van Genuchten equation by taking $\theta_r = 0$ (even if the residual volumetric water content can have an important role as a curve fitting parameter). In the following, θ stands for $(\theta - \theta_r)$, then $\theta(h) \rightarrow 0$ as $h \rightarrow \infty$. The van Genuchten (1980) model becomes:

$$\theta_{vG} = \theta_s \left[\frac{1}{1 + (\alpha h)^n} \right]^m \quad [7]$$

where θ_{vG} is the volumetric water content estimated with van Genuchten equation and m , n and α correspond to fitted parameters of the equation. The authors defined a parameter $C(h)$ as the specific capacity:

$$C(h) = \theta_s m n \alpha (\alpha h)^{n-1} \left[\frac{1}{1 + (\alpha h)^n} \right]^{m+1} \quad [8]$$

Using the van Genuchten equation to fit data of the main wetting curve, the main drying curve can be expressed as:

$$\theta_d = \theta_{vG}(h) + (h - h_d)C(h) \quad [9]$$

where θ_d is the calculated (predicted) volumetric water content on the main drying curve and h_d is the water pressure at the inversion point from wetting to drying. Braddock et al. (2001) have also presented other equations that allow predicting primary and secondary hysteresis curves.

3.3 Universal Mualem model

Mualem (1977) presents a general relationship between the two main curves (wetting and drying). The calibration of the model needs only one curve. By using the experimental data obtained during the main wetting process, the main drying curve can be calculated using the Universal Mualem model. This relationship is expressed as:

$$\theta_d(h) = [2\theta_s - \theta_w(h)] \frac{\theta_w(h)}{\theta_s} \quad [10]$$

where θ_s , θ_d , and θ_w are the volumetric water content at saturation, and during the drying and wetting processes, respectively.

4. EXPERIMENTAL SETUP AND MATERIALS CHARACTERISTICS

Different tests were performed to evaluate the hysteresis phenomenon on the WRC of sandy soils. The experimental setup was described in details by Maqsoud et al. (2002). It consisted of: (i) a plexiglas cylinder (70 cm in height and 14 cm in diameter) containing the soil sample at a known void ratio and (ii) a set of tensiometers and TDR probes inserted at the same elevation in the soil sample to measure θ and h . Calibration curves of the pressure transducers were made in the laboratory before the tests. Water pressure and volumetric water content profiles were recorded at constant intervals and measured data were transmitted at the end of the test to a microcomputer for analysis.

Three materials with different grain size distributions were tested: a uniform fine sand, a uniform coarse sand, and a well graded silty sand. The grain size distributions of these materials are presented in Figure 2.

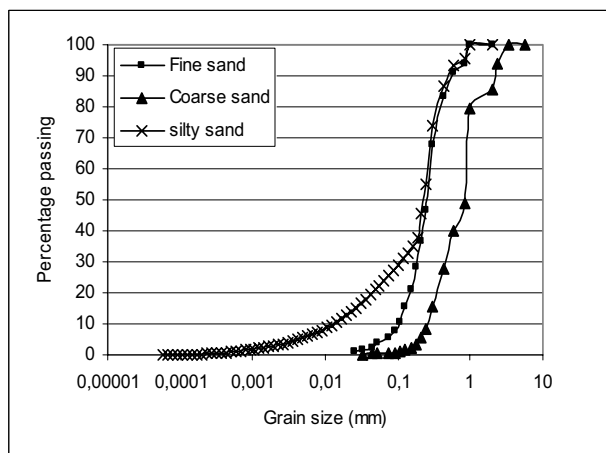


Figure 2. Grain size distribution of the fine sand, coarse sand and silty-sand used for the determination of WRC with hysteresis effects

The parameters D_{10} (diameter of particles at 10 % passing) is 0.278, 0.105 and 0.0135 mm for the coarse, fine and silty sand respectively. The uniformity coefficient C_u (D_{60}/D_{10}) for the three materials is 3.2, 2.6 and 21 for the coarse, fine and silty sand.

For each material, the following tests were performed. First, the main wetting curve was obtained by wetting gradually the soil sample from a dry condition to almost full saturation. The soil was then allowed to gradually drain; this defines the main drying curve. Second the soil is wetted to a known volumetric water content and then dried to determine a primary wetting and primary drying curves (these curves are not presented here due to space limitation).

