A METHOD FOR THE OPTIMAL LOCATION OF PIEZOMETERS TO ESTIMATE THE CONTRIBUTING AREA OF A PUMPING WELL

Michael Verreault, Université du Québec à Chicoutimi, Les Laboratoires S.L. inc., Chicoutimi, Québec, Canada
Alain Rouleau, Université du Québec à Chicoutimi, Québec, Canada
Réal Daigneault, Université du Québec à Chicoutimi, Québec, Canada
Denis W. Roy, Université du Québec à Chicoutimi, Québec, Canada

ABSTRACT
The contributing area of a pumping well can be determined using several investigation tools, including field methods and mathematical simulation. Piezometric mapping is a relatively direct field method that implicitly captures important factors such as the recharge by vertical infiltration and the heterogeneity of the medium. The number of observation wells, their location and the interpolation method affect the estimation results. An optimization method is proposed in order to minimize the number of piezometers to be installed, while obtaining a valid estimation of the contributing area. Two distinct areas must be considered in this analysis, namely the vicinity of the pumping well and the area situated further upstream. This optimization method has been tested using numerical simulation of groundwater flow at virtual sites. Good results are obtain for sites in porous and heterogeneous media.

RÉSUMÉ
L'aire d'alimentation d'un puits de pompage s'estime à l'aide de plusieurs méthodes d'investigation, incluant les travaux de terrain et la modélisation numérique. La cartographie piézométrique est une méthode directe qui tient compte implicitement d'importants facteurs tels la recharge par infiltration verticale et l'hétérogénéité du milieu. Le nombre de puits d'observation, leur emplacement et la méthode d'interpolation influent sur le résultat de l'estimation. Une méthode d'implantation optimale de piézomètres est proposée afin de minimiser le nombre de piézomètres à implanter lors de l'analyse tout en obtenant un résultat satisfaisant. Deux secteurs sont à considérer dans l'analyse, soit la partie proximale du puits et la partie plus loin en amont. Cette méthode d'optimisation a été évaluée par simulation numérique sur des sites virtuels. Des résultats satisfaisants sont obtenus pour les scénarios virtuels en milieux poreux et hétérogènes.

1. INTRODUCTION

Several methods of analysis are used to estimate the contributing area of a pumping well, including analytical methods (Bear and Jacobs, 1965; Grubb, 1993), semi-pic and homogeneous porous medium, and an aquifer of infinite lateral extend. The semi-analytical methods consider supplementary factors, but the actual computer programs limit the possibilities of this method. Numerical simulation of groundwater flow allows for the integration of several factors not considered in analytical methods such as the heterogeneity and the anisotropy of the medium, the vertical recharge and the hydrographic network. Piezometric mapping is based on field data that capture the effects of important factors such as the aquifer heterogeneity, the flow field perturbation and the vertical recharge.

All of these methods do require some field investigation work in order to get an accurate estimation of the contributing area. Analytical methods require at least three observations wells to estimate the regional piezometric gradient. More field information is required for the semi-analytical and the numerical ground water flow simulation methods. This information can be obtained by various investigation works such as piezometric monitoring, stratigraphic surveying, pumping test, tracer analytical methods (Strack, 1989), piezometric mapping and ground water flow simulation. The analytical methods consider very limiting assumption such as an isotro test, seepage meter test, and fracture system analysis (Chevalier et al., 2003; see chapter 6).

It has been demonstrated that a contributing area estimated by analytical methods is often affected by important uncertainties (Bahtt, 1993, Jacobson et al., 1995; Verreault, 2003). These uncertainties are caused by many factors inherent with the complexity of an aquifer. Semi-analytical and numerical methods are also affected by uncertainties unless all of these factors are taken into account based on sound and important field data.

Piezometric measurements represent a relatively direct method for obtaining reliable information on the groundwater flow system and the contributing area of a pumping well. However, the construction and the monitoring of piezometers represent important costs in a groundwater management program. For this reason, we propose an Optimal Piezometer Positioning (OPP) method to estimate the contributing area of a pumping well using the piezometric mapping method. Optimization methods have been proposed for the characterization and the control of a contaminant plume in ground water (French et al., 2000; Hudak, 2000). The OPP method that is proposed here pays a special attention to the determination of the lateral boundaries of the contributing area.

