Visualization and modelling of dynamic paste tailings flows

Julio Henriquez and Paul Simms
Department of Civil and Environmental Engineering – Carleton University, Ottawa, Ontario, Canada

ABSTRACT
The overland flow of tailings during deposition is a critical element in managing the geometry of paste and thickened tailings impoundments. Various small-scale flows of a gold paste tailings are imaged using high-speed cameras. The visualized flows are modelled using a transient numerical non-Newtonian flow code. The code is based upon “lubrication theory”, a simplification of the Navier-Stokes equations. The experimental and modelled results compare well, and theory shows promise for upscaling to the field. The theory explains the scale-dependence of paste and thickened tailings flows, which has commonly been observed in practice.

RÉSUMÉ
l'écoulement par voie de terre des résidus miniers pendant le dépot est un element essentiel en maniant la géometrie de pâte et les dépot des residus épaissis. Plusieurs écoulements en petite échelle de pate en or sont images en utilisant un camera de rapidité. Les écoulements rendus visibles sont modelès en utilisant un code numèrique de passage de l'écoulement non-Newton. Le code est basé sur la theorie de lubrication, une simplification des équations de Navier Stokes. Les résultants experimentaux et modelès se comparent bien, et la thèorie montre la promesse pour utilization à la grande measure. La theorie explique la dependence échelle de la pâte et les ecoulements des residus epaissis, ce qu'on à observé ordinairement dans la pratique.

1 INTRODUCTION

Disposal of thickened or paste tailings is an attractive option for surface disposal, as it eliminates or reduces some of the risks associated with conventional tailings disposal, most importantly reducing reliance on dams and the associated risk of catastrophic failure associated with conventional impoundments (ICOLD, 2001). Thickened or paste tailings offers other comparative advantages, such as increased water recycling within the mining operation, and reduced groundwater seepage out of the tailings impoundment. Recent advances in technology have permitted economic dewatering and pumping of thicker tailings to the point where they behave as a non-Newtonian fluid, and form gently sloping stacks during deposition, thus reducing reliance on confinement by dams. Tailings thickened to the extent that no segregation of particle size occurs during transport and relatively small amount of settling occurs post-deposition are often called “paste” (Cincilla *et al*. 1997). Paste deposition has been employed at a full-scale at the Bulyanhulu Gold Mine in Tanzania (Simms *et al*. 2007, Suttleworth *et al*. 2005, Theriault *et al*. 2003). It has been shown at Bulyanhulu, that if the deposition is cycled between a number of points in the impoundment and the geometry of the flow is properly controlled (Shuttleworth *et al*. 2005), the tailings will densify and gain significant strength through desiccation, allowing the development of stable stacks with up to a 6 degree slope.

While the behaviour of thickened or paste tailings in the pipeline have been studied to a significant degree (Nyugen and Boger 1998, Pullam *et al*. 2007, Sofra 2007), the relation between rheology and deposition geometry has received less attention. Most studies have focused on charactering the geometry using a single angle (Kwak *et al*, 2005, Sofra and Boger 2003) at the laboratory scale. However, it has widely been observed in practice that the overall angles of deposits in the field are typically less than angles measured in the laboratory (Oxenford and Lord 2006, Engman *et al*. 2004). This was partially thought to be attributed to shear thinning occurring during transport (For example, Houman *et al*. 2007). However, Simms (2007) proposed, based on non-Newtonian flow theory that had been previously applied to mud and lava flows (Liu and Mei 1989), that the equilibrium beach profile is not characterized by a unique angle, and that the overall angle of the deposit is a function the size of the flow. Other researchers have begun tackling this problem (Fitton *et al.* 2008, Pirouz and Williams 2007) using a different approach, one based on the critical deposition velocity for turbulent overland flow. This paper pursues the method proposed in Simms (2007), and examines the applicability of “lubrication theory” for predicting both equilibrium profiles and the dynamic flow behaviour of a gold paste tailings during multilayer deposition in small scale flume tests

2 THEORY

Equations for the equilibrium profiles of yield stress fluids may be derived using “Lubrication Theory”, in which the continuity and momentum equations for fluids are simplified by assuming the slow spreading of a thin layer or film. The simplifying assumptions are:

1. The ratio of thickness to horizontal extent of the flow are small, and

2. The velocity of the material is slow, such that terms that include the ratio of inertial to viscous forces will vanish from the Navier-Stokes equations.

