

## REGRESSION ANALYSIS TO DETERMINE THE STATISTICAL PROPERTIES OF INDIVIDUAL FRACTURES FROM CORE LOG AND HYDRAULIC TESTING DATA

Anthony C.F. West, Kent S. Novakowski

Department of Civil Engineering, Queen's University, Kingston, Ontario, Canada

**ABSTRACT** The density of fracturing (i.e. fracture spacings) and the distribution of flow among a set of fractures (i.e. fracture apertures) both play a crucial role in governing groundwater flow in bedrock. One way to determine these parameters is through hydraulic testing of discrete intervals along boreholes. When the density of fracturing is high compared to the density of test intervals, however, the statistical properties of the fractures are masked by the averaging effect of the testing method. Fracture data collected from the core log can alleviate this problem, but caution must be used in its interpretation. In this study, regression analysis is carried out to distinguish among classes of core-log-noted features and to determine the mean of the logarithms of their individual transmissivities. By treating the location of the logged features as Laplace-distributed random variables, estimates of the mean error in their location, relative to the location of hydraulic tests, are determined. The relative efficacy of a descriptive logging technique vs. a numerical logging technique based on a feature's probability of being permeable is also assessed. It is found that so long as only strictly permeable features are included as independent variables, a non-linear regression model based on the theory of effective permeability in a layered horizontal system is most appropriate for determination of fracture apertures. When indicators of reduced permeability are included as independent variables, a linear regression model is most appropriate. The parameters from the non-linear model are shown to be of physical significance, while those of the latter are not.

**RÉSUMÉ** La densité des fractures (i.e. l'espacement des fractures) et la distribution de flux parmi une série de fractures (i.e. les ouvertures de fracture) jouent un rôle crucial dans le gouvernement du flux d'eau souterraine. Une méthode pour déterminer ces paramètres sont par l'essai hydraulique d'intervalles discrets le long de boreholes. Quand la densité des fractures est haute en comparaison de la densité d'intervalles de test, cependant, les propriétés statistiques des fractures sont masquées par le fait l'effet moyenne de la méthode d'essai. Des données des fractures recueillies par la diagraphie des carottes peuvent alléger ce problème, mais la prudence doit être utilisée dans son interprétation. Dans cette étude, une analyse de régression est exécuté pour distinguer parmi les classes de caractéristiques notées dans la diagraphie des carottes et déterminer les moyens des logarithmes de leur transmissivités individuel. En traitant l'emplacement des caractéristiques notées comme des variables aléatoire laplace\_distribué, les estimations de l'erreur moyenne dans leur emplacement, relatif à l'emplacement de tests hydrauliques, sont déterminées. L'efficacité relative d'une technique notant descriptive contre une technique notant numérique basée sur un "probabilité d'être perméable" d'une ouverture est aussi évalué. Il est trouvé que pourvu que seulement les caractéristiques strictement perméables sont inclu comme les variables indépendantes, un modèle de régression non linéaire basé sur la théorie de perméabilité effective dans un système horizontal est plus approprié pour la détermination d'ouvertures de fracture. Quand les indicateurs de perméabilité réduite sont inclu comme les variables indépendantes, un modèle de régression linéaire est plus approprié. Les paramètres du modèle non linéaire sont montrés d'avoir de signification physique, pendant que ceux du modèle linéaire ne sont pas.

### 1. INTRODUCTION

If a fractured bedrock aquifer is regarded as a large water filter, purifying contaminated ground water on its journey from recharge to discharge at, for example, a well, it is easy to see why hydrologists are eager to understand how this process occurs and how to predict its effectiveness at specific locations. Intuition tells us, along with our experience with so many better-characterized filtering processes, that it is a maximization of the surface area of the rock to which the flowing water is exposed that results in maximum "purification" capacity. Research has further shown that it is diffusion of solute from the small volume of "flowing" water in the cracks and fissures of the aquifer into the vast reserves of "stationary" water trapped in the less permeable matrix that provides the most powerful purification mechanism. With this in mind, it is natural that

we should be interested in characterizing the networks of fissures and cracks which make up the flowing portion of the aquifer, and furthermore in understanding how the total flow through the aquifer is distributed among these features. However, this viewpoint guides us away from a determination to characterize every one of the features - we only need to determine their statistics.

