APPLICATION OF CENTRIFUGE FOR UNSATURATED SOIL PARAMETER DETERMINATION

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ABSTRACT
Experimental methods to simulate unsaturated flow in a geotechnical centrifuge have been attempted but the validity of these experimental methods is unclear. One-dimensional drainage tests were performed at the Idaho National Engineering and Environmental Laboratory to investigate the unsaturated flow process under centrifugal acceleration. Cumulative outflow and capillary pressure at several soil depths were monitored during the drainage process and were used as input data to determine unsaturated soil parameters by an inverse simulation using HYDRUS-1D. Cumulative outflow measurements showed reasonably good agreement when scaled by centrifugal acceleration although a slight dependency of drainage by the magnitude of centrifugal acceleration was identified. van Genuchten parameters determined from the inverse analysis showed consistent results with some degree of scatter. The quality of the soil parameter estimation depends on accuracy of the experimental observation. When analyzed correctly, the increased acceleration by the centrifuge method does not appear to adversely affect parameter estimation.

RÉSUMÉ
Les problèmes Geo – environnementaux ont été étudiés en centrifugeuse géotechnique . La validité de ces essais demeure incertaine. Dans cette étude, des essais unidimensionnels de drainage ont été effectués afin d'examiner le procédé d'écoulement insaturé sous un champ d'accélération centrifuge. L'écoulement cumulé et le changement de la pression capillaire ont été mesurés et enregistrés à plusieurs profondeurs de sol pendant le processus de drainage.
Les résultats de mesure enregistrés pendant l'essai ont été employés comme données d'entrée d'une simulation inverse, ceci dans le but de déterminer les paramètres du sol non saturé. La variation temporelle de l'écoulement cumulé à l'échelle prototype, montre une assez bonne conformité pour tous les tests en centrifugeuse. On considère que l'instabilité dans l'évaluation des paramètres dépend à la fois de la technique d'analyse inverse et de la précision des observations faites à partir des essais en centrifugeuse.

1. INTRODUCTION

Mathematical analysis of unsaturated flow and soil mechanics for a porous medium requires knowledge of unsaturated hydraulic properties, i.e., relationship between suction (h), volumetric water content (θ), and conductivity (K). The inverse analysis technique is one experimental method that has been used to determine soil hydraulic parameters (e.g., Zachmann et al. 1982, Kool et al. 1985, Parker et al. 1985, Eching and Hopmans 1993). Under the premise that a flow equation and hydraulic functions of θ(h) and K(h) used in a numerical model properly express the actual water flow behavior in the soil, the principle of the inverse method is relatively straightforward; perform unsaturated flow observations under defined initial and boundary conditions, obtain auxiliary measurements such as outflow, water content, and/or capillary pressure, and use the measurements as input data to optimize the hydraulic parameters. The inverse analysis attempts to minimize sums of squared deviations between the test measurements and solutions derived from the numerical model. The flow equation is repeatedly solved using subsequently improved parameters until the sum of squared deviations becomes less than user-defined threshold values. The inverse method is attractive because it has flexibility in initial and boundary conditions and can yield results quickly under a transient condition with a wide range of water content. However, it is also recognized that the inverse method is often ill posed and may not yield a unique solution.

To minimize ill-posedness and non-uniqueness problems for a one-step drainage process in which pore water is vertically drained from the lower boundary of an initially saturated homogeneous soil, Kool et al. (1985) determined that the following conditions should be met: (i) capillary pressure applied on the sample is large enough to yield a low final reduced water content, (ii) outflow measurements are extended over an adequate period of time, (iii) initial parameter estimates are reasonably close to their true values and (iv) experimental error is relatively small. Parker et al. (1985) verified the conclusions of Kool et al. (1985) with results from one-step tests using four different soils types. Toorman et al. (1992) concluded that including capillary pressure data in the objective function in addition to the outflow data could minimize the uniqueness problems of the inverse solution.

