

A CLOSER LOOK AT THE 'BRAZILIAN' TEST AND ITS MODE OF FAILURE

Arvid Landva, GEMTEC Limited, Fredericton, New Brunswick, Canada

ABSTRACT

Timoshenko (1934) showed that a compressive line load applied perpendicularly to the axis of a solid cylinder and in a diametral plane generates a tensile stress over that plane. Carneiro (1947) applied Timoshenko's theory to a compression test on a solid cylinder of concrete (length L and diameter D) loaded in a compression testing device with the cylinder axis parallel to the load plates. It was tacitly assumed that the tensile stress σ_h set up by the applied vertical load P would be constant and equal to $2P/\pi LD$. However, the writer has concluded that the Brazilian mode of failure is one of shear rather than tension and that the failure is progressive. It is also concluded that the tensile stress calculated from $2P/\pi LD$ is considerably lower than the tensile strength of the material.

RÉSUMÉ

Timoshenko (1934) a démontré qu'une charge de compression en ligne qui est appuyé perpendiculaire à l'axe d'un cylindre solide et dans un plan diamétral engend une contrainte de traction sur ce plan.

Carneiro (1947) a utilisé la théorie de Timoshenko pour un essai de compression sur un cylindre solide de béton (longueur « L », diamètre « D ») qui a été mis dans un dispositif de contrôle de compression avec l'axe du cylindre parallèle aux plaques de charges. C'était pris à charge que la contrainte de traction σ_h créée par la charge verticale appuyé « P » serait constante et égale à $2P/\pi LD$. Par contre, l'auteur a conclu que la mode de rupture brésilienne est par cisaillement et non par traction et que cette rupture est progressive. Il a aussi conclu que la contrainte de traction calculé par $2P/\pi LD$ est considérablement moins que la résistance de traction du matériel.

1. INTRODUCTION

Timoshenko (1934) showed that a compressive (positive) load applied perpendicularly to the axis of a solid cylinder and in a diametral plane generates a tensile (negative) stress over that plane. A simplified treatment of this problem was later presented by Frocht (1948) and by Wright (1955).

Timoshenko and Goodier (1951) recognized that, at the point of application of the (line) load, the stress is infinitely large because a finite force is acting on an infinitely small area. Plastic flow will therefore occur immediately below the loaded area. However, they consider the local area of plastic flow to be infinitely small and stated that the equations of elasticity could still be applied to the cylinder.

The Timoshenko findings were applied by Carneiro (1947) to a test referred to as the 'Brazilian' test, in which a solid cylinder of concrete was loaded vertically in a horizontal position in a compression testing frame, i.e. it was loaded perpendicularly to its axis and in a diametral plane by positioning it with its axis parallel to the load plates. In this mode the vertical stress component (compressive) becomes (Timoshenko 1931, Frocht 1948)

$$\sigma_v = \frac{2P}{\pi LD} \left(\frac{D}{r} + \frac{D}{D-r} - 1 \right) \quad [1]$$

and the horizontal stress component (tensile)

$$\sigma_h = -\frac{2P}{\pi LD} \quad [2]$$

where

P is the applied (total) load,
 D is the diameter of the cylinder,
 L is the length of the cylinder, and
 r is the depth from the loaded (i.e. top and bottom) surfaces to the point considered.

Equation 2 is conventionally used to calculate the "splitting tensile strength" as proposed by Carneiro (1947) and as specified in ASTM D 3967 (for rock core specimens), ASTM C 496 (for concrete specimens) and CSA A23.2-13C (for concrete specimens). Equation 2 is also generally quoted in textbooks on concrete, e.g. Neville (1995) and on rock mechanics, e.g. Goodman (1989). In the latter reference it is pointed out that "the actual cause of failure may also reflect the action of the vertical stress along the vertical diameter in concert with the horizontal tension . . .".

2. PRACTICAL ANALYSIS

Whereas the theory is based on a line load acting along two opposite generators of the cylinder, the load is actually distributed over a finite width. This width is a function of the strength of the material if no loading

strips are used. If loading strips are used, the actual width of the loaded areas is a function of the applied load and the compressibility of the loading strips.