The OPP method is tested on virtual sites. These sites that are developed using the numerical flow model MODFLOW (McDonald and Harbough, 1988). The contributing areas estimated with backward particles tracking using MODPATH (Pollock, 1994) are considered as the actual contributing areas. The use of virtual sites is necessary because the validation of the OPP results on a real site would require an extremely important effort of field investigation in order to make comparisons with the real contributing area of the well.

Only one virtual site is presented here, in order to illustrate the development of the OPP method. Application to other virtual sites are presented in Verreault (2003).

2. THE VIRTUAL SITE

The virtual site presented here is composed of various units of porous material (figure 1). A sand and gravel unit deposited by a high energy river underlies a less permeable clay layer and a silty sand layer. The hydraulic conductivity (K) values are 1 X 10^{-3} m/s for the sand and gravel unit, 1 x 10^{-4} m/s for the silty sand and 1 x 10^{-8} m/s for the clay. The cristalline bedrock located under the porous deposits has a K value of 1 X 10^{-7} m/s.

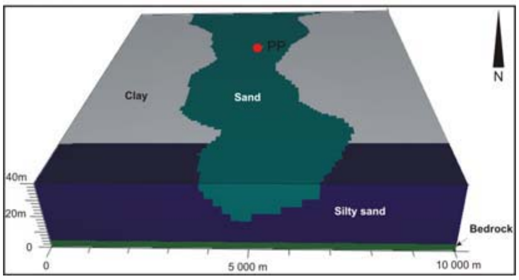

Figure 1: 3D view of the fossil valley virtual site.

Two constant head boundaries, one at the north (elevation 27 m) and the other at the south (elevation 39 m), generate a groundwater flow direction to the north. The average hydraulic horizontal gradient in the permeable units is 0.0011 under natural flow conditions.

A water well pumping at 2000 m^3/day is located in the northern part of the site, in the sand and gravel unit. Well is screened at the depth interval 12.4 to 17.5 meters, almost reaching the base of the aquifer.

The contributing area created by this system is open to the south with a very large angle due to flow lines that are converging towards the fossil valley (figure 2). A slightly lower pumping rate would result in a considerably narrower contributing area restricted to the more permeable fossil valley. At the considered pumping rate, a significant portion of the water is captured outside of the fossil valley, with a non negligible potential impact on the quality of the pumped water.

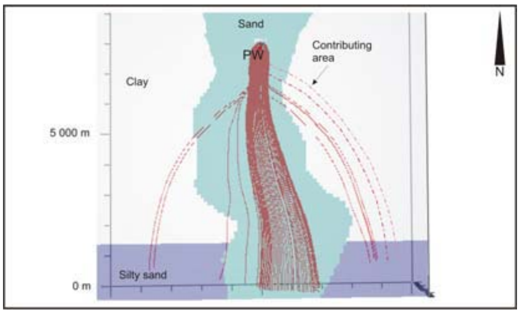

Figure 2: Plan view of the contributing area generated by backward particules tracking using MODPATH.

3. THE OPP METHOD

The Optimal Piezometer Positioning (OPP) method was developed with the *Mapinfo geographic information system* (MAPINFO, 2003), using a combination of vectorial surface production and gridding interpolation techniques. Vectorial surface production techniques used are triangular irregular network (TIN) and Thiessen polygons. Several gridding interpolation methods are available with the utility software *Vertical MAPPER* (MAPINFO, 2003) but natural neighbour analysis and triangulation were selected mainly because the grid results are conditioned to the exact values of initial points. As for more over, the natural neighbor method considers the anisotropy in the spatial distribution of the measurement points and it can create surfaces with overshoots and undershoots, generating a much smoother surface.

3.1 Basic concepts

The aquifer material is considered isotropic, resulting in floor lines crossing perpendicularly the equipotential lines. Only the flow direction is considered in the present analysis, the groundwater flow velocity is neglected. The OPP method is sensitive to the fact that the downstream boundary of the contributing area corresponds to a geometric singularity taking the form of a saddle in the piezometric water table (figure 3). Backward particle tracking is used to define the two lateral boundaries of the contributing area, using only two particles. Only the particles corresponding to the lateral boundaries are considered, as the particles traveling within the contributing area do not represent flow paths of interest in this analysis. For this reason, the determination of an initial point at the downstream boundary of the contributing area is a critical element of the method.