Better parameter estimation can be achieved by including the capillary pressure data in inverse data set (e.g., Eching and Hopmans (1993), Eching et al. (1994)). Eching and Hopmans (1993) conducted both one-step and multi-step outflow experiments with four different textured soils. The authors found that with inclusion of soil water pressure head data in the optimization, the optimized curves obtained from both one-step and multi-step experiments agreed well with the independently measured θ(h). The authors also found that optimization with both measured outflow and capillary pressure head data resulted in almost identical final parameter estimates for each method. Based on a comparison of one-step and multi-step methods performed by Eching and Hopmans (1993), the one-step method is more attractive because multi-step experiments take twice as long to perform if the capillary pressure data are included.

In the field of geotechnical engineering, geotechnical centrifuges have been used extensively to study soil mechanics problems. More recently, the geotechnical centrifuge has been used to study multi-phase (air-water or air-water-oil) flow problems (e.g., Cooke and Mitchell 1991, Culligan et al. 1997, Esposito et al. 1999). Advantages of the geotechnical centrifuge include its ability to mimic variably saturated pore fluid movement in reduced size and time under well-controlled initial and boundary conditions. However, to date, the validity of scaling similitude for these applications is still not clearly defined.

Recently, researchers have investigated scaling relationships to evaluate if multi-phase flow is modeled properly using established scaling laws (Garnier et al. 2000). Bunkhart et al. (2000), Crançon et al. (2000), and Rezzoug et al. (2004) investigated the validity of scaling laws for centrifuge modeling of capillary rise. The authors monitored the rise of a sharp wet-dry boundary during the wetting process and analyzed the temporal change and final height of the boundary. The results showed that the scaled height of the wet-dry boundary was reasonably accurate. This indicates that centrifuge modeling can scale forces acting on pore water when the governing control length is macroscopic. The visible wet-dry boundary typically appears close to the capillary fringe where pore space is nearly saturated by capillary force. Under a high degree of saturation, a majority of pore water forms a continuous phase; thus, forces controlling pore water movement is both capillary and gravitational in macroscopic length scale. Culligan and Barry (1996) pointed out that centrifuge modeling might not scale pore fluid movement in multiphase conditions properly when the governing control length is microscopic (e.g., pore scale). In their study, the governing control length on pore fluid movement changes depending on the pore liquid conditions. When pore liquids exist in a discontinuous condition as ganglia, blobs or at pendular saturation, governing controlling length is microscopic. For these conditions, similitude for relative magnitude of viscous force and body force to capillary force acting on pore water is violated. The authors thus concluded that it is impossible for centrifuge modeling to exactly simulate multiphase flow processes since both macroscopic and microscopic length scales are involved in the processes. For drainage of an air-water system, as outflow progresses, pore water may change from a continuous to

Table 1. Scaling relationship of a centrifuge model with a scale factor N (Arulanandan et al. 1988).

Parameter		Prototype/model ratio
Gravity,	g	1/N
Macroscopic length,	l	N
Microscopic length,	l_p	1
Pore fluid velocity,	v	1/N
Time,	t	N^2
Fluid pressure,	p	1
Hydraulic conductivity,	K	1/N
Intrinsic permeability,	k	1
Soil porosity,	n	1
Fluid density,	ρ	1
Fluid viscosity,	μ	1
Fluid interfacial tension,	σ	1

discontinuous phase. As this occurs, the governing length scale changes from entirely macroscopic to a combination of macroscopic and microscopic. Under these conditions, scaling similitude is not completely satisfied. However, the magnitude of deficiency of the scaling law is not clear, especially for the purpose of soil parameter determination. Therefore, application of the scaling law may still be reasonable in a practical manner.

The work by Cooke (1994) gave insight into the usefulness of the centrifuge approach for soil parameter determination. Cooke (1994) conducted centrifuge one-step drainage tests to determine the hydraulic properties of two types of sands. Pore water in the initially saturated sand in a column was allowed to drain from the base of the column by operating a valve. The cumulative outflow was continuously monitored using a pressure transducer installed in a drainage tank. An inverse analysis was performed for three scenarios, i.e., (1) estimate three parameters used in constitutive models to describe the θ(h) and K(h) relationships; (2) determine only one parameter for the K(h) relationship using known parameters for the θ(h) relationship determined from pressure plate tests; and (3) determine one parameter for the K(h) relationship but using θ(h) data from sectioned samples that were taken after the centrifuge test. Good agreement with parameters from standard tests was obtained when only one parameter was estimated; whereas, the three parameters estimation showed somewhat poorer agreement.