If the width of the loaded areas is a and the loads are assumed to be uniformly distributed over this width, Wright (1955) has shown that if $a < 0.1 \cdot D$, the stresses on the vertical diameter approximate the values below with sufficient accuracy:

$$\text{Vertically, } \sigma_v = \frac{2P}{\pi LD} \left[\frac{D}{2a} (\alpha + \sin \alpha) + \frac{D}{D-r} - 1 \right] \quad [3]$$

$$\text{Horizontally, } \sigma_h = \frac{2P}{\pi LD} \left[1 - \frac{D}{2a} (\alpha - \sin \alpha) \right] \quad [4]$$

where a is the width of the loaded areas and α is the angle subtended by the loaded areas at depths r below the applied loads.

The stress distribution given by Eqs. 3 and 4 is shown in Fig. 1, which is calculated for $a/D = 1/20$. It is seen that the tensile stress is practically constant within the inner three-quarters of the vertical plane through the cylinder, i.e. between $r \approx 0.13 \cdot D$ and $r \approx 0.50 \cdot D$. It then gradually decreases to zero at $r \approx 0.06 \cdot D$ from the loaded surfaces. At this level the lateral stress becomes compressive, and the compressive stress increases very rapidly towards the top and bottom surfaces.

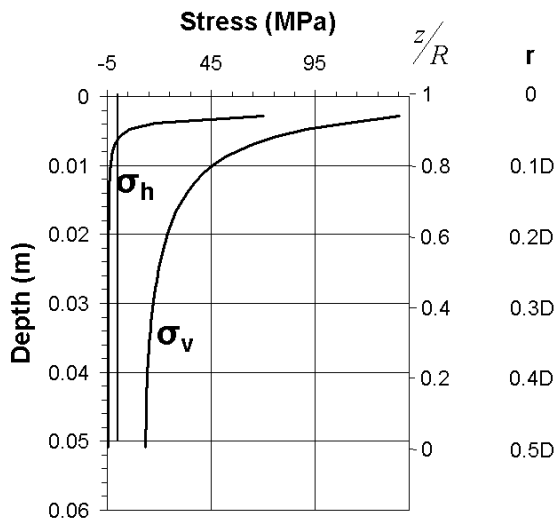


Figure 1. Variation of vertical and horizontal stresses with depth below loaded generator of cylinder (Wright 1955)

On investigating the effects of dimensions and nature of the packing strips through which the load is conventionally applied to the Brazilian test specimens, Wright (1955) concluded that these effects are relatively small and are due partly, if not entirely, to the effectiveness with which the strips are (i) able to conform to the irregularities of the surface of the

specimens and (ii) able to distribute the load uniformly. It is possible to estimate the contact area between the packing strip and the concrete and between the steel load plate and the concrete. In the case of a plywood packing strip having a compression modulus of 10,000 MPa, the width of the contact area was determined to be about 5 mm. In the case of steel on concrete, the bearing capacity of a high-strength (61 MPa) concrete was determined to be (Fig. 2) 493 MPa, which would correspond to a contact width steel/concrete of about 1.5 mm.