The use of down-hole hydraulic tests is an effective method for characterizing variability of hydraulic properties within an aquifer. Typically, isolated sections of the borehole are systematically tested to ascertain their hydraulic conductivity. Naturally, the fewer the number of permeable "discrete" features there are within these test intervals, the more the statistics of the test results tell us about the statistics of the features themselves. As an example, see Snow (1970) in which the frequency of tight

intervals was used to determine the average fracture frequency, and this frequency was used to infer the mean and variance of fracture transmissivities from those same statistics of the hydraulic test results. Thus, the size of the test intervals should be customized to match the typical spacing of the features within the aquifer under consideration. Unfortunately, these hydraulic tests are cumbersome and expensive to carry out, and a test interval length of 2 m is often cited as the lower end of what can be reasonably expected.

In the many cases in which the frequency of fractures is much greater than the frequency of test intervals, we are left to wonder about the distribution of permeability among the many features contained within each test interval. A natural source of information that can be brought to bear on this problem is the data collected during the drilling of the borehole - the core log. A variety of techniques have been devised to relate core-log information to fracture statistics and/or permeability (Fransson, 2002 Priest, 1970). Recently, techniques to distinguish among logged features based on their individual characteristics have been more commonly seen. Typically, the features might be ranked, based on their appearance, on their "likelihood of being permeable" (Nativ, 2003).

## 2. CONSTANT HEAD INJECTION TESTS

In this analysis we conceptualize an aquifer as a collection of quasi two-dimensional horizontal openings separated by a perfectly impermeable matrix. We assume that permeability within the plane of the openings is isotropic. In this context flow within the aquifer is obviously contained within the horizontal openings. Furthermore, the aquifer can be conceptualized as being made up of arbitrarily chosen layers whose horizontal transmissivities are related to the transmissivities of the openings contained within them. The transmissivity of a layer between elevations  $\zeta_1$  and  $\zeta_2$  (see Figure 1) is

$$T(\zeta_1, \zeta_2) = \int_{\zeta_1}^{\zeta_2} K(z) dz = \sum_i T_i \quad (1)$$

The values  $T(\zeta_1, \zeta_2)$  are commonly measured using a technique known as a constant head injection test (CHIT). In this test, a pair of inflatable packers is lowered into the borehole which, when inflated, isolate the portion of the borehole which lies between them from the remainder (see Figure 1). The upper packer is run through by a steel tube which allows access to the isolated interval for injection of water and measurement of pressure within it. Once the packers are inflated, water is injected into the interval in such a way as to maintain a constant pressure within it. The injection rate typically decreases monotonically towards a steady rate,  $Q$ . Once this steady (or quasi-steady) rate is achieved, it is noted along with the hydraulic head increase above ambient conditions,  $\Delta H$ , caused by the injection. The determination of  $T(\zeta_1, \zeta_2)$  is then determined using the well known Theim equation

$$T(\zeta_1, \zeta_2) = \left( \frac{Q}{\Delta H} \right) \frac{\ln(r_e / r_w)}{2\pi} \quad (2)$$

where  $r_e$  is the radius of influence of the hydraulic test and  $r_w$  is the radius of the well.

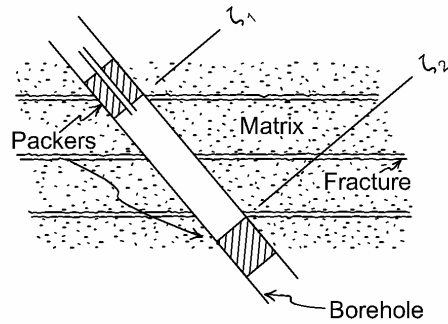


Figure 1 Downhole apparatus for a constant head injection test

## 3. REGRESSION ANALYSIS

In this study regression analysis is used to find relationships between the numbers of certain types of features as noted in core logs which lie within hydraulic test intervals and the logarithms of transmissivities as determined in these tests. In order to determine meaningful counts of features within a hydraulic test interval it is necessary to account for error in the relative locations of the openings noted in the core and the bounds of the hydraulic test intervals, the former usually being referenced to ground surface during drilling, and the latter being reference to borehole casing elevation at the time of hydraulic testing.