In this study, the unsaturated soil parameters were determined from a series of centrifuge drainage tests. The cumulative outflow and capillary pressure were monitored during the tests and used as auxiliary variables for the one-step method. The centrifuge tests were treated as scaled models. Based on the scaling relationship as shown in Table 1, a prototype of the centrifuge scaled model represents a 2.54m high uniformly packed sand column. Parameters calculated from the series of centrifuge tests are compared to assess the suitability of the scaling law for the one-step method.

2. PARAMETER DETERMINATION USING INVERSE ANALYSIS

One-dimensional unsaturated flow may be mathematically expressed using Richards equation with vertical distance x taken as positive downward:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(K(h)\frac{\partial h}{\partial x} \right) - \frac{\partial K(h)}{\partial x} \qquad [1]$$

where t = time. For the gravity driven drainage process, the initial and boundary conditions are:

$$h = h_0(x) \qquad t = 0, \qquad 0 \le x \le L \qquad [2a]$$

$$\frac{\partial h}{\partial x} = 1 \qquad t > 0, \qquad x = 0 \qquad [2b]$$

$$h = h_L \qquad t > 0, \qquad x = L \qquad [2c]$$

where x = L is the bottom of the porous plate and h_L = pressure head at the bottom of the porous plate. Solving Eq.1 with Eq.2a-c requires constitutive relations describing the unsaturated soil properties. The soil hydraulic properties are assumed to be of the form described by the van Genuchten model (van Genuchten, 1980):

$$\overline{S} = \begin{cases} \dfrac{1}{\left[1 + (-\alpha h)^n\right]^{1-1/n}} & h < 0 \\ 1 & h \ge 0 \end{cases} \qquad [3]$$

$$K = K_s \overline{S}^{1/2}\left[1 - \left(1 - \overline{S}^{n/(n-1)}\right)^{1-1/n}\right]^2 \qquad [4]$$

where

$$\overline{S} = \frac{\theta - \theta_r}{\theta_S - \theta_r} \qquad [5]$$

is an effective saturation, θ_r and θ_s are the residual and saturated water content, respectively, K_S is the saturated hydraulic conductivity, and n and α are van Genuchten's fitting parameters. The objective function E(b) to be minimized by the inverse method is

$$E(b) = \sum_{i=1}^{I}\left\{ w_i\left[Q(t_i) - \hat{Q}(t_i, b) \right]\right\}^2 + \sum_{j=1}^{J}\left\{ v_j\left[h(t_j) - \hat{h}(t_j, b) \right]\right\}^2 \qquad [6]$$

where b is a vector containing the parameters to be optimized, i.e., θ_r, α, n, and K_s. $Q(t_i)$ is observed cumulative outflow per unit area at a specific time, t_i. $\hat{Q}(t_i,b)$ and $\hat{h}(t_j,b)$ are numerically calculated values of the cumulative outflow and the pressure head respectively. i and j are the number of the observations of the cumulative outflow and the pressure head. w and v are weighting factors. In this study, w and v were set to 1. The optimization was implemented using HYDRUS-1D (Simunek et al., 1998).

3. EXPERIMENTAL METHODS AND PROCEDURES

The test system consists of a cylindrical sample container, 10.2 cm in diameter and 43.2 cm high, and an outflow collector, Figure 1. A 1.0 cm thick porous plate with filter paper (Whatman #1) on the top face was installed directly above the reservoir to allow free drainage and to prevent migration of soil particles into the reservoir. Miniature tensiometers were used to measure capillary pressure at several depths. For all the experiments, fine Ottawa sand (US Silica, F110), having a mean particle diameter (D_{50}) of 0.01 cm and uniformity coefficient (C_u) of 1.6, was used as the porous medium. After the tensiometers were installed to ports on the test container wall, dry sand was uniformly packed by pluviation. Bulk dry density was occasionally measured during pluviation to ensure uniformity of the sample. After the soil sample was packed to a specified height, the container was placed in a vacuum chamber. A vacuum was applied to the sample for approximately 3 hours and de-aired water was then slowly introduced to the sample from a port located below