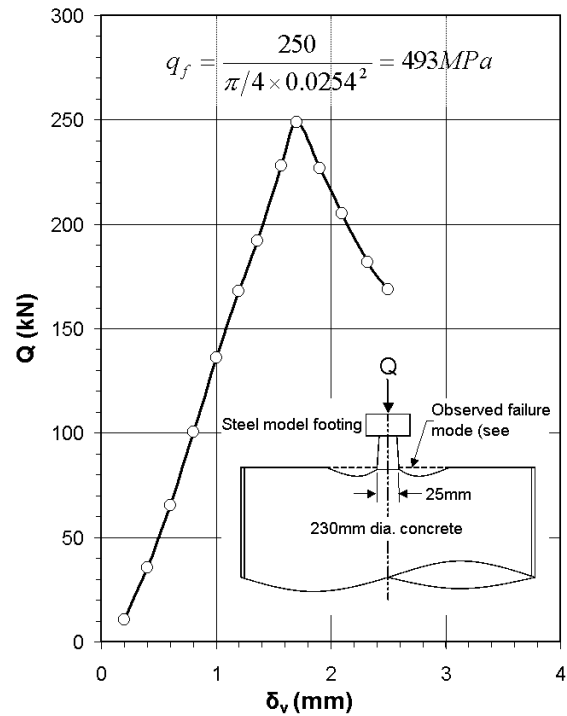


Figure 2. Model footing test on high-strength (61 MPa) concrete

A comparison between Brazilian test specimens brought to failure with and without packing strips showed that there were no significant differences between the appearances of specimens tested with and without packing strips. Concrete specimens brought to failure in unconfined compression gave friction parameters (determined from the relationship $\alpha = 45^\circ + \phi/2$) of $\phi = 56^\circ$ for $\alpha = 73^\circ$ and $\phi = 32^\circ$ for $\alpha = 61^\circ$. Comparisons between Brazilian and unconfined compression test (UCT) failure surfaces showed that there were no significant differences between the appearances of the Brazilian and the UCT failure surfaces.

3. MODES OF FAILURE

Trollope (1968) distinguishes between elastic stresses and effective stresses (Fig. 3) claiming that the effective tensile stress (σ_x) is considerably greater than σ_h , the stress normally assumed to be the lateral stress (Eqs. 2 and 4). For a value of Poisson's ratio of $\mu =$

0.25 he shows that σ_x is approximately twice the value of σ_h . He also states that tests carried out on concrete with restrained loading had given clear evidence of initial cracking occurring in the regions of maximum effective tensile strength.

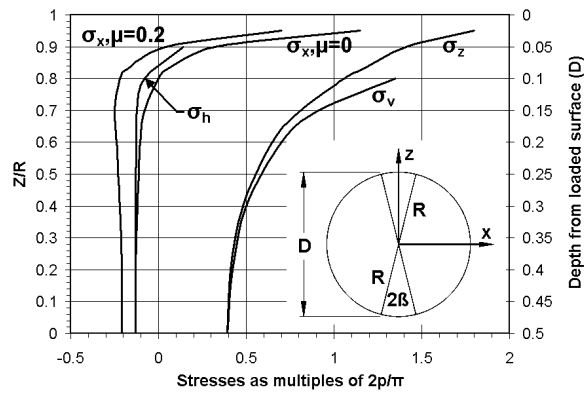


Figure 3. Variation of vertical and horizontal stresses with depth below loaded generator of cylinder (Trollope 1968)

The horizontal-stress curves in Fig. 3 marked $\sigma_x, \mu = 0$ and $\sigma_x, \mu = 0.20$ and the vertical-stress curve marked σ_z are those originally presented by Trollope (1968). The

horizontal-stress curve marked σ_h and the vertical-stress curve marked σ_v were prepared by the writer on the basis of Fig. 1, which again is based on Eqs. 3 and 4. The stresses $\sigma_h = -4.5$ MPa and $\sigma_v = 13.5$ MPa at $z/R = 0$ correspond, respectively, to $\sigma_x = -0.13 \cdot (2p/\pi)$ and $\sigma_z = -0.39 \cdot (2p/\pi)$ at $z/R = 0$, σ_x and σ_z being given in Fig. 3 as multiples of $2p/\pi$, and p is a contact force in Trollope's (1968) elastic model of the material considered.

Failure in the Brazilian test is conventionally taken to mean that the tensile stress σ_h corresponding to the failure load is the tensile strength. However, as may be seen from the Mohr-Coulomb diagrams in Figs. 4 and 6, the Brazilian test specimens are actually failing in shear, not in tension. Also, the restrained loading tests referred to above showed that the failure is progressive, starting at a depth of about $0.1 \cdot D$ below the applied load and proceeding towards $0.5 \cdot D$, i.e. toward the centre of the cylindrical sample. One consequence of this would be that any violent failure of the inner zone could be expected to be protected by the outer un-failed zones. This is in agreement with all observations made at the GEMTEC laboratory, i.e. failure of a Brazilian test specimen is typically non-violent, even for high-strength brittle concrete and rock.