Error in the precise location of individual features (relative to ground surface) during core logging comes about mainly due to movement of core within the core-barrel during recovery, especially in sections of the borehole where the size of the openings or the weakness of the rock structure leads to less than 100% recovery. Error in the location of CHIT intervals (relative to the casing top) come about mainly due to cable stretch. Lastly, error in the measurement of the casing top elevation relative to ground surface comes about mainly due to fluctuation in ground surface elevation during, and as a result of, drilling. To account for these errors it seems reasonable to assume the location of the CHIT intervals to be correct relative to ground surface and the location of each feature noted in the core log to be a random variable with an assumed (in this case the Laplace) density function.

The probability of the  $k$ 'th feature lying between two points  $\zeta_1$  and  $\zeta_2$  is the integral between these bounds of the assumed density function,  $f(z)$

$$P(\zeta_1 \leq z_k \leq \zeta_2) = \int_{\zeta_1}^{\zeta_2} f(z)dz \quad (3)$$

which is easily calculable and depends only on the relative locations of the bounds of the CHIT and the feature located at  $z_k$ . Using the probabilities as defined in (3), we can introduce the term "probable number of features of type  $j$  located in interval  $(\zeta_1, \zeta_2)$ ". This is the sum of the probabilities that each of the  $n(j)$  feature of type  $j$  noted in a borehole lies within the interval  $(\zeta_1, \zeta_2)$

$$PN(j|\zeta_1 \leq z_k \leq \zeta_2) = \sum_{k=1}^{n(j)} P(\zeta_1 \leq z_k \leq \zeta_2) \quad (4)$$

In this study multiple linear and non-linear regression is used to determine relationships between the probable numbers of various sets of features falling within a hydraulic test interval, and the log of the transmissivity measured within that interval. The linear regression model hypothesized in this study is

$$y_i = \sum_{j=0}^k x_{ij} \beta_j + \varepsilon_i \quad i = 1, \dots, n \quad (5)$$

where  $n$  and  $k$  are the number of CHITs and the number of types of feature under consideration, respectively,  $y_i = \log(T_i)$  is the logarithm of the transmissivity measured in the  $i$ 'th test,  $x_{ij} = PN(j|\zeta_{i1} \leq z \leq \zeta_{i2})$  are probable numbers of features of type  $j$  located in the  $i$ 'th test interval,  $\beta_j$  are unknown constants (parameters) to be estimated and  $\varepsilon_i$  is error. The advantage of this model is its simplicity and the uniqueness of its solution. We use ordinary least squares regression to find the estimates  $b_j$  of the parameters  $\beta_j$ .

The non-linear regression model hypothesized is

$$y_i = \log\left[\sum_{j=0}^k x_{ij} 10^{\beta_j}\right] + \varepsilon_i \quad i = 1, \dots, n \quad (6)$$

where all the terms are defined as for (5). In this model, due to the congruency of (6) with (1), we can further define  $\beta_j$  as the expectation of the logarithm of the transmissivity of the  $j$ 'th type of feature

$$\beta_j \equiv E(\log T_{f_j}) \quad (7)$$

Thus, the parameter estimates  $b_j$  determined from the non-linear model have the advantage of having physical significance. The disadvantage is that iterative techniques must be used to determine estimates  $b_j$ . We use the Levenberg-Marquart procedure as implemented in PEST (Watermark Computing, 1994).

As a measure of fit between the observed values of  $\log(T)$  and the predicted values determined from either (5) or (6) we use  $R^2$  which is defined as the square of the sample

correlation coefficient between the observed and predicted values of  $\log(T)$ .