From these assumptions equations describing equilibrium profiles for certain geometries can derived (Yuhi and Mei, 2004; Liu and Mei 1989; Balmforth *et al*, 2002; Coussot and Proust; 1996). For example, for two dimensional flow across an inclined bed, the distribution of shear stress may be written as:

$$\tau = (h - z)\left(\rho g \cos\theta\left(\tan\theta - \frac{\partial h}{\partial x}\right)\right) \quad (1)$$

Where p is pressure, ρ is the bulk density, θ is the angle of the inclined surface from the horizontal, *g* is the acceleration due to gravity, τ is the shear stress, the *x* axis is the direction of the inclined plane, and the *z* axis is perpendicular to the inclined plane. Setting z=0 and τ to the yield stress τ_y, we can find a equilibrium profile as a function of \x for a given θ. For example, if θ=0, we can find the equilibrium profile explicitly:

$$h^2 - h_0^{\ 2} = \frac{2\tau_y}{\rho g}\left(x - x_0\right) \quad (2)$$

Where h_0 is the height at x_0.

A depth integrated transient solution based on lubrication theory may be similarly derived (Yuhi and Mei 2004):

$$\frac{\partial h}{\partial t} = \frac{g\rho}{\mu}\frac{dh}{dx}\frac{1}{6}(3h - h_y - 2H)\left(h_y - H\right)^2 \quad (3)$$

Where μ is the dynamic viscosity, h is the vertical height of flow, h_y is the depth above which plug flow occurs, and H is the bed elevation. Equation 3 is based on the assumption of laminar flow in sheared zones and plug flow in regions where the yield stress is not exceeded.

3 MATERIALS AND METHODS

The tailings used were nonplastic (Plastic limit of 20) gold tailings from the Bulyanhulu mine in Tanzania. These tailings are predominantly silt-sized, and have a specific gravity of 2.9. The tailings are prepared at a gravimetric water content of 40%, corresponding to the pumping water content in the field. Tailings were shipped at the pumping water content, but arrived at the laboratory in a compacted state with bleed water on top. The tailings were prepared to the pumping water content using this bleed water and a mechanical mixer. The mechanical mixer is applied to the sample for at least ten minutes. Though tests were conducted over the period of a year, ph measurements ranged from 6.5 to 7 showing not trend with time. Tailings were stored underwater to minimize oxidation. Pore-water analyses were conducted at different times and no trend with time was observed. The tailings pore-water is dominated by iron, calcium, magnesium, and sulphate ions.

For this study flume tests were performed on horizontal planes and on successive layers. A high speed camera, Model IN250 from High Speed Imaging was used to record the tests. The camera has a capture rate of 50 frames per second. For multilayer tests, each layer was left for about 24 hours to allow it to reach the post-bleed GWC of 30%, before the next layer was added. This water content was confirmed by sampling. Pore-pressures were monitored using a tensiometer to ensure no significant suction developed. At this water content the older layers remain saturated and no cracking develops. Flume tests were performed by pouring the tailings through a funnel at one end of the flume. The funnel had an opening diameter of 2.4 cm. Once prepared at the appropriate water content, the paste was placed in a bucket and mixed with a stirring machine for approximately 10 minutes prior to being deposited in the flume. The flume is made of see-through acrylic and is 30 cm tall, 15 cm wide, and 2.5 m long.

Initial tests were performed to examine the effect of width of the flume and lubrication of the walls. A hydrophobic grease was used as a lubricant, and the walls were narrowed from 15 to 10 cm. It was found that neither greasing nor changing the flume width had any significant effect on the final equilibrium profile of the flows (using the same volume of material per unit width).

The parameters of interest are the yield stress, the viscosity-shear stress relationship (flow curve), and the bulk density. Yield stress and viscosity were obtained from flow curves measurements at different water contents using a Brookfield RS rheometer using a cone and plate fixture. Yield stresses were also determined based on slump tests, using the interpretation of Palshias et al. (1996).

4 RESULTS

4.1 Rheological testing

In this paper we only report the flow curves measured at the pumping water content, shown in Figure 1.