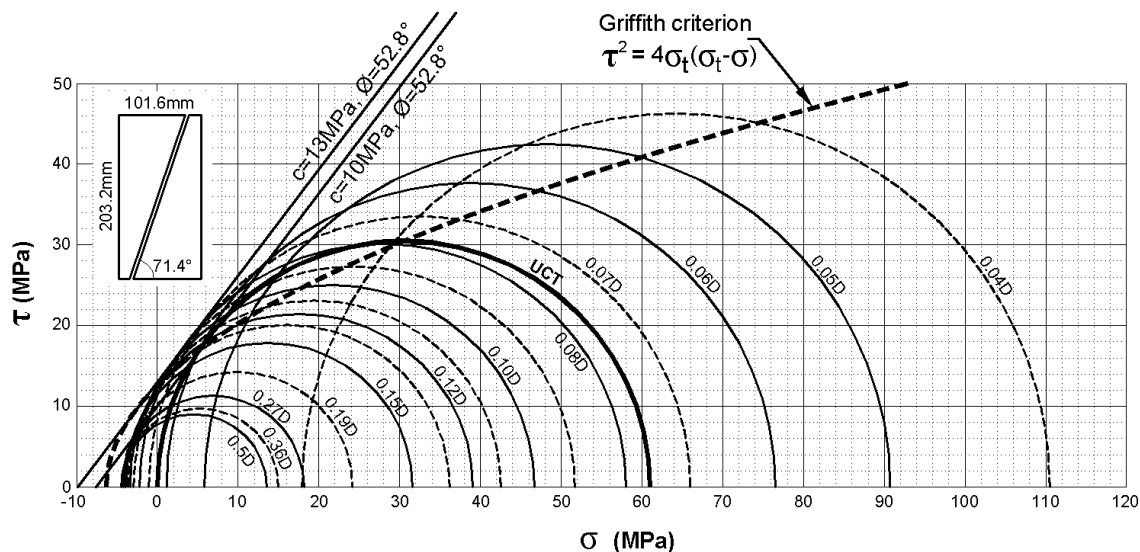


Figure 4. Brazilian and unconfined compression tests on 61MPa concrete (UCT = Unconfined compression test)

The failure zone in a sample brought to failure in a Brazilian test is a forced zone because, simply for reasons of symmetry, the Brazilian failure zone located diametrically between the applied loads is the only location where failure could possibly occur in practice.

Reference is finally made to Hoek's (1968) rupture modes and rupture criteria for brittle rock and to his corresponding Mohr rupture envelopes, reproduced in Fig. 5. According to Hoek, all Mohr stress circles which fall within the circle of radius $2\sigma_t$ can touch the rupture

envelope at one point only, and this point is defined by $\tau = 0$ and $\sigma_3 = -\sigma_1$. This implies that pure tensile rupture occurs only when $\sigma_1 \leq -3\sigma_3$. Referring to Eqs. 1 and 2, the equality $\sigma_1 = -3\sigma_3$ may be written

$$\frac{D}{r} + \frac{D}{D-r} - 1 = -3$$

from which it will be found that $r = D/2$, i.e. the equality occurs at the centre of the cylindrical sample only. It

follows that, in practice, there is really no pure tensile rupture anywhere in the sample.

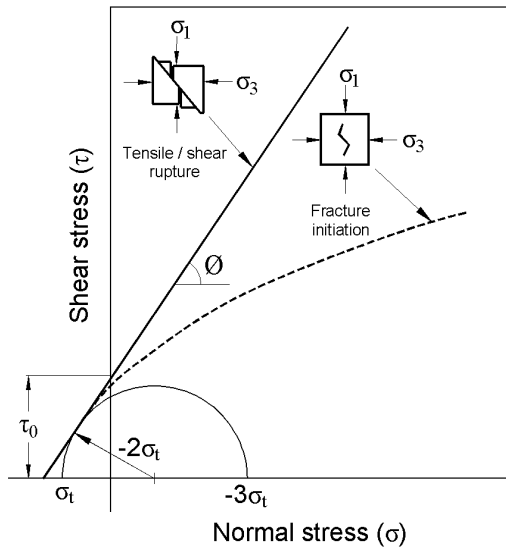


Figure 5. A rupture criterion for brittle rock (tensile stresses shown positive) (after Hoek 1968)

4. FAILURE CRITERIA

Goodman (1989) discusses the Mohr-Coulomb failure criterion for rock and states that t_0 is "the uniaxial tensile strength" as determined by the Brazilian test, i.e.