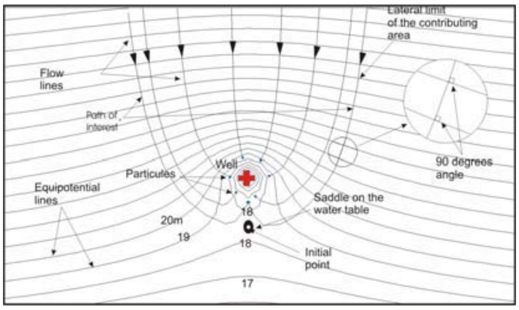

Figure 3: Plan view of the OPP method, showing the initial point and particle the paths of interest.

We have seen that the OPP method considers only the flow paths corresponding to the lateral boundaries of the contributing area. These boundaries are composed of two arms ; we define the left arm and the right arm (figure 3), when looking of upstream in the direction the contributing area. The left arm and the right arm do converge at the singular point in the piezometric saddle located downstream from the pumping well.

3.2. Development of the method

An infinite number of piezometer patterns can be used to estimate the contributing area of pumping well. We do propose an approach to minimize the number of observation points while obtaining the desired accuracy in the estimation of the contributing area. A "*proximal pattern"* is proposed within the vicinity of the pumping well while a *"distal pattern*" applies further upstream.

As the contributing area is estimated using backward particle tracking, it is essential to properly evaluate the downstream boundary of the contributing area. If this initial point is not correctly estimated, the accuracy of the contributing area as estimated by the OPP method shall be substantially reduced.

3.2.1 Proximal pattern

The proximal pattern of piezometers is shown in figure 4. The distance between piezometers is proportional to the distance between the pumping well and the downstream boundary of the contributing area. This proportionality of the distances eliminated the need for a preliminary determination of the entire domain of investigation. When this distance is known, all the others become known as well. As well, with this correspondence between distances, preliminary domain area evaluation was not necessary.

Equipotential lines arising from the proximal pattern of the OPP method are interpolated with the natural neighbor method. The Voronoï diagram created by the natural neighbor method shows Thiessen polygons of similar shapes and dimensions (figure 4). Other interpolation methods poorly react to the important variations of the hydraulic head generally observed within the vicinity of the pumping well. The triangulation method for instance, would require quite a few more observation points to estimate properly the contributing area within the proximal zone.

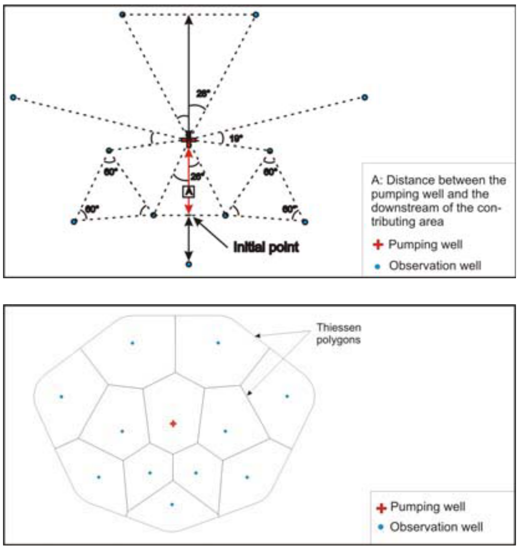

Figure 4: Plan view of the proximal pattern distribution (a), and the distribution of the Thiessen polygon (b).

A radially symetric distribution of piezometers crossing the pumping well would not give an optimal interpolation of the equipotential lines, because of the rather elongated drawdown cone along the axis of the natural groundwater flow. For instance, the anisotropy of the pattern of observation points should be similar to the anisotropy of the observed parameter would result in an estimated cone that is elongated cone in the direction opposite to the anisotropy of the observation point pattern. As the piezometric field is generally anistropic, particularly near a pumping well, the optimal pattern of piezometers in the proximal zone cannot be isotropic.