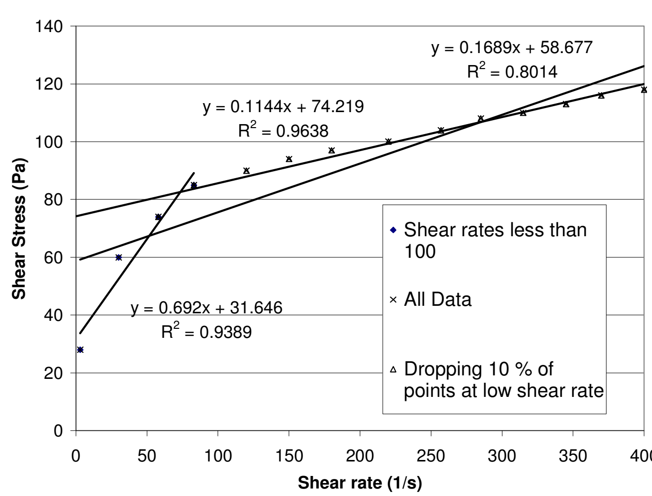


Figure 1 Flow curve at 40% GWC

Figure 1 shows the measured flow curve fitted with three possible fits with the Bingham model (the Bingham assumes a constant viscosity and therefore a constant slope to the flow curve). It will be shown that the best fit to the data at the low shear rate range gives the best agreement with theoretical predictions. However, yield stresses determined from the slump test at the pumping water content ranged between 70 and 100 Pa, which agree better with yield stresses determined using a Bingham fit to the whole flow curve.

4.2 Flume test static profiles

Figure 2 presents 3 equilibrium profiles for flows of different volumes. This figure shows that the overall angle of the deposit is scale-dependent, the angle decreasing as the deposit gets larger. Each of these flows can be fitted using Eqn 2 using the same density and yield stress. This result is important as it explains why direct extrapolation of flume angles to field scale deposition has not worked.

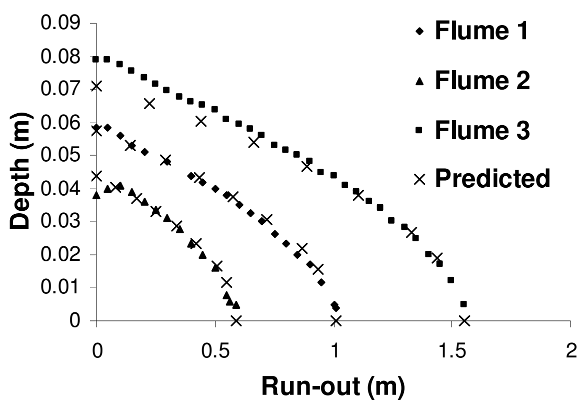


Figure 2. Equilibrium profiles of three flows of different volumes, all fitted using Eqn 2, employing a yield stress of 36 Pa.

Figure 3 presents equilibrium profiles for multilayer deposition.

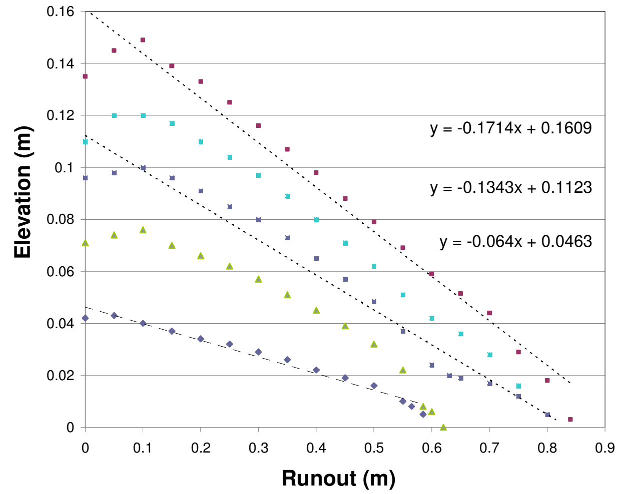


Figure 3 Sequential deposition, each layer initially placed at 40% GWC

It can be seen that with each layer the layer thickness decreases and the overall angle of the slope increases.

It was found that the multilayer static profiles could be iteratively determined using Eqn. 1 by calculating dh/dx in discrete steps back from toe, considering that θ= θ(x). An example of the prediction from this test is shown in Figure 4.