5. MAIN RESULTS

This section presents the main fitting parameters obtained for different infiltration and drying tests performed in the column. The parameters presented are the ones of the van Genuchten (1980) and Brooks and Corey (1964) equations. The results obtained by using three predictive hysteresis models for the different materials are also presented and analysed.

5.1 Water retention parameters

The WRC obtained during different wetting and drying cycles were fitted with the van Genuchten (1980) and Brooks and Corey (1964) equations presented above. These parameters were obtained by using the RETC computer program (van Genuchten et al. 1991). For all curves, the residual water content was considered nil because the first test (wetting) was performed on a dry material. This hypothesis is considered realistic for coarse soils as the ones tested in this study. Fitting results are presented in Table 2. In this table one can find the water content at saturation (θ_s), α which is a fitting parameter that gives an estimate of the air entry pressure for MDC ($\alpha^{-1} \approx$ air entry pressure), and the r^2 value which gives the sum-of-squares of the vertical distances of the points from the fitted equation (a value of 1 indicates a perfect correlation between the fitted and observed volumetric water content values).

The results presented in Table 2 show that the air entry pressures (from the MDC) of these three sands, estimated by using α^{-1} , are between 16 and 52 cm of water. The r^2 for the main wetting and drying curves (for both van Genuchten and Brooks and Corey equations) are between 0.88 and 0.99, which corresponds to a good correlation between fitted and experimental data. These fitting curves will be used to compare the experimental and predicted data using the three chosen hysteresis models.

TABLE 2. Main hydraulic characteristics of the materials studied in the numerical simulation (MWC: main wetting curve and MDC: main drying curve)

Soil	Test	van Genuchten parameters				Brooks and Corey parameters			
		θ_s	α (cm^{-1})	n	r^2	θ_s	h_{xe}^* (cm^{-1})	λ	r^2
Fine sand	MWC	0.2795	0.0385	3.5379	0.97	0.2766	0.0616	1.5494	0.98
Fine sand	MDC	0.2836	0.0192	6.0186	0.93	0.2699	0.0257	1.6983	0.94
Coarse sand	MWC	0.2645	0.1635	4.7000	0.90	0.2622	0.2451	1.7400	0.90
Coarse sand	MDC	0.2655	0.0632	3.6098	0.98	0.2655	0.0836	1.7500	0.96
Silty sand	MWC	0.3000	0.1240	3.0000	0.88	0.2797	0.0301	0.9750	0.90
Silty sand	MDC	0.2926	0.0549	4.0522	0.93	0.2978	0.0199	0.9000	0.99

*: h_{xe} : x equal to a (h_{ae}) for MDC curves and equal to w (h_{we}) for MWC curves

5.2 Fine sand

The fitted main wetting and main drying curves (based on the experimental data) (see Figure 3) exhibit hysteresis effects. For a given water pressure, the difference between volumetric water content during wetting and drying processes can exceeds $0.18 \text{ cm}^3/\text{cm}^3$.

The Parlange model modified by Braddock et al. (2001) was first used to predict the main drying curve (MDC Pred. in the Figures) using the main wetting curve (MWC Fitted). The MDC predicted using Equation [8] and [9] exhibits an unrealistic behaviour when θ_d is greater than θ_s for water pressures between h_{we} and h_{ae} (see Figure 1 for the location of h_{we} and h_{ae}). This behaviour was presented and explained by Braddock et al. (2001). For the other part of MDC curve (when h is greater than h_{ae}), the correlation between fitted and predicted θ is very good and the deviation is typically lower than measurement errors of TDR probes (0.025).