The pattern of piezometers shown in figure 4 takes into consideration the anisotropy of the piezometric field in the vicinity of a pumping well. Piezometers are distributed symmetrically with respect to the axis of the natural groundwater flow. They are slightly closer in the downstream area of the well ; where the geometric saddle of the water table is located. The location of the piezometers includes an interpolation domain that is necessary for a proximal estimation of the contributing area. The results obtained with this particular pattern reproduce very well the shape of the contributing area even in a complex medium and with relatively low number of piezometers. A number of others patterns would also give valid results.

The first estimation of the contributing area can be made after the construction of piezometers following the proximal pattern. This first estimation is limited to the interpolated domain, but a slight extrapolation is made for each arm of the contributing area in order to determine the location of further observations.

To estimate the location of the initial point, several methods can be used, such as the analytical solutions or the positioning of lined up piezometers. A minimum of three points is required to create a second degree interpolation but the positionning of four or five piezometers will give more accurate results. Piezometers should be implanted in a specific order to avoid the insertion of unnecessary piezometers. Of these three required piezometers, one should be situated outside the contributing area to make it easier to target its downstream boundary. Because this interpolation is only on two dimensions, Newton polygon, cubic spline or Lagrange interpolation could be used (Fortin, 2000). The integration of pumping well in the analysis allow to get only one solution.

3.2.2 Distal pattern

The estimation of the distal contributing area is carried out using the triangulation as the interpolation method for two reasons : 1) the shape of the lines created at the boundaries of the interpolation zone and 2) the interpolation is conditioned to the measured values. At each step only three observations points are used in general, resulting in straight isopiezometric lines forming a plane.

Both arms forming the boundaries of the contributing area should generally be estimated independently from one another. The implantation of the boundaries for the distal portion of the contributing area is done using a sequence of piezometers triangles overlapping as much as possible each one of the two arms of the contributing area as much as possible. Ideally, the considered arm of the contributing area should pass through the center of the sequential triangles (figure 5). Following to the addition of each new piezometer, the location of the corresponding arm should be re-estimated and slightly extrapolated further upstream, using three or four piezometers including the new one. The piezometers previously emplaced for the proximal estimation shall therefore be ignored as the user go upstream in the estimation of the contributing area.

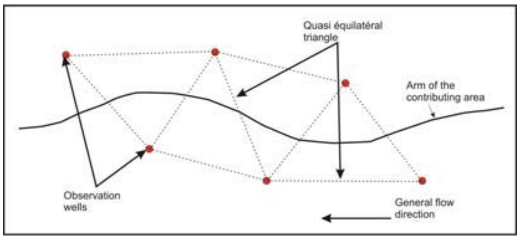

Figure 5: Plan view of the observation wells distribution in the distal zone of the contributing area.

3.2.3 Transition between proximal and distal patterns

The transition between the proximal and the distal pattern of piezometers require two types of supplementary points, called “artificial points” and “exit piezometers”. The numeral transition between the two interpolation methods that were used, namely the natural neighbor and the triangulation methods, is done using “artificial points” introduced in the proximal zone. These points are not actually emplaced on the site and they do not provide new piezometric values. Instead they consider values that were obtained through the natural neighbor method (Figure 6). The distribution of these points can be random, but their number should be quite large (figure 6).

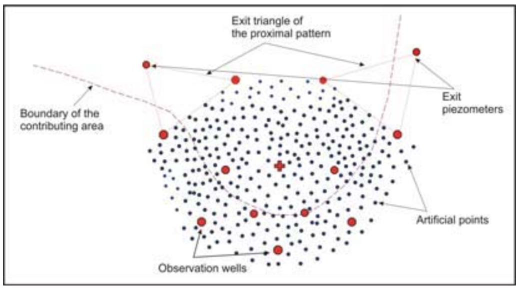

Figure 6: Plan view of the exit and the artificial points.

When the proximal interpolation is done, two other piezometers (Ro and Lo) should be added outside the proximal interpolation zone ; they are called exit piezometers. Each exit piezometers should form a quasi-equilateral triangle with two piezometers of the proximal pattern. Exit piezometers should be used along with the proximal piezometers pattern during process in the proximal zone using the natural neighbor method. The interpolation is repeated with the triangulation method and a second estimation is obtained.

3.2.4 Increasing the dimension of the triangles

When the contributing area is small and in line with the well, the distance between each piezometer in the distal area shall be small as well. This distance could be increased progressively if the desired estimation domain is large. However, the accuracy of the estimation of the contributing area is inversely proportional to the distance between observation points.