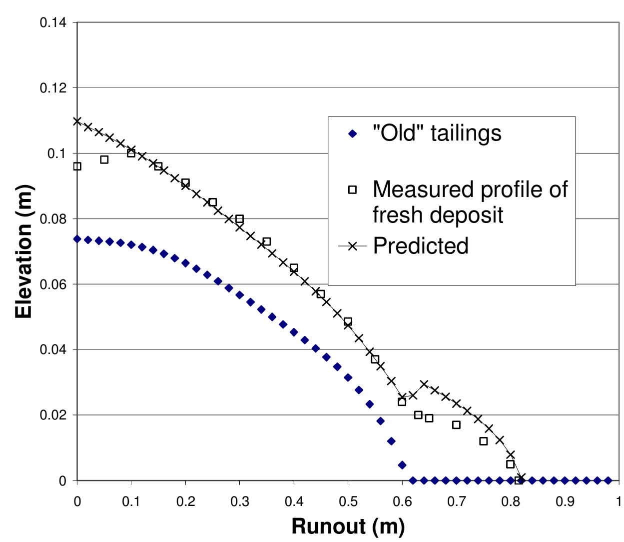
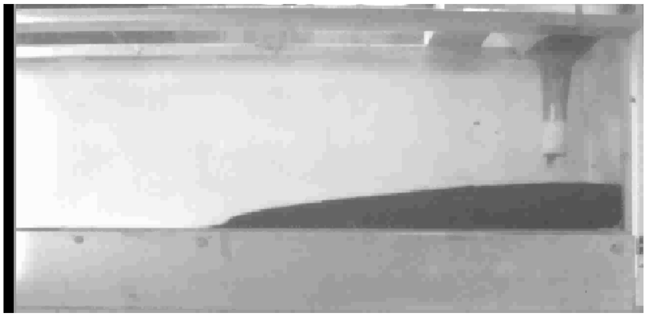


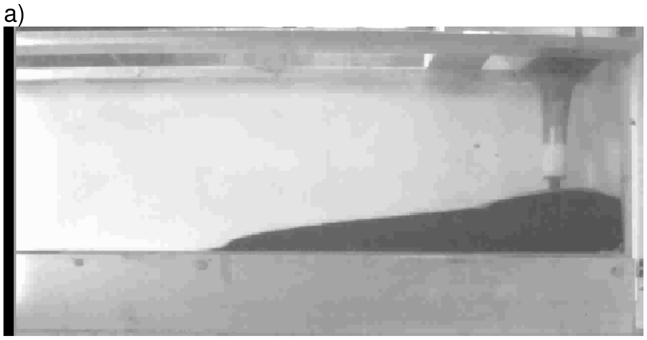
Figure 4. Predicted and measured equilibrium profiles for multilayer deposition.

4.3 Dynamic flow profiles

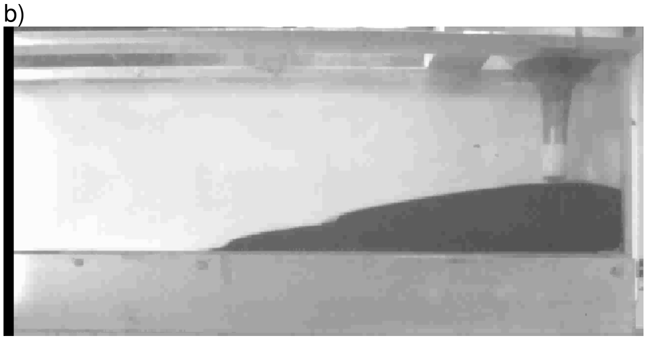
Figure 5 shows an example of dynamic imaging from a flume test.



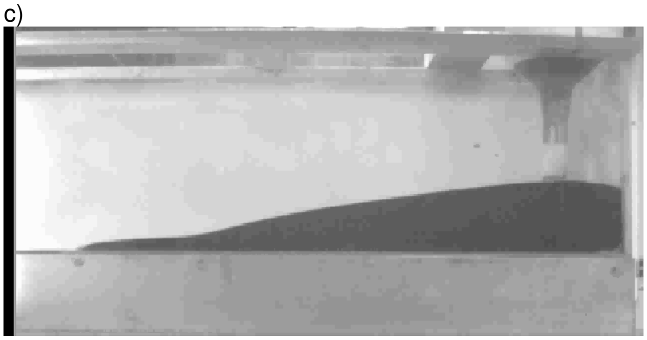
a)



b)



c)



d)