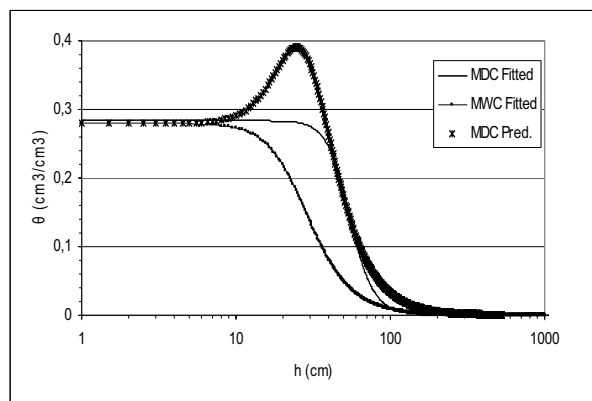


Figure 3. Predicted and measured hysteresis curves obtained for fine sand by Parlange model modified by Braddock et al. (2001)

The application of Universal Mualem model to the experimental data obtained for the fine sand shows that the predicted main drying curve is quite different than the experimental data (see Figure 4). This Figure also shows that the MDC predicted by the Universal Mualem model has a shape similar to the fitted curve. This model

underestimates the h_{ae} value and for this reason the fitting is not as good as with the Parlange model.

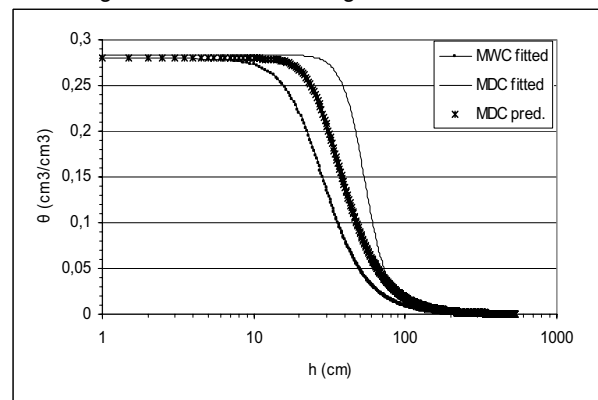


Figure 4. Predicted and fitted hysteresis curves obtained for fine sand using the Universal Mualem model

For the Parlange Model modified by Hogarth et al. (1988), the main wetting curve for $h_{we} < h < h_{ae}$ is given by an horizontal line (constant water content) and θ_{ae} is calculated with Equation [4]. Figure 5 shows the two fitted curves (MWC and MDC) and the predicted main curve. The agreement between the fitted and predicted main drying curve is seen to be good.

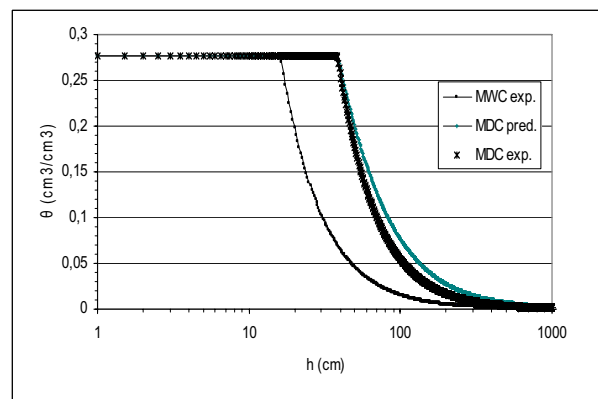


Figure 5. Predicted and fitted hysteresis curves obtained for fine sand using Parlange model modified by Hogarth et al. (1988)

5.3 Coarse sand

The three models were tested using the experimental data obtained on the fine sand. Figure 6 shows hysteresis effects on the water retention curve, with a difference between volumetric water content during wetting and drying processes.

With the Parlange model modified by Braddock et al. (2001), the trend seen with the fine sand is also observed (see Figure 6). The higher than expected θ values is again present. θ values on the predicted MDC for water pressure smaller than the h_{ae} value exceed the full saturation value. It can also be seen that for this material, the agreement between predicted and fitted curves is not as good as that for the fine sand. This difference between predicted and fitted values may be due, at least in part, to measurement errors and also to errors generated by the fitting procedure ($r^2 = 0.90$ for the main wetting curve).

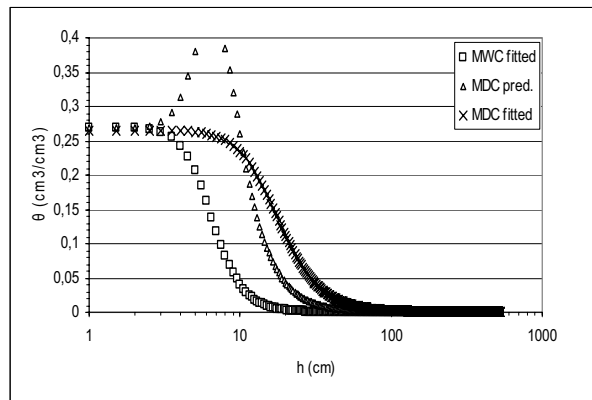


Figure 6. Predicted and fitted hysteresis curves obtained for coarse sand using the Parlange model modified by Braddock et al. (2001)

For the Universal Mualem model, results are fairly similar to the ones obtained for fine sand (see Figure 7), where the predicted MDC underestimates the θ value for most of the water pressure range. Again the predicted h_{ae} is lower than the experimental value.