#### 4. DATA COLLECTION AND ORGANIZATION

The data used for this study were collected on a property where efforts are ongoing to mitigate the effects of spills from a former chemical waste management facility on a shallow bedrock aquifer. The site is located on the Niagara Peninsula in Smithville, Ontario, Canada. The shallow bedrock beneath the site is the Silurian aged Lockport Formation, an approximately 40 m thick, flat lying, succession of dolomites underlying less than 10 m of clayey overburden and overlying the Decew dolomite and Rochester shale. The Lockport Formation, which dips gently towards the southwest, is made up, from bottom to top, of the Gasport, Goat Island, Vinemount and Eramosa members, distinguishable through differences in colour, mineralization, and bedding thickness. The flow of groundwater in the Lockport Formation is primarily through large-aperture horizontal fractures of significant lateral extent (Zanini et al., 2000), though the abundance of fractures detected in recovered core and the lack of specific "signatures" of permeability makes quantification of flows within individual fractures difficult.

The data is derived from two separate loggings of core from five boreholes drilled at the Smithville site as well as two sets of hydraulic tests conducted in each of the same boreholes. The boreholes were drilled in the fall of 1995 and spring of 1996. The 76 mm diameter boreholes were drilled using diamond drilling techniques, and were inclined with approximately 56 degree dip. Core recovery, using triple tube techniques was greater than 95% in all boreholes. Immediately following removal of the core barrel, each core run was photographed, along with a tape showing distance from the top of the borehole, and logged.

##### 4.1 Descriptive Core Logging and Hydraulic Testing

In this logging of the core, described henceforth as the "descriptive logging", the length along the borehole of all relevant features was noted, along with a classification and a brief description. Breaks in the core were identified as either mechanical or open fractures. Open fractures were identified by the presence of infilling (generally calcite or gypsum), rough surfaces, or evidence of weathering. Based on the classification and on the description, the feature was later classified into one of approximately 120 distinct types, examples of which include "bedding plane fracture", "vug", "vuggy zone", "stylolite", etc.

Following drilling, a steel casing was installed in each borehole through the overburden and into the upper portion of the dolomite. The length of the stick-up of the casing top above ground surface was measured. A series of CHITS were performed in the boreholes with the upper and lower depths of the intervals being measured below casing top (bct).

CHITs were performed with a separation distance ( $\zeta_2 - \zeta_1$ ) of 2 m every 2 m, with no overlap, over the entire thickness of the Lockport Formation in each borehole. The total number of these 2 m tests was 116. A set of 469 CHITs was performed with a 0.5 m interval, again across the entire Lockport Formation with no overlap between tests. The magnitude of  $\log(T)$  in each of the 2 m and 0.5 m test intervals in a typical borehole are shown in Figure 2. Also shown in the figure are locations of certain types of features from both the descriptive and the numerical logging.

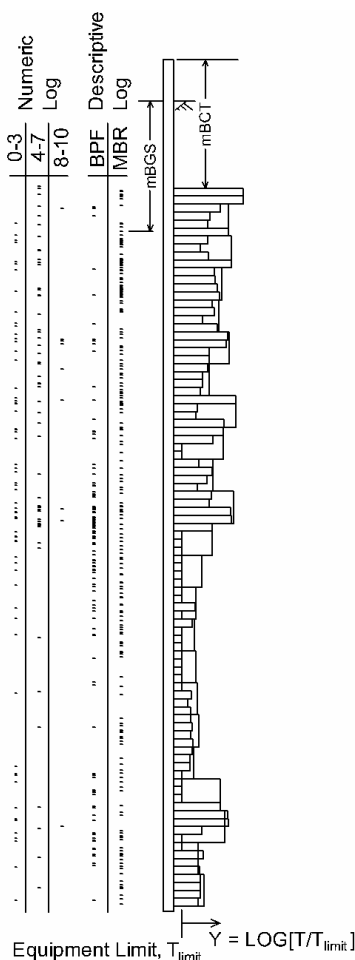


Figure 2 Typical Core Log and Hydraulic Testing Data

#### 4.2 Classification of Core Log Entries into Groups

In order to further organize the descriptive corelog data into a form suitable for regression analysis it was necessary to determine which of the 120 types of features were relevant to the analysis. To carry out this task the entries in the corelog were tallied up by type and only features that were observed on average once or more per

borehole were included in the analysis. This reduced the number of types of features to 30. These 30 features were then classified into one of eight thematic groups, as shown in Table 1.