Figure 5. Example of a dynamic visualization a) 0s b) 2s c) 4s d) Stopped at 18s

A 1-D finite difference solution of Eqn. 3 was employed to simulate the dynamic flows. A spatial step of 1 cm and a time step of 0.0001 s were employed. A constant viscosity of 0.65 Pa s, corresponding to the fit of the flow curve in the low shear rate zone, was used. Further details can be found in Henriquez (2008). Predictions of the transient model are shown in Figure 6 for a single layer and Figure 7 for multilayer flow. In general, the model tended to predict faster flows than were observed. This may be due to the assumption of constant viscosity.

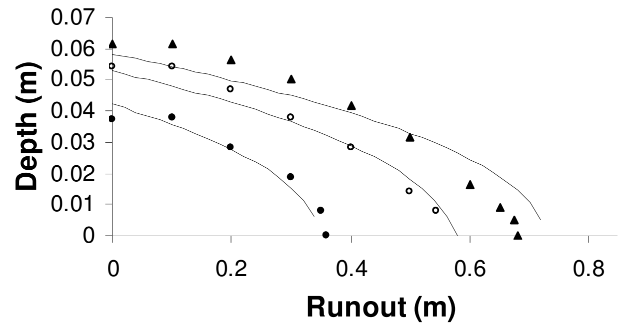


Figure 6. Predicted and measured flow profiles at 3, 6, and 9 s during single layer deposition.

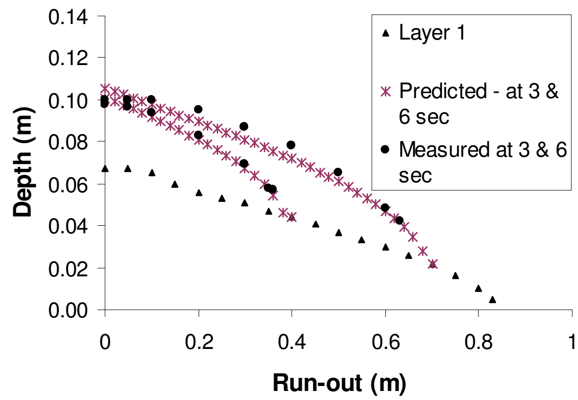


Figure 7. Predicted and measured flow profiles of a fresh layer flowing on top of an older layer

5 DISCUSSION

5.1 Significance of the "best fit" yield stress

It was observed that the yield stress found from fitting the flow curve in the low shear rate range (~32 Pa) agreed with the yield stress that gave the best fit of the lubrication theory predictions to the measured data (36 Pa). This is a reasonable result, as the Bingham model fit to the data closest to the origin would most likely give the best approximation of the actual yield stress. Unfortunately, the low shear rate region is also the most susceptible to measurement error, and indeed is often neglected in interpreting flow curves to derive model parameters for pipe flow analysis. However, flume tests themselves may serve to obtain the yield stress pertinent to deposition modelling thought their interpretation using Eqn. 2.

It was found that the yield stress predicted from the slump tests (70 - 100 Pa) conformed to the best fit of the Bingham model to the flow curve for shear rates greater than 50 s^{-1}. This suggests that this yield stress is more appropriate for determining friction losses in pipe flow rather than characterizing the stress at which the tailings stop flowing during deposition.

5.2 Application of theory to the field scale

The theory shows promise as it predicts the scale-dependency of the overall angle of the deposit observed in the field. While the flow coming out of the pipe at deposition point may have considerable inertia, the flow away from the deposition point is very likely to be dominated by viscous forces, and the authors expect that lubrication theory will be applicable. Initial comparisons with field results have been shown elsewhere (Simms 2007).

5.3 Relevance to geotechnical slope stability.

Note that we have assumed that the older layers possess infinite strength in our analyses and that the stability of the older layers does not influence stack geometry. While at Bulyanhulu, the undrained strength of the tailings, as measured by a field vane, increases above 100 kPa within a few days, this is in part to the desiccation of the tailings. In wet climates, the gain in strength may not be as large, and the therefore the stability of the older layers should be checked by conventional slope stability calculations.