$$-t_0 = -\sigma_t = -2P/\pi LD \quad [5]$$

which is the same as Eq. 2 above. He also states that "the minor principal stress can never be less than t_0 and that failure is presumed to occur because of tensile stress whenever σ_3 becomes equal to $-t_0$, regardless of the value of σ_1 ". It should be noted, however, that Goodman (ibid.) does not provide any evidence to support these statements, nor are they in agreement with other estimates of the maximum tensile stress (i.e. the tensile strength). For example, Trollope (1968) introduces a "critical effective tensile stress criterion of failure" or "tensile cut-off" and concludes that, for a brittle material with a Poisson's ratio μ in the range 0.15 - 0.22 (e.g. high-strength concrete), the maximum effective tensile stress $\sigma_{x\max}$ is about 1.6 to 2.0 times greater than σ_h of Eq. 5. As shown in Fig. 3, $\sigma_{x\max}$ is reached at a depth of about $0.15 \cdot D$ below the loaded surface.

Another example of an evaluation of the tensile strength is the following relation (Hoek 1968) between the "intrinsic shear strength" τ_0 (in soil mechanics

terms, τ_0 is the same as the cohesive intercept c) and the normal stress intercept $-\sigma_t$:

$$\tau_0 = -\sigma_t \left(\frac{1}{\tan \phi} + \tan \phi \right) \quad [6]$$

where ϕ = angle of friction (Fig. 6). For values of ϕ between 30° and 60° , $\sigma_t = -0.43\tau_0$ to $-0.50\tau_0$. It is noted that Eq. 6 is based on Griffith's (1924) criterion of failure in brittle materials. Plotted as a Mohr envelope (i.e. on σ , τ Cartesian co-ordinates), Griffith's criterion is a parabola:

$$\tau^2 = 4\sigma_t(\sigma_t - \sigma) \quad [7]$$

In the case of the test results in Fig. 4, where $\phi = 52.8^\circ$, Eq. 6 gives

$$\sigma_t = \frac{13}{\frac{1}{\tan 52.8^\circ} + \tan 52.8^\circ} \approx -6.3 \text{ MPa}$$

as compared with σ_h = the tensile stress = -4.5 MPa from Eq. 4. In Trollope's (1968) diagram (Fig. 3) the abscissa is labelled "stresses", but in a footnote preceding the diagram the term used by Trollope (ibid.) is "maximum effective tensile strength", and it would be in the range $\sigma_{x\max} = (1.6 \text{ to } 2.0) \cdot (-4.5) = -7.2 \text{ to } -9.0$ MPa. Thus both σ_t and $\sigma_{x\max}$ are referred to as tensile strengths and they both yield values considerably higher than σ_h . It seems clear therefore that the tensile stress σ_h induced by the vertical load P does not represent the tensile strength of the material.

The Mohr stress circles in Fig. 4 are calculated from Eqs. 3 and 4 and a recorded failure load of 145 kN on a 101.6 mm diameter by 203.2 mm length high-strength concrete specimen. The shear strength parameter $\phi = 52.8^\circ$ was determined from a compression test on a twin specimen of concrete. The inclination of the failure plane $\alpha = 71.4^\circ = 45^\circ + \phi/2$ gave $\phi = 52.8^\circ$, and the unconfined compressive test (UCT) strength was 61 MPa. The corresponding Mohr stress circle is shown in Fig. 4, and a cohesion intercept of 10 MPa was obtained by tracing the $\phi = 52.8^\circ$ failure envelope tangential to the UCT circle. A somewhat higher intercept ($c = 13$ MPa) was obtained by tracing the failure envelope parallel to the UCT envelope but tangential to the circle that yields the maximum intercept, i.e. $c = 13$ MPa. This particular circle corresponds to the stresses at a depth of about $0.1 \cdot D$ below the applied load.