Table 1 Make up and number of occurrences of each thematic group, sample correlation coefficient between the number of occurrences per 2 m CHIT and  $\log(T)$

Group Name	Count	$R_{xy}$	GROUP MEMBERS
Brc	99	0.25	"broken core - possibly open" "broken core – mechanically broken" "lost core"
Bpf	438	0.11	"bedding plane fracture (open/closed)" "bedding plane fracture possibly associated with drilling" "bedding plane fracture – uncertain"
Min	731	0.05	"machine break" "machine break possible bedding plane fracture" "machine break possible vertical fracture" "machine break uncertain"
Vfr	107	0.32	"vertical fracture" "vertical fracture possibly associated with drilling" "vertical machine break"
Vug	50	0.33	"vug(s)" "vuggy zone" "porous zone" "fossil/corral zone" "crinoid(s)" "fossil related feature"
Sty	239	0.17	"stylolite" "zone of more than one stylolite"
Fil	253	-0.4	"gypsum feature" "gypsum zone" "chert feature" "chert zone" "argillaceous" "clay"
Min	13	0.24	"calcareous band, bed, or infill" "galena" "mineralization"

The members of these groups were selected based on obvious similarity between members ("stylolite" and "zone of more than one stylolite") and on their correlations with  $\log(T)$ . For instance, "vugs" and "fossil/coral zone" were grouped together because they were both few in number and both positively correlated with  $\log(T)$ . Similarly chert and gypsum were grouped together because of their negative correlation with  $\log(T)$ . The sample correlation coefficients between the number of occurrences of members of each group within a given 2 m test interval and the  $\log(T)$  measured in that test are shown in Table 1

along with the total number of observations of each group. The fact that the largest magnitude of sample correlation shown in Table 1 is 0.4 indicates that none of the groups individually are strongly correlated with  $\log(T)$ .

#### 4.3 Numerical Logging

From the time of the original (descriptive) logging until the summer 2003, the core lay undisturbed in a warehouse at the Smithville site. In the summer 2003, in response to the lack of correlation between the original core log and  $\log(T)$ , the core was logged a second time. In this logging, described henceforth as the "numerical logging", the coordinates of all breaks in the core were determined from the original corelog. Rather than providing a description of each feature, each break was ranked according to its likelihood of being permeable prior to its disturbance by the drill rig. Table 2 shows the characteristics of the breaks which were evaluated in order to determine this likelihood. They are listed in approximate order of importance. Based on these criteria a number between 0 and 10 was assigned to each break in the core, with the geologist attempting to mentally balance the various characteristics and their order of importance. During this logging of the core the hydraulic test results were not available to the geologist (who in fact had relatively little prior exposure to the data).

For the purposes of regression, each break in the core was further classified into one of three groups depending on its ranking. The groups, decided upon through trial and error, judgement and on numbers of occurrences, were "fractures with rankings between 1 and 3", "between 4 and 7", and "between 8 and 10". The choice of only three groups reflects the subjectivity of the method used to determine the rankings, and the principle of parsimony which is an important guiding principle in regression analysis.

#### 5. RESULTS

An analysis of location error was carried out by determining the parameter of the laplace distribution that maximized correlation between observed and predicted  $\log(T)$  using the 0.5 m test intervals and linear regression. This analysis suggested that 63% of the time the actual location of fractures were within 0.33m of the noted location in the core log. The regression analysis described in this section was carried out using the correlation-maximizing laplace parameter. The majority of these analyses were carried out using linear regression (5) principally because of its ease of use, and because it was found to be an accurate indicator of the presence of a relationship between the dependent and independent variables. Where relationships were found, regression was carried out using the non-linear model (6).