A drastic increase in the dimension of the triangles would result in non-equilateral triangles a less accurate interpolation. The addition piezometers and the resulting piezometer of triangles to estimate the boundaries of the contributing area might be pursued up to the desired location, and ultimately to the upstream boundary of the contributing area. Table 1 summarizes the process of the method.

Table 1 : Summarizes the process of the method

Process step number	Process	Estimation step number
1	Determination to the initial point	
2	Emplacement of piezometer of the proximal pattern	1st step of estimation
3	Insertion of artificial points and exit piezometers	2nd step of estimation
4	Emplacement of piezometers of the distal pattern	Subsequent steps of estimation

4. EXAMPLE AND RESULTS

An application example of the OPP method is given for the site shown in section 2. Only the estimation of the left arm is described in detail. Results for other sites can be view in Verreault (2003).

4.1 Exemple

The downstream boundary of the contributing area is estimated as a first step by the emplacement of lined-up piezometers. Then, the proximal pattern is applied and a first step of the estimation of the contributing area is carried out (figure 7). Artificial points are then inserted in order make the transition of the interpolation method from natural neighbors to triangulation (figure 8). The two exit piezometers (Lo and Ro) are emplaced forming a triangle with piezometers situated along the boundary of the proximal pattern, allowing for the second step of the estimation of the contributing area using the triangulation method (Figure 8).

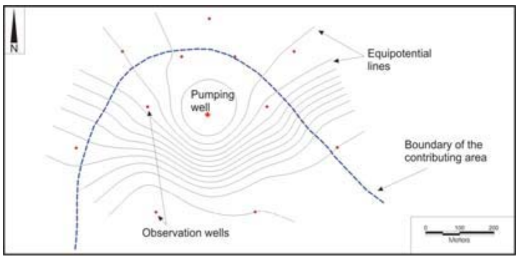

Figure 7: Results of the interpolation in the proximal zone of the contributing area.

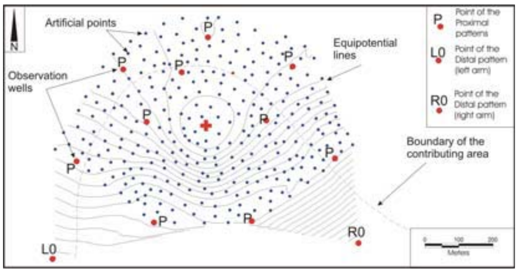

Figure 8: Plan view of the second step of the estimation of the contributing area, including exit piezometers and artificial points.

Piezometers of the distal pattern are emplaced sequentially, each step resulting in a extension further upstream of the estimation of the considered arm of the contributing area (figure 9). Two new piezometers (G1 and G2) are emplaced to form a quasi-equilateral triangle with the exit observation point previously emplaced. Only one piezometer could have been added in the north-east direction from the exit observation point, but the addition of new piezometers at this step in the upstream zone reduces the number of piezometers that are necessary in total for the contributing area estimation. On the other hand, the accuracy is not as great. The positioning of the next piezometer much further upstream (G3) (figure 9) creates an isosceles triangle which reduces the accuracy of the estimation. The emplacement of a supplementary piezometer (G4) result in an estimation of this arm that is very different from the one previously obtained. Final result of this example is show on figure 10.

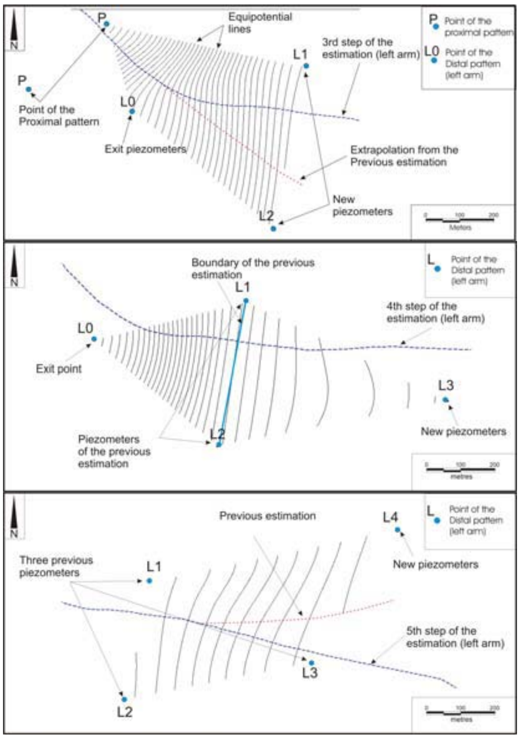

Figure 9: Plan view of the 3rd step (a), the 4th step (b), and the 5th step (c) of the estimation of the contributing area.