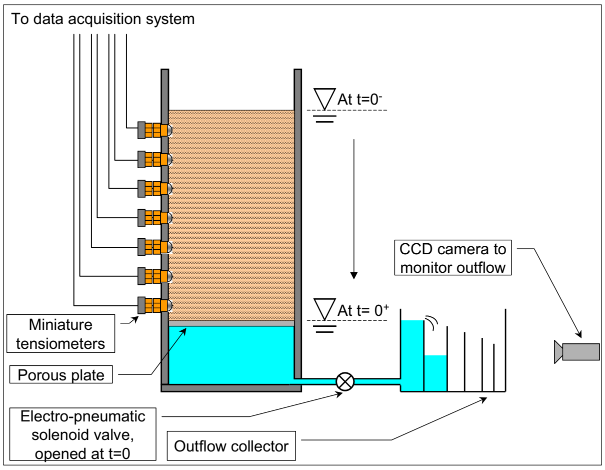

Figure 1 Centrifuge test setup

Table 2. Centrifuge test condition

Test Code	Sample height H [cm]	Applied gravity [N-g]
OS1	25.4	10
OS2	12.7	20
OS3	6.4	40
OS4	25.4	10
OS5	12.7	20
OS6	6.4	40

the soil sample. Once the water level rose above the soil surface, the vacuum pressure was released. After excess water above the soil surface was removed, the test specimen was weighted to determine the degree of saturation. The soil filled container was then placed on the centrifuge platform and the reservoir port was connected to the outflow collector. The test container and the outflow collector were connected through plastic tubing and an electro-pneumatic valve. A valve with a relatively large orifice opening (flow coefficient = 6) was chosen to minimize energy loss when water passed through the valve. The outflow collector consists of 5 sections connected in series, with a reduction in cross-sectional area of each successive section for the purpose of increasing resolution of the outflow measurements.

An outflow collection system was designed to ensure sufficient flux resolution through out the experiment. To initiate an experiment, the valve was opened to begin drainage and the cumulative outflow discharged from the bottom of the sample was collected in the outflow collector. A constant lower boundary condition was set by allowing water to overflow from the first section of the outflow collector. As the water reached the top of each section, water would overflow into the successive section. As drainage progressed, enough rise of the water level in smaller sections could be visualized from a video camera placed in front of the collector to accurately quantify the flux. From captured images of the outflow collector, temporal change of the water level was obtained. Drainage was monitored for approximately 2 hours for each test. After the centrifuge was stopped, the sample height was measured to see if significant settlement occurred. Measured settlements were 0.1 cm or less for all tests.

Six tests were performed. The test conditions are listed in Table 2. Porosity of the test samples was approximately 0.37, ranging between 0.362 and 0.377, and saturation was more than 97% for all the tests.

4. RESULTS AND DISCUSSION

Figure 2 shows the cumulative outflow vs. time from the experiments. A water leak was found in the outflow collector during tests OS1 and OS6 that prevented collection of complete outflow data for these two tests. All the curves are smooth as would be expected for continuous outflow measurements using the outflow collector and visualization measurement technique. In general, good agreement is seen from each pair of the tests conducted under the same acceleration field, confirming test repeatability. Because centrifugal acceleration enhances the gravity-driven drainage process, the fastest outflow was observed for tests OS3 and OS6 at 40g. The outflow for OS6 approached the equilibrium condition while the outflow was ongoing at the end of measurement for OS2 and OS5 conducted at 20g and more significantly for OS1 and OS4 conducted at 10g.