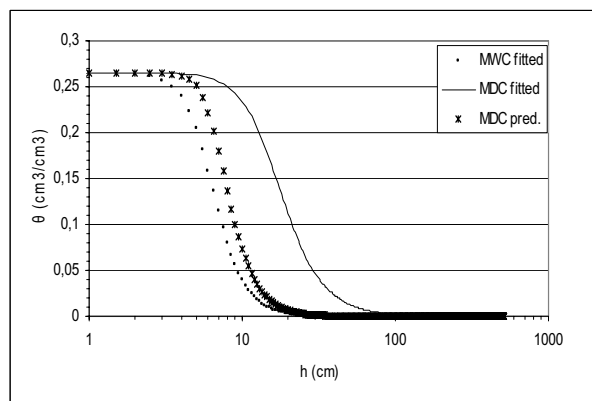


Figure 7. Predicted and fitted hysteresis curves obtained for coarse sand using Universal Mualem model

The predicted main drying WRC obtained with Parlange model modified by Hogarth et al. (1988) is shown in Figure 8. The Figure shows that the agreement between the fitted and predicted main drying curves is excellent.

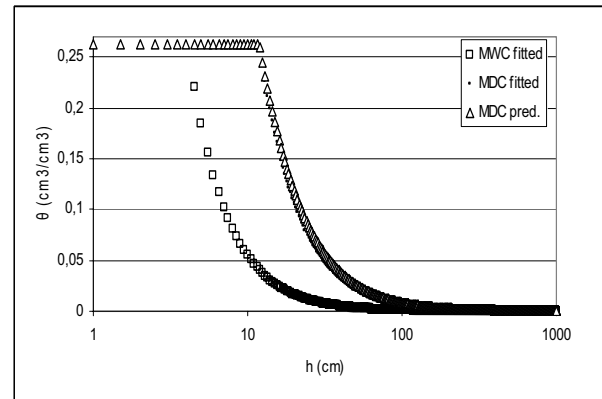


Figure 8. Predicted and fitted hysteresis curves obtained for coarse sand using Parlange model modified by Hogarth et al. (1988)

5.4 Silty sand

The fitted water retention curve obtained with the wetting and drying processes are finally compared to predictive WRC obtained with the predictive models, for the silty sand. This material also shows hysteresis effects on the WRC (see Figure 9). Prediction with the Parlange model modified by Braddock et al. (2001) for MDC gave excellent results (except for the water pressure range between $h_{we} < h < h_{ae}$ when an overshoot of the water content is again seen on Figure 9).

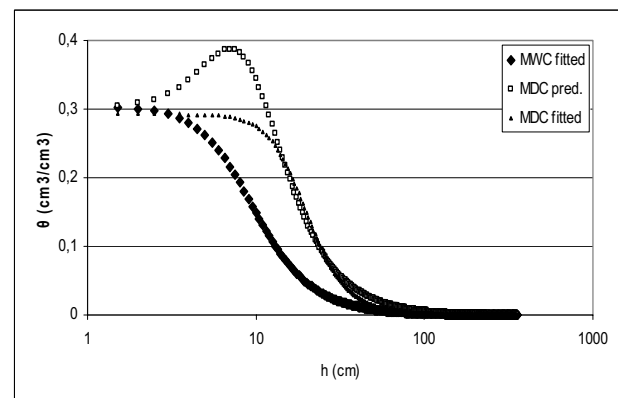


Figure 9. Predicted main drying curve (MDC pred.) and fitted main drying curve (MDC fitted) obtained for silty-sand; the Parlange model modified by Braddock et al. (2001) model is used.