Table 2 Characteristics of core breaks used to evaluate ranking (in approximate order of importance)

less likely $\Leftarrow$ likelihood $\Rightarrow$ more likely of being permeable	
Fresh crystalline appearance of break surfaces	Weathered rounded appearance
No calcite on break surface	Calcite on break surface
Complete closure of two surfaces possible	Surfaces cannot be closed manually
Similar break not apparent at depth in other boreholes	Similar break apparent at same depth in other boreholes
Oriented perpendicular to borehole	Oriented with bedding
Core edges which match perfectly	Core edges are rounded
No staining on surface	Staining on surfaces
Clay or mud present	No infilling
Previously classified as machine break	Previously classified as vertical or bedding plane fracture

Linear regression was carried out using (5) using both the numerical and descriptive logging and both the 2 m and 0.5 m test interval transmissivity data. For each combination (of the possible 4), the regression was carried out with data from each of the five boreholes individually ( $n \approx 23$  and 92 for the 2 m and 0.5 m test intervals, depending on borehole, respectively), and with data from all five boreholes simultaneously ( $n = 117$  and 468 for the 2 m and 0.5 m test intervals, respectively). The least squares  $R^2$  values for each of these linear regression models are shown in Table 3.

The results shown in Table 3 indicated that, on the whole, descriptive logging provided a better fit to the measured  $\log(T)$  data than did the numerical logging, while both core logs were more useful at predicting 2 m transmissivities than 0.5 m transmissivities. Higher values of  $R^2$  were achieved when boreholes were considered individually relative to when the data from all five boreholes were considered simultaneously.

Table 3  $R^2$  values determined from linear regression

	Individual Boreholes	All 5 Together
	Numerical Log	
2 m CHITs	0.36-0.61	0.24
0.5 m CHITs	0.18-0.41	0.22
	Descriptive Log	
2 m CHITs	0.47-0.66	0.38
0.5 m CHITs	0.28-0.5	0.30

Figure 4 shows a plot of observed vs. predicted 0.5 m test interval  $\log(T)$ . The plot indicates a concentration of predicted  $\log(T)$ 's about the mean of the observed data. The concentration of data indicates that  $E(\epsilon_i) i=1,n$  are not uniformly equal to zero, and the first Gauss-Markov condition has not been met by this regression model. While parameter estimates obtained from this model are unbiased, certain of their statistical properties, such as their variance, cannot be ascertained.

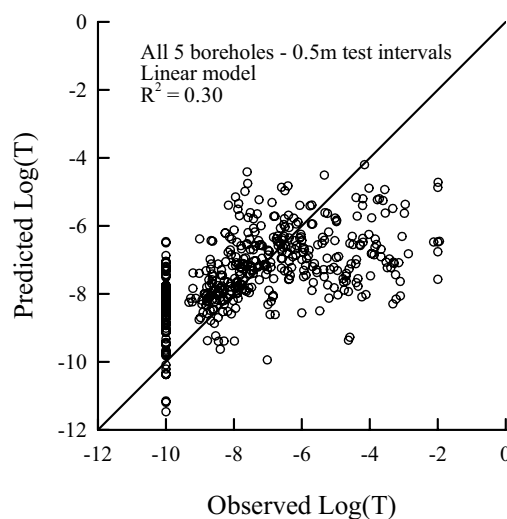


Figure 4 Observed 0.5 m test interval  $\log(T)$ 's from all five boreholes plotted against values predicted using the numerical log and the linear model

Table 4 shows  $R^2$  values and parameter estimates determined using both linear and non-linear regression between both the 0.5 m and 2 m  $\log(T)$  data and the numerical core log from Borehole 57. Table 5 shows  $R^2$  values and parameter estimates determined using both linear and non-linear regression between both the 2 m and 0.5 m  $\log(T)$  data and the descriptive core log from Borehole 57. These individual boreholes were selected for non-linear regression because of the high  $R^2$  values achieved in the linear regression analysis.

Table 4  $R^2$  values and parameter estimates from both the linear and non-linear models determined using the 2 m and 0.5 m  $\log(T)$  data and the numerical log from Borehole 57

	Parameter Estimates ( $b_0$ to $b_3$ )				
	$R^2$	Intercept	1-3	4-7	8-10
2 m Linear	0.61	-8.3	-0.43	-0.45	-0.24
Non-linear	0.65	-12	-8.6	-7.1	-4.9
0.5 m Linear	0.40	-8.9	0.15	2.1	-0.82
Non-linear	0.44	-10	-9.5	-7.7	-5.3

Inspection of Table 4 and Table 5 indicates that predictions of both 2 m and 0.5 m test interval transmissivities based on numerical logging are improved with the use of the non-linear model. In contrast, predictions of transmissivity from the descriptive logging are less likely to be accurate using the non-linear model than the linear model. The reason for this result is that only non-zero ranked breaks in the core are included in the numerical logging and the individual non-negative transmissivities of these entities are naturally additive as in (1) and (6). In contrast, features possibly associated with lower transmissivity such as infill material (see Table 1) are included in the descriptive logging. The contribution of these features to the regression model should be negative making the linear model more suitable than the non-linear model.