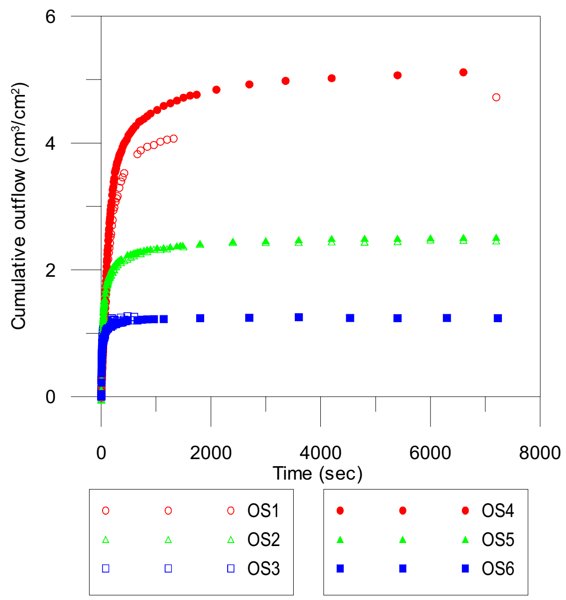

Figure 2. Relationship between cumulative outflow and time

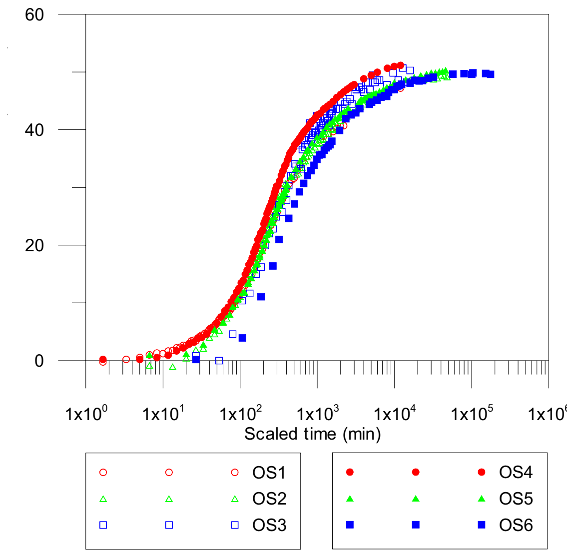

Figure 3 Relationship between cumulative outflow and time in prototype scale

Figure 3 shows the cumulative outflow data plotted using the scaling relationships shown in Table 1. In general, all six tests showed good agreement until outflow reached approximately 60% of the final drainage (30 cm^3/cm^2 in the cumulative outflow). The final scaled cumulative outflow was also fairly consistent ranging from 47.2 cm^3/cm^2 to 51.1 cm^3/cm^2. However, it was noticed that the outflow rate becomes slightly slower at higher gravity. In addition to the experimental accuracy, flow rate dependency of the unsaturated flow process and the non-scalability of capillary force in centrifuge models may need to be considered to interpret the test results. Although the unsaturated hydraulic characteristics are commonly assumed to be identical for steady or transient conditions, a number of researchers have suggested that this assumption is not always justifiable (e.g., Topp et al. 1967, Smiles et al. 1971, Vachaud et al. 1972). Wildenschild et al. (2001) conducted one-step and multi-step outflow experiments as well as quasi-static experiments on identical disturbed samples to evaluate the influence of flow rate on the calculated unsaturated hydraulic parameters. The authors found that soil water retention for Lincoln sand, which has relatively uniform pore size distribution, increases as the number of pressure steps decreases, with the largest retention and residual water content for the single step experiment, and the lowest retention and residual water content for the quasi-static syringe pump and low-pressure multi-step outflow experiment. In contrast, no apparent rate dependency was observed from tests for fine textured sandy loam. Among five factors which Wildenschild et al. (2001) suggested to explain the rate dependency, entrapment of water may be plausibly applied to the centrifuge test observation as well. Water entrapment is thought to occur through hydraulic isolation of water-filled pores by draining surrounding pores. The larger the drainage rate, the less opportunity exists for all pores to drain concurrently leading to an increased water retention value. This agrees with the results from the centrifuge tests since drainage rate is larger at higher centrifugal acceleration.

While the entrapped water will reduce the drainage volume, violation of similitude between the forces acting on the pore water could enhance the drainage process and outflow volume with respect to prototype scale. Until water content decreases and the pore water is in a discontinuous phase at pore scale during the drainage process, pore water is continuous and thus the control length scale is macroscopic, i.e., relative elevation from the fixed water table in the outflow collector. Violation of the similitude may be seen at later stages of the drainage process, as more pore water exists in a discontinuous phase. However, the final cumulative outflow results in comparable values; hence, it is considered that the violation of the scaling similitude was not significant for the centrifuge test configuration designed in this study.