The predicted curves using the Universal Mualem model show a relatively poor correlation; the volumetric water content is lower for almost the entire pressure range when compared with the fitted curve deduced from the experimental data (see Figure 10). However the difference

between the fitted and predicted water content values is less significant than the results obtained for the coarse sand.

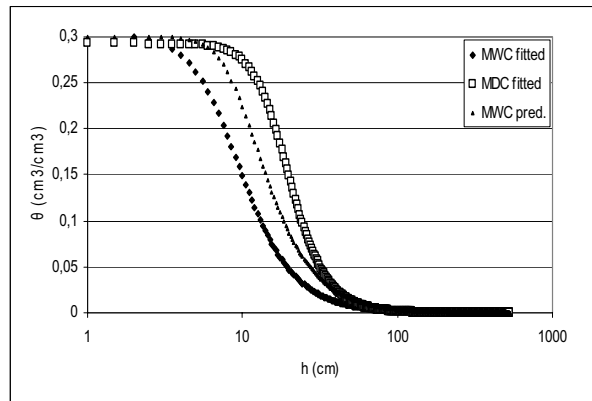


Figure 10. Predicted and fitted hysteresis curves obtained for silty sand and using universal Mualem model

The predicted results with the Parlange model modified by Hogarth et al. (1988) are very close to the measured values obtained from the column tests (MWC fitted in Figure 11) for the entire curve.

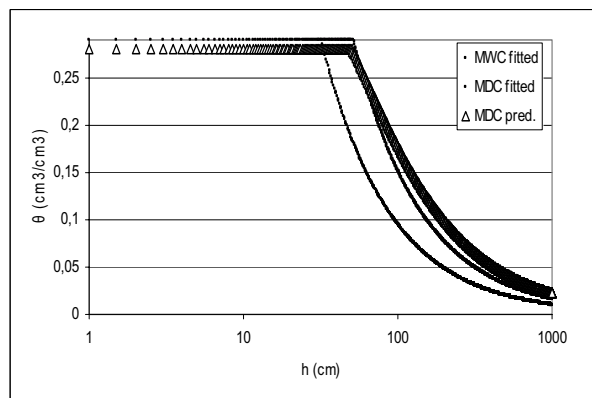


Figure 11. Predicted and fitted hysteresis curves obtained for silty sand using Parlange model modified by Hogarth et al. (1988)

6. SUMMARY AND CONCLUSION

Three hysteresis models taken from the literature were described and tested to predict water retention curves and making an assessment of their predictive capabilities using experimental data. These models were chosen in part because of their relative simplicity and ease of application. They require a knowledge of only one of the main curves for calibration. The water retention curves of three sandy materials that can be used as the basic materials for the construction engineered covers were measured in laboratory during different drying and wetting processes. All the first wetting tests were performed on dry soils. For the studied materials, the hysteresis effect is more significant (area between the MWC fitted and the

MDC fitted) for the fine sand and the silty sand than for the coarse sand. The results obtained by using different models indicate that the Universal Mualem model did not predict adequately the WRC. However, this model predicted correctly the shape of the drying curve and could be improved by incorporating a better estimate of the h_{ae} parameter. The Parlange model modified by Braddock et al. (2001) exhibited an unrealistic response at suction close to the air entry pressure. This behaviour could be eliminated easily, so this model may still remain promising. For the two versions of the Parlange model, the results are different from one material to another. For the silty sand and the fine sand, the Braddock et al. (2001) version was the best one to predict the MDC, while for the coarse sand the Hogarth et al. (1988) version gave better predictions. The main difference between the two versions of the Parlange model comes from the equation selected to describe the WRC. The best fitting for coarse sand is obtained with the Brooks and Corey equation, and this explains the good prediction obtained using the Hogarth et al. (1988) version of the Parlange Model. On the other hand, for the silty sand and the fine sand, the van Genuchten equation gave the best fit with the experimental results (see Table 2) and a better prediction of the MDC by the Braddock et al. (2001) version of the Parlange model.

The Parlange models tested for different materials provided good predictions of the main drying water retention curves. Because of their relatively simple formulation, they could easily be incorporate into numerical models for a unsaturated water flow and/or contaminant transport. In the future, other physically based models developed to predict the WRC, such as the MK model (Aubertin et al. 2003), will be considered in the ongoing investigation to obtain a suitable model for general application.