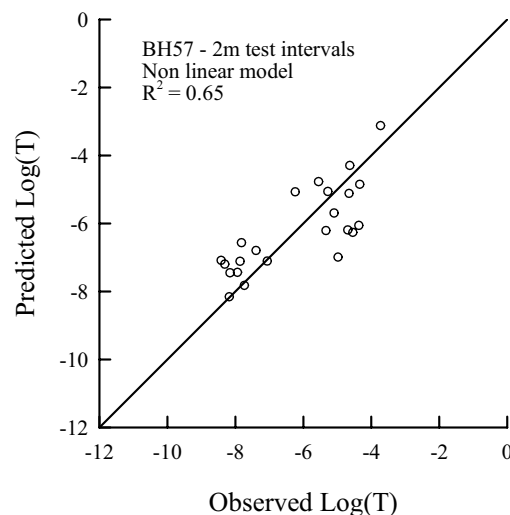


Figure 5 Observed 2.0 m test interval  $\log(T)$ 's from Borehole 57 plotted against values predicted using the numerical log and the non-linear model

Table 5  $R^2$  values and parameter estimates from both the linear and non-linear models determined using the 2 m and 0.5 m log(T) data and the descriptive core log from Borehole 57

	$R^2$	Parameter Estimates ( $b_0$ to $b_8$ )								
		Int.	brc	bpf	Mbr	vfr	vug	sty	fil	min
2 m Linear	0.66	-6.3	-0.03	0.1	-0.13	0.54	0.39	-0.04	-0.15	1.8
Non-linear	0.52	-10	-9.8	-8.3	-20	-6.3	-4.9	-8.0	-11	-6.2
0.5 m Linear	0.40	-8.6	-2.0	0.07	-0.1	1.8	-0.87	-1.0	-0.49	-2.0
Non-linear	0.36	-20	-20	-9.1	-17	-6.9	-5.4	-8.1	-16	-6.1

Figure 5 shows a plot of observed vs. predicted 2 m test interval log(T)'s. In contrast to Figure 4, the  $\varepsilon_i$ 's appear to have zero mean and to be uncorrelated. The variances of the parameter estimates from this model may be ascertained.

Examination of the non-linear parameter estimates provides some insight into the reliability of the numerical vs. the descriptive logging. In the case of the numerical logging Table 4 indicates that the transmissivity of the aquifer in absence of any macro-porosity (i.e. matrix transmissivity), as estimated by the model intercept, is estimated as  $1 \times 10^{-12}$  and  $1 \times 10^{-10}$  m<sup>2</sup>/s from the 2 m and 0.5 m test interval models respectively. The respective mean transmissivity estimates for the 1-3 ranked fractures are  $3 \times 10^{-10}$  and  $3 \times 10^{-9}$  m<sup>2</sup>/s, for the 4-7 ranked fractures are  $8 \times 10^{-8}$  and  $2 \times 10^{-8}$  m<sup>2</sup>/s, and for the 8-10 ranked fractures are  $1 \times 10^{-5}$  and  $5 \times 10^{-6}$  m<sup>2</sup>/s. The consistency of these estimates between models, and the trend of increasing transmissivity with increasing rank reflects well on the accuracy of the estimates and on the numerical logging methodology in general. In the case of the descriptive logging, reasonable consistency is observed in non-linear parameter estimates derived from the 2 m vs. the 0.5 m test data, but the parameter estimates are more difficult to reconcile with our understanding of what they represent. For instance in both the 2 m and 0.5 m test interval models, the estimates of bedding plane mean transmissivity are less than  $5 \times 10^{-9}$  m<sup>2</sup>/s – virtually impermeable by the standards of the aquifer. In fact, the only features with appreciable transmissivity are estimated to be vertical fractures, vugs, and mineralization. With the exception of mineralization, none of these features appeared during the informed logging of the core to be permeable.