The parameters optimized by the inverse analysis module of HYDRUS-1D are listed in Table 3. Reference values in Table 3 were obtained from hanging-column tests and constant head permeability tests performed for soil samples having the same porosity as the centrifuge tests. As input data for the inverse analysis, the cumulative outflows with approximately every 5 cm^3/cm^2 increment in prototype scale and the final pressure measurements were used. Figure 4 shows retention curves using the estimated parameters. Although the retention curves are not identical, they show reasonable agreement except for OS5. An air-entry value, which is defined as the pressure head when pore water can begin to drain from the soil, was approximately 60 cm except for test OS5. This can also be recognized from similar values in the optimized α values. The saturated hydraulic conductivity from OS6 is approximately 40% less than the reference value. However, hydraulic conductivity from other tests show good agreement, and the value from OS6 has the same order of magnitude. The residual water content ranged between 0.050 and 0.081. For the estimated parameters, distinctive gravity tendency was not identified, rather the values seems to be scattered having similar values to the reference values. It is considered that the gravity dependency seen in the cumulative outflow data was not sensitive as much as accuracy of the inverse technique with input data having certain degree of experimental errors.

Table 3. Optimized parameters using cumulative outflow and final pressure distribution

	α [1/cm]	N	θ_r [m^3/m^3]	K_s [cm/s]
Reference	0.0107	13.0	0.06	3.22×10^{-3}
OS1	0.0115	9.62	0.081	3.19×10^{-3}
OS2	0.0121	8.50	0.081	3.20×10^{-3}
OS3	0.0117	9.42	0.059	2.98×10^{-3}
OS4	0.0114	9.53	0.050	3.89×10^{-3}
OS5	0.0154	5.26	0.086	2.87×10^{-3}
OS6	0.0126	6.27	0.071	1.89×10^{-3}

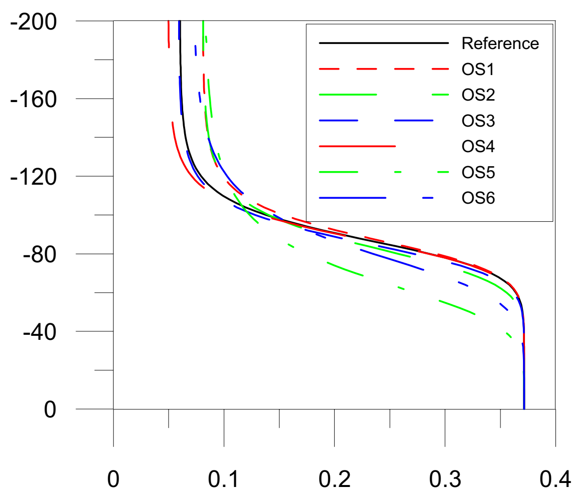

Figure 4. Retention curves from optimized parameters

5. CONCLUSIONS

In this study, one-step drainage tests were performed at different centrifugal accelerations to investigate the suitability of the centrifuge technique for unsaturated soil parameter determination. Each of the centrifuge tests was treated as a scaled model simulating one-dimensional gravity drainage through a 2.54 m high uniform sand. The test results show good repeatability and reasonable agreement. It was noticed that there was slight flow rate dependency in the cumulative outflow. One explanation for this may be entrapped water. However, within the limited number of tests conducted in this study, it is difficult to conclude that the water entrapment certainly occurred. Pore water likely moves as a continuous phase during most of the drainage process, hence the dominant governing length scale is macroscopic. Therefore, it is considered that the violation of scaling similitude did not play a significant role during the tests conducted in this study. Parameters optimized from the inverse analysis using HYDRUS-1D show good consistency. Since the flow rate dependency observed in the cumulative outflow data was not exhibited in the optimized parameters, scatter of the optimized parameters is thought to be mainly due to precision limitation of the inverse analysis technique and experimental accuracies.

Results from this data set indicates that the one-step drainage process can be scaled using published scaling relationships and that the experimental technique developed in this study was successfully applied to calculate unsaturated soil parameters. To use the technique with more confidence, further study is necessary especially for conditions where soil is compressible.

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