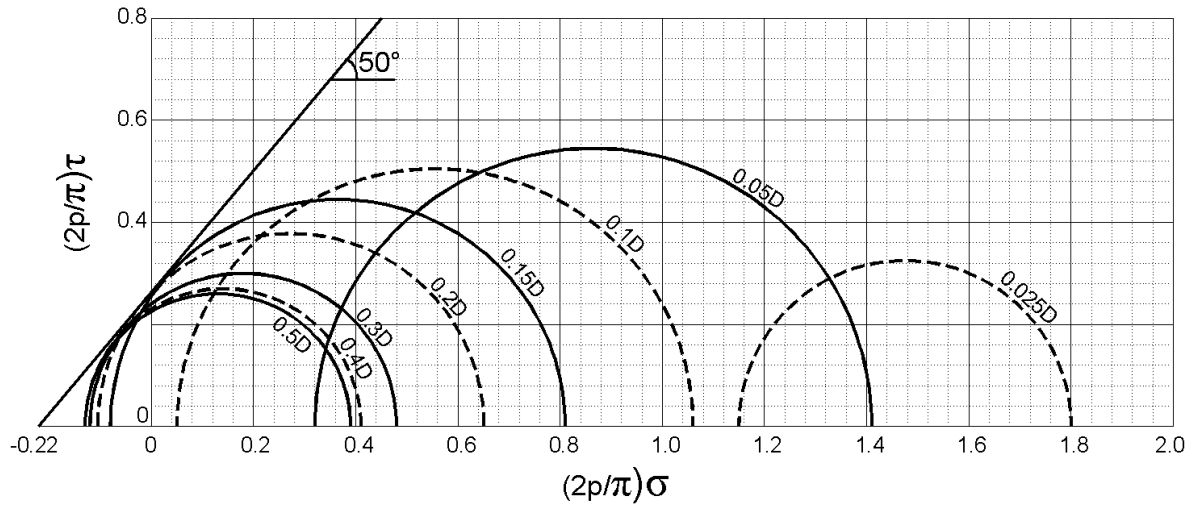


Figure 6(a). Mohr stress circles corresponding to σ_z and σ_x , $\mu=0$ from figure 3