4.2. Results

Figure 10 illustrates the «true» contributing area of the fossil valley site in comparison with the one estimated using the OPP method, as well as the one estimated using the analytical method developed by Grubb (1993). The respective surface area of each of the contributing areas is 41 540 000 m^2 for the true on 39 040 000 m^2 with the OPP method and 8 054 000 m^2 with analytical method. The standard, i.e. the average distance between the estimated boundary area and the true boundary is 193 m with the OPP method and 2 171 m with the analytical method. In this case, the estimation done with the OPP method yield better results than the analytical method.

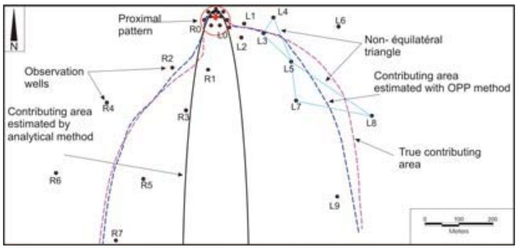

Figure 10: Plan view of the different results of the estimation of the contributing area.

Numerical modeling of groundwater flow would not necessarily give a more accurate estimation of the contributing area than the estimation that was obtained with the analytical method. One reason is that the dimensions of the normal section of the groundwater flow field must be well known in order to estimate properly the quantity of water coming from a given portion of the aquifer. A slight overestimation of the quantity of groundwater that circulates through a given portion of the aquifer would narrow down the estimated width of the contributing area, giving results similar to that obtained above with the analytical method.

5. DISCUSSION

In this section, we discuss a number of factors to be considered during the application of the optimal piezometer positioning method (OPP).

Because the contributing area is estimated by backward particle tracking (OPP), the accuracy of this estimation depends largely on the accuracy of the positioning of the starting-off points for the particles (initial point). The injection of the particles directly at the pumping well is not appropriate because of the abrupt changes in the direction of the flow lines in the downstream zone area from the well and particularly near the downstream boundary of the contributing area. For this reason, particles are introduced on the piezometric saddle situated at the downstream boundary of the contributing area. A proper positioning of saddle (initial point) is required for an accurate estimation of the contributions area.

A series of lined-up piezometers are emplaced to locate the downstream boundary of the contributing area. These lined-up piezometers cannot be considered for the flow analysis in the proximal area. In fact, no piezometer can be considered to estimate the proximal contributing area, other than the observation points that are includes in the proximal pattern, as these extra points would result in an heterogeneous Voronoï diagram. Dimensions and forms that are very different from the Thiessen polygons yield an interpolation that does not give the desired results. For a higher accuracy, several other points shall have to be added in order to create a homogenous Voronoï diagram. In such a case, the proximal pattern shall be completely reorganized.

During the distal estimation of the contributing area, the triangles should be as equilateral as possible. An elongated triangle results in a poor interpolation of the considered arm of the contributing area. The triangulation method with smoothing uses several triangle and takes into account the pattern neighbors. Factors that are taken into account in the interpolation with the smoothing include the following : location of the center of gravity of each triangle, the area of the triangle, the slope of the triangle in comparison with the general slope and the statistically derivated slope of the triangle vertex. These factors result in a different weight being given to each one of the interpolated triangles. Non-equilateral triangles have variable weight which alters the quality of the desired results ; It is often preferable to neglect the observation point that creates such a triangle.

Should there be an important local deviation of the direction of the groundwater flow, one point of the triangles may be located on, or very close to, one of the arms of the contributing area. In this event, it might be difficult to restore equilateral shape of the triangle along this arm the contributing area and the creation of an irregular polygon is necessary. The subsequent observation points should once again form triangles that are almost equilateral and that overlap considered the arm of the contributing area.