Examination of the linear model parameter estimates determined from the descriptive logging further indicates ambiguity in the relationships between the presence of features in the core and enhanced transmissivity. In this model the sign of the parameter indicates the sign of the correlation between the feature and log(T), while the magnitude of the parameter indicates the strength of this correlation. The fact that the magnitude of parameter estimate for broken core (brc) is the smallest of those determined from the 2 m data and is the largest of those determined from the 0.5

m data indicates the above-noted ambiguity. Similarly, the large magnitude but opposite sign of the two estimates of the mineralization parameter points to ambiguity.

In light of the ambiguities in parameter estimates associated with the descriptive logging, it is worth considering why the  $R^2$  values are similar, or higher, than those from the numerical logging. The likely reasons are the greater number of parameters in the former model relative to the latter, and the greater number of features included in the descriptive log (breaks in the core plus lithological features) relative to the numerical log (breaks in the core only).

## 6. CONCLUSIONS

Regression analysis was carried out with the independent variables being the probable numbers of classes of feature present within hydraulic test intervals, and the dependent variable being log(T)'s measured in these tests. The probable numbers of features within the test intervals were determined by considering the locations noted in the core log to be Laplace distributed random variables, whose parameter was determined as part of the study. Regression was carried out using both a linear and non-linear model, with the advantage of the former being its simplicity and the uniqueness of its solution, while the advantage of the latter being that its parameters are the mean log(T)'s of each class of feature included in the model. Estimates of individual fracture transmissivities are useful in predicting the movement of solute within fractured bedrock aquifers.

Classes of feature were made up from two separate loggings of the core from five boreholes diamond-drilled in Smithville, Ontario. The first logging of the core was carried out during drilling and was descriptive in nature. Every relevant feature in the core was classified according to its geological characteristics and estimated provenance. The second logging of the core, carried out in response to the lack of correlation between the descriptive logging and the transmissivity, was carried out some eight years later. It was numerical in nature, with each break in the core being ranked according to its likelihood of being permeable.

It was found that accounting for the location error of feature noted in the core log provided some improvement in correlation between the core log and  $\log(T)$ , with the mean error being approximately 0.3 m. It was found that classes of feature derived from the descriptive logging were best related to  $\log(T)$  using the linear model due to its ability to account for both positively and negatively correlated independent variables. In contrast, classes of feature derived from the numerical logging, all of which were theoretically associated with some positive value of transmissivity, were best related to  $\log(T)$  through the non-linear model. The reason for this is that this model reflects the additivity of transmissivities in a horizontally-bedded aquifer.

In general, fits between the observed and predicted  $\log(T)$ 's were poor, and the statistics of parameter estimates could not be ascertained. However, the estimates of mean  $\log(T)$ 's for classes of numerically logged fractures were consistent whether the regression was carried out using individual boreholes or the five boreholes simultaneously. In one particular borehole, an  $R^2$  of 0.65 provided some confidence that the

statistics of parameter estimates might be determinable. The reason for the overall difficulty in determination of parameters is attributed to the large variance in  $\log(T)$  of the overall population of fractures intersected by the boreholes at Smithville.

## 7. REFERENCES

- Fransson A., 2002. Nonparametric method for transmissivity distributions along boreholes. *Groundwater*, 40, 201-204.
- Nativ R, Adar L, Assaf L, and Nygaard E., 2003. Characterization of hydraulic properties of fractures in chalk. *Groundwater*, 41, 532-543.
- Priest S.D. and Hudson J.A., 1970. Discontinuity spacings in rock. *Int J Rock Mech Min Sci*, 13, 135-148.
- Snow D.T., 1970. The frequency and apertures of fractures in rock. *Int J Rock Mech Min Sci*, 7, 23-40.
- Watermark Computing, 1994. PEST, Model Parameter Estimation, Australia.