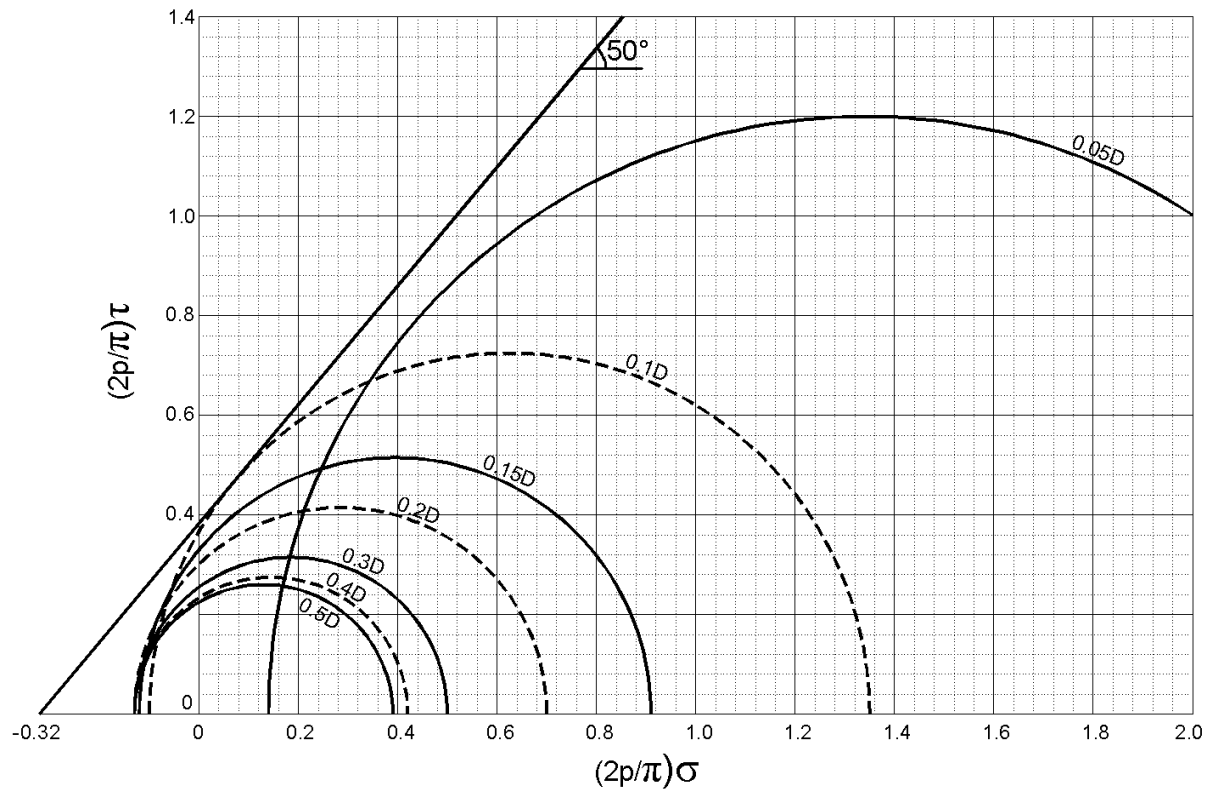


Figure 6(b). Mohr stress circles corresponding to σ_v and σ_h from figure 3

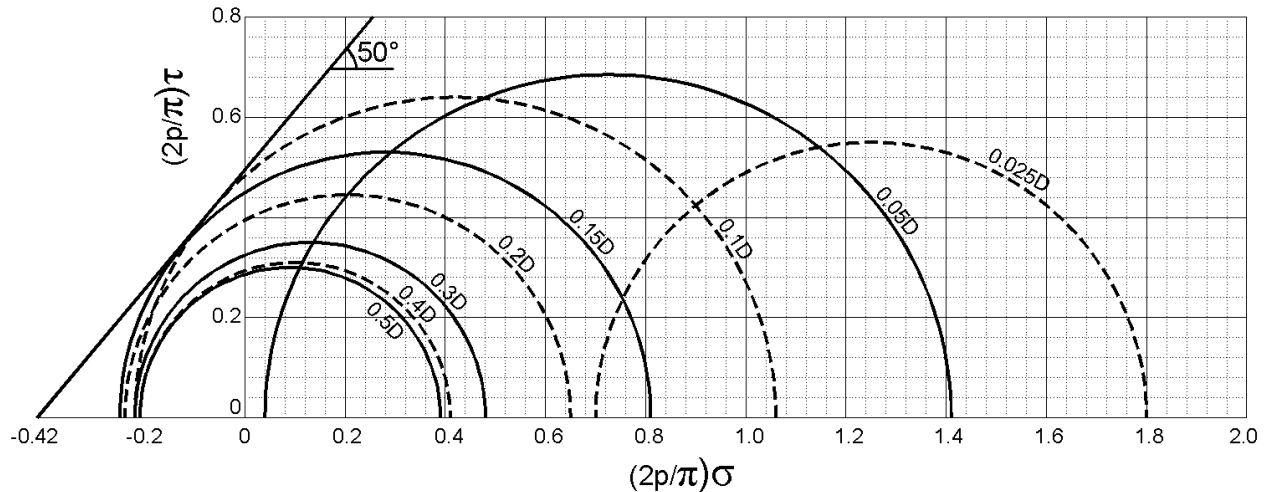


Figure 6(c). Mohr stress circles corresponding to σ_z and σ_x $\mu=0.2$ from figure 3

The stresses calculated by Trollope (1968) and shown in Fig. 3 are plotted as three sets of Mohr stress circles in Fig. 6: (a) these circles correspond to Trollope's elastic stresses σ_z and σ_x , $\mu = 0$, (b) these circles correspond to the σ_v (Eq. 4) and the σ_h (Eq. 3) stresses, and (c) these circles correspond to the σ_z and σ_x , $\mu = 0.20$ stresses. The friction parameter $\phi = 50^\circ$ in this case is an assumed value since no shear strength parameters are given. However, the corresponding failure plane inclination of $45^\circ + \phi/2 = 70^\circ$ is typical of the numerous concrete specimens tested by the writer and can be considered to be a reasonable approximation. For all three sets of stress circles progressive shear failure can be seen to start at a depth range of $0.1 \cdot D$ to $0.15 \cdot D$ below the applied load

Jaeger and Cook (1979) state that (i) it is generally assumed that failure [in the Brazilian test] is the result of the uniform tensile stress normal to the loaded diametral plane, (ii) exceptional cases are found with a single diametral fracture which may not even extend to both platens, (iii) there are generally several fractures branching from the diametral plane, some of which appear to be wedges near the contact, (iv) these subsidiary effects have given rise to doubts about the mechanism of failure in this test, and (v) it has been suggested that failure starts by shear fracture in the region of high compressive stresses near the contacts. Referring to Figs. 4 and 6, and as observed by Wright (1955), it is seen, however, that the high vertical compressive stresses near the contact are accompanied by sufficiently high compressive lateral stresses that failure in compression would not occur in this region. On the other hand, Figs. 4 and 6 show that shear failure starts at a depth range of $0.1 \cdot D$ to $0.15 \cdot D$ below the applied load and then spreads progressively toward the centre of the cylindrical specimen, i.e. to $0.5 \cdot D$ below the applied load.