The OPP method is valid for a constant pumping flow rate. A change in flow rate results in a change of the contributing area boundaries which might come out of the corridors investigated by the piezometer pattern.

The piezometers are systematically positioned near the boundaries of the distal contributing area. Consequently, all of the piezometer are of equal use with the OPP method and no piezometer is useless. The aspect is different from other systematic patterns of piezometers. For example, with a regular grid pattern for instance, several piezometers may be useless especially if the contributing has an open shape. Several piezometers located at the center of the contributing area would be useless to locate the contributing area.

6. CONCLUSIONS

Several methods of analysis can be used to estimate the contributing area of a pumping well. An accurate estimation of the contributing area require some field investigation work. Piezometric mapping is based of field data that capture the effects of very important factors such as the heterogeneity of the aquifer.

An optimization method has been proposed in order to minimize the number of piezometers to be installed, while obtaining an accurate estimation of the contributing area

Two distinct areas must be considered in this analysis, namely the zone in the vicinity of the pumping well and the area situated further upstream. In the proximal zone, a fixed pattern of piezometers is efficient and the interpolation is done with the natural neighbour analysis method. In distal zone, the piezometers are installed progressively. At each step, a new piezometers position is determined from the previous estimation of the contributing area. These piezometric data are interpolated using the triangulation method.

An important number of piezometers may be required for the full implementation of the proposed method. Nevertheless, two factors reduce considerably the number of useless piezometers: 1) their sequential emplacement, and 2) their systematic location near the boundaries of the contributing area

REFERENCES

Bhatt K., 1993. Uncertainty in wellhead protection area delineation due to uncertainty in aquifer parameter values. Journal of Hydrology, 149 no.1 / 4, p.1.

Bear J. et Jacobs M., 1965. On the movement of the water bodies injected into aquifers. Journal of Hydrology, 3, pp. 37-57.

Chevalier S., Cousineau P.A., Rasmussen H., Rouleau A., Roy D.W., Tremblay M.L. et Verreault M., 2003. Guide de détermination d'aire d'alimentation et de protection de captages d'eau souterraine ; Chapitre 6 : Méthodes d'investigation sur le terrain. Publication Québec, Québec, Ministère de l'Environnement du Québec, disponible à www.menv.gouv.qc.ca.

Fortin, 2000. Analyse numérique pour ingénieurs. École polytechnique de Montréal, Montréal, 448 p.

French H.K., Van S.M. and Leijnse A., 2000. Prediction uncertainty of plume caracteristics derived from a small number of measuring points. Hydrogeology Journal, 8, pp. 188-199.

Grubb S., 1993. Analytical model for estimation of steady state capture zones of pumping wells in confined and unconfined aquifers. Ground Water, 31, pp. 27-32.

Hudak P.F., 2000. Procedure for upgrading contaminant detection networks in aquifers. Bulletin of Environment and Contaminant Toxicology, 65, pp. 62-69.

Jacobson E., Andricevic, R. et Hultin T., 1994. Wellhead Protection Area Delineation Under Uncertainty. University of Nevada, Las Vegas Publication no. 45118, 63p.

MAPINFO, 2003. Mapinfo Professional, user guide. Mapinfo corporation, Troy, New York, 496p.

McDonald M.G. et Harbaugh A.W., 1988. A modular three dimensionalsfinite-difference ground water flow model. U.S.G.S, Book 6.

Polock W.D., 1994. User guide for MODPATH/MODPATH-PLOT, version 3: A Particle Tracking Post-Processing Package for MODFLOW, the USGS Finite-Difference Ground Water Flow Model. United States Geological Survey, Open Report 94-464.

Strack O.D.L., 1989. Groundwater Mechanics, Prentice Hall.

Verreault M., 2003. Étude méthodologique pour l'estimation de l'aire d'alimentation de captage d'eau souterraine en milieu complexe. Mémoire de maîtrise, Université du Québec à Chicoutimi, Chicoutimi, 131p.

ACKNOWLEDGMENT

This work is funded by the *Programme d'aide à la recherche en environnement* (PARDE) of « Ministère de l'environnement du Québec » (MENV), and by the « Fondation de l'Université du Québec à Chicoutimi ». Special thanks to Yves Chiricota for his support on the interpolation method and to Sylvie Chevalier for frutfull discussion.