7. REFERENCE

- Aubertin, M., Mbonimpa, M., Bussière, B., and Chapuis, R.P. 2003. A model to predict the water retention curve from basic geotechnical properties. *Canadian Geotechnical Journal*, 40(6): 1104-1122.
- Basile, A., Ciollaro, G., and Coppola, A. 2003. Hysteresis in soil water characteristics as a key to interpreting comparisons of laboratory and field measured hydraulic properties. *Water Resour. Res.*, Vol. 39, pp. 1301-1312.
- Braddock, R.D., Parlange, J.-Y., and Lee, J. 2001. Application of a soil water hysteresis model to simple water retention curves. *Transport in Porous Media*, Vol. 44, pp. 407-420.
- Brooks, R.H. and Corey, A.T., 1964. Hydraulic properties of porous media. Hydrology paper no. 3. Colorado State University, Fort Collins.
- Bussière, B., Aubertin, M., Mbonimpa, M. and Benzaazoua, M. 2004. In situ test cells for the evaluation of silty covers with capillary barrier effects. Submitted to *Canadian Geotechnical Journal* (Decembre 2003).

- Dane, J. H. and Wierenga, P. J. 1975. Effect of hysteresis on the prediction of infiltration, redistribution and drainage of water in a layered soil, *J. Hydrology*, Vol. 25, pp. 229-242.
- Enderby J. A. 1955. The domain of hysteresis. Part I, independent domains, *Fraday Soc. Trans.* Vol. 51, pp. 835-848.
- Everett, D. H. and Whitton, W. I. 1952. A general approach to hysteresis, *Trans. Fraday Soc.*, Vol. 48, pp. 749-757.
- Everett, D. H. 1954. A general approach to hysteresis – Part 3 : A formal treatment of the independent domain model of hysteresis, *Fraday Soc. Trans.* Vol. 50, pp. 1551-1557.
- Everett, D. H. 1955. A general approach to hysteresis – Part 4 : An alternative formulation of the domain model. *Faraday Soc. Trans.* Vol. 50, pp. 187-197.
- Everett, D. H., and Smith F. H. 1954. A general approach to hysteresis – Part 2 : Development of the domain theory, *Fraday Soc. Trans.* 50 : 187-197.
- Gandola, F., Debionne, S., Varado, N., Haverkamp, R., Ross, P.J., Sander, G., and Parlange, J.Y. 2004. Simple soil water hysteresis prediction model based on theory and geometric scaling. *Proc. of EGU conference*, Vol. 6.
- Haines, W. B. 1930. Studies in the physical proprieties of soil : V. The hysteresis effect in capillary proprieties, and the modes of moisture distribution associated therewith. *J. Agricultural Sci.*, 20, pp. 97-116.
- Hanks, R. J., Klute, A. and Bresler, E. 1969. A numerical method for estimating infiltration, redistribution, drainage and evaporation of water from of water from soil, *Water Resour. Res.*, Vol. 5, pp. 1064-1069.
- Haverkamp, R., Reggiani, P., Ross, P.J., and Parlange, J.Y. 2002. Soil water hysteresis perdition model based on theory and geometric scaling. *Environmental Mechanics; Water, Mass and Energy Transfer in the Biosphere*, Geophysical Monograph Series, Vol. 129.
- Haverkamp, R. and Parlange, J-Y. 1986. Predicting the water-retention curve from particle size distribution: 1. Sandy soils without organic matter, *Soil Sci.*, Vol. 142, pp. 325-339.
- Hillel, D., 1980. *Fundamentals of soil physics*. Academic Press Inc., New York, 413 p.
- Hoa, N-T., Gaudu, R. and Thirriot, C. 1977. Influence of the hysteresis effect on transient flows in saturated-unsaturated porous media, *Water Resour. Res.*, Vol. 13, pp. 992-996.
- Hogarth, W., Hopmans, J. and Parlange, J-Y. 1988. Application of a simple soil water hysteresis model, *J. hydrol.*, Vol. 98, pp. 21-29.
- Jaynes, D. 1992. Estimating hysteresis in the soil water retention function. In *Proceedings of the International Workshop on Indirect Methods for Estimating the Hydraulic Proprieties of Unsaturated Soils*, University of California, Riverside, California, pp. 219-232.