5. CONCLUSIONS

Equation 2 has been generally accepted by ASTM as well as by most textbooks on rock mechanics and is generally used to analyze the results of the 'Brazilian' test. However, Hoek (1968) showed that pure tensile failure occurs only at the centre of the cylindrical specimen, i.e. there is really no pure tensile rupture anywhere in the specimen. Trollope (1968) used a clastic mechanics model to analyze the stress distribution set up in a Brazilian test specimen and showed that the maximum effective tensile stress is considerably greater than σ_h of Eq. 2 or 4.

On the basis of (i) a series of unconfined compression tests and Brazilian tests on concrete cylinders, (ii) an analysis of the corresponding Mohr stress diagrams, and (iii) a literature review, the writer has concluded that the Brazilian mode of failure is one of progressive shear, starting at a depth of about one tenth of the diameter below the loaded surfaces and proceeding towards the centre. Another conclusion is that the tensile strength of the specimen is considerably higher than the tensile stress σ_h normally calculated from the Brazilian test results. It follows that the Brazilian test cannot be considered to yield the real tensile strength of the material. The tensile stresses generated in the failure zone can only be considered to represent a conservative estimate of the tensile strength.

6. REFERENCES

- ASTM D3967, Standard Test Method for Splitting Tensile Strength of Intact Rock Core Specimens.
- ASTM C496, Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens.
- Carneiro, F. (1947). Une Nouvelle Methode d'Essai pour Determiner la Resistance a la Traction du Beton. Paris, *Reunion des Laboratoires d'Essai De Materiaux*.
- CSA A23.2-13C, Splitting Tensile Strength of Cylindrical Concrete Specimens.

- Frocht (1948). *Photoelasticity*, vol. 2, pp. 121-129. John Wiley & Sons, New York. Chapman and Hall Ltd., London.
- Goodman, R.E. (1989). *Introduction to Rock Mechanics*. John Wiley & Sons, 562 pp.
- Griffith, A.A., (1924). *Theory of Rupture*. Int. Cong. Appl. Mech., 1st, Delft, pp. 55-63.
- Hoek, E. (1968). Brittle Fracture of Rock. In *Rock Mechanics in Engineering Practice* (chapter 4). Edited by K.G. Stagg and O.C. Zienkiewicz. John Wiley & Sons, 442 pp.
- Jaeger, J.C. and Cook, N.G.W. (1979). *Fundamentals of Rock Mechanics*. Chapman and Hall, London. 593 pp.
- Landva, A. and La Rochelle, P. (1983). Compressibility and Shear Characteristics of Radforth Peats. *ASTM Special Technical Publication 820*, pp. 157-191.
- Neville, A.M. (1995). *Properties of Concrete*, 4th ed. Longman Group, 844 pp.
- Timoshenko, S. (1934). *Theory of Elasticity*, pp. 104-108. McGraw-Hill, New York.
- Timoshenko, S. and Goodier, J.M. (1951). *Theory of Elasticity*, 2nd ed. McGraw-Hill, 506 pp.
- Trollope, D.H. (1968). The Mechanics of Discontinua or Clastic Mechanics in Rock Problems. In *Rock Mechanics in Engineering Practice* (chapter 9, Fig 9.20). Edited by K.G. Stagg and O.C. Zienkiewicz. John Wiley & Sons, 442 pp.
- Vidal, H (1969). The Principal of Reinforced Earth. *Highway Research Record No. 282*, Highway Research Board, NRC, National Academy of Sciences, National Academy of Engineering, pp. 1-16.
- Wright, P.J.F. (1955). Comments on an Indirect Tensile Test on Concrete Cylinders. *Road Research Laboratory, Department of Scientific and Industrial Research. Magazine of Concrete Research*, July, pp 87-96.