- Jaynes, D. 1984. Comparison of soil-water hysteresis models, *J. Hydrology*, Vol. 75, pp. 287-299.
- Maqsoud, A., Bussi re, B., and Aubertin, M. 2002. L'hyst r sis des sols non satur s utilis s dans les recouvrements avec effet de barri res capillaires, *Proceeding of 55 Canadian Geotechnical Conference - 3rd Joint IAH-CNC/CGS : Ground and Water : Theory to practice*, pp 181-188.
- Mualem, Y. 1973. Modified approach to capillary hysteresis based on a similarity hypothesis, *Water Resour. Res.*, Vol. 9, pp. 1324-1331.
- Mualem, Y. 1974 A conceptual model of hysteresis, *Water Resour. Res.*, Vol. 10, pp. 514-520.
- Mualem, Y. and Dagan, G. (1975) A dependence domain model of capillary hysteresis, *Water Resour. Res.* 11, 452-460.
- Mualem, Y. 1977. Extension of the similarity hypothesis used for modeling the soil water characteristics, *Water Resour. Res.*, pp. 773-780.
- Mualem, Y. and Morel-Seytoux, H. J. 1978. Analysis of a capillary hysteresis model based on a one variable distribution function, *Water Resour. Res.*, 14, 605-610.
- Mualem, Y. 1984. A modified dependent-domain theory of hysteresis, *Soil Sci.*, pp. 283-291.
- N el, L. 1942-1943. Th ories des lois d'aimantation de Lord Raileigh, *Cah. Phys.*, Vol. 12, pp. 1-20, Vol. 13, pp. 19-30.
- O'Kane, J.P., Pokrovskii, A., and Flynn, D. 2004. The fest model for testing the importance of hysteresis in hydrology. *Proc. of EGU conference*, Vol. 6, 07303.
- Parlange, J-Y. 1976. Capillary hysteresis and relationship between drying and wetting curves, *Water Resour. Res.*, Vol. 12, pp. 224-228.
- Parlange, J-Y. 1980. Water transport in soils, *Annual Review of Fluid Mechanics*, Vol. 12, pp. 77-102.
- Pickens, J. F. and Gillham, R. W. 1980. Finite element analysis of solute transport under hysteretic unsaturated flow conditions, *Water Resour. Res.*, Vol. 16, pp. 1071-1078.
- Poulovassilis A. and Childs E. C. 1971. The hysteresis of pore water : the non-independence of domains, *Soil Sci.* Vol. 112, pp. 301-312.
- Poulovassilis, A. 1962. Hysteresis of pore water, an application of concept of independent domains, *Soil Sci.* Vol. 93, pp. 405-412.
- Poulovassilis, A. 1970. Hysteresis of pore water in granular porous bodies, *Soil Sci.*, Vol. 109, pp. 5-12.
- Preisach P., 1938.  ber die magnetische Nachwirkung. *Zeitschrift f r Physik*, Vol. 94, pp. 277-302.
- Scott, P. S., Farquhar, G. J. and Kouwen, N. 1983. Hysteretic effects on net infiltration, *Adv. in Infiltration*, ASAE, St Joseph, MI, pp. 163-170.
- Si, B. C. and Kachanoski, R. C. 2000. Unified solution for infiltration and drainage with hysteresis : Thery and field test, *Soil Sci. Soc. Am. J.* Vol. 64, pp. 30-36.
- Topp, G. C. 1971. Soil-water hysteresis : the domain theory extended to pore interaction conditions, *Soil Sci. Soc. Amer. Proc.*, Vol. 35, pp. 219-225.
- Topp, G.C. and Miller, E. E. 1966. Hysteresis moisture characteristics and hydraulic conductivities for glass-bead media, *Soil. Sci. Amer. Proc.* 30 : pp. 156-162.
- van Genuchten, M. TH. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, *Soil Sci. Am. J.* 44 : 892-898.
- van Genuchten, M. TH., Leij, F. J. and Yates, S. R. 1991. The RETC code for quantifying the hydraulic functions of unsaturated soils, EPA document EPA/600/2-91/065.
- Viane, P., Vereecken, H., Diels, J. and Feyen, J. 1994. A statistical analysis of six hysteresis models for the moisture characteristics, *Soil Sci.* Vol. 157, pp. 345-355.