Some may wonder at the physical mechanism behind the curvature in the lubrication theory equations. The relationship between depth, *h*, and runout, *x*, in Eqn. 2 is not a straight line. The curvature arises out of the treatment of stresses: the material is considered a fluid, and so the driving shear stresses are generated by the hydrostatic pressures. For a fluid, the horizontal force exerted by one vertical slice on another is the integration of the hydrostatic pressures with depth: namely $\rho g H^2/2$.

6 SUMMARY AND CONCLUSIONS

Static profiles and dynamic flows of multilayer deposition of paste tailings are simulated in the laboratory, and compared to analytical and numerical predictions developed from lubrication theory of non-Newtonian flow. The results compare well and the theory shows promise for upscaling to the field. The theory shows that a single angle cannot be used to characterize the geometry of freshly deposited flow, and that the overall angle is scale dependant. This at least partially explains the discrepancy noticed between laboratory flume tests and field scale thickened tailings flows.

It is observed that the best fit yield stresses agree with rheometry results when a Bingham model is fit to data on a flow curve measured at low shear rates. This is a problem since these points are often discarded due to large measurement errors that occur at lower shear stresses. However, an alternative method to obtain the yield stress pertinent to deposition modelling is to fit flume tests using the theory outlined in the paper.

ACKNOWLEDGEMENTS

This work was funded by a collaborative grant between Ontario Centres of Excellence and Golder Associates. Ryan Francoeur and Frank Palkovits at Golder Paste Tec provided technical assistance.

REFERENCES

Balmforth,N.J., Craster, R.V., and Sassi, R. 2002. Shallow viscoplastic flow on an inclined plane. *Journal of Fluid Mechanics*, 470: 1-29.

Cincilla, W.A., Landriault, D.A., Verburg, R. 1997. Application of paste technology to surface disposal of mineral wastes. *Proceedings of the Fourth International Conference on Tailings and Mine Waste '97*. pp. 343–356.

Clayton, S.; Grice, T.G.; Boger, D.V. 2003. Analysis of the slump test for on-site yield stress measurement of mineral suspensions. *International Journal of Mineral Processing,* 70(1-4): 3-21.

Coussot, P. and Proust, S. 1996. Slow, unconfined spreading of a mudflow. *Journal of Geophysics Research*, 101: 25217-25229.

Engman, M. ,Sellgren, A.,Sundquist, A., Wennberg, T. and Goldkuhl, I. 2004. Users perspectives on the design of high density base metal tailings handling systems. *Proceedings of the eleventh tailings and mine waste conference*, 10-13 October 2004, Vail, Colorado, USA, pp 45-54.

Fitton, T.G., Bhattacharya, S.N. and Chryss, A.G. Three-dimensional modeling of tailings beach shape. *Computer-aided Civil and Infrastructure engineering*, 23: 31-44.

Gawu, S.K.Y. and Fourie, A.B**.** 2004. Assessment of the Modified Slump Tests as a Measure of the Yield Stress of High-Density Thickened Tailings. *Canadian Geotechnical Journal*, 41: 39 – 46.

Houman, J., Paterson, A.J.C., and van Sittert, F. 2007. Quantifying the effect of pump shear on slurry rheology. *Proceedings of the tenth international seminar on paste and thickened tailings,* Perth, Australia, March 13th-15th 2007, pp. 163-174.

Hulme, G. 1974. The interpretation of lava flow morphology. *Geophysics Journal of the Royal Astronomical Society*, 39: 361-383.

Kwak, M. James, D. F., and Klein, K. A. 2005. Flow Behaviour of Tailings Paste Disposal. *International Journal of Mineral Processing*, 77(3): 139 – 153.

Liu, K.F., and Mei C.C. 1989. Slow spreading of a sheet of Bingham fluid on an inclined plane. *Journal of fluid mechanics*, 207: 505-529.

Nguyen, Q.D., and Boger, D.V. 1998. Application of rheology to solving tailings disposal problems. *International Journal of Mineral Processing*, 54 (3-4): 217-233.

Oxenford, J. and Lord, E.R. 2006. Canadian Experience in the Application of Paste and Thickened Tailings for Surface Disposal. *Proceedings of the ninth international seminar on paste and thickened tailings*, Limerick, Ireland, 3-7 April 2006, pp. 93 – 105.

Pashias, N., Boger, D.V., Summers, J., and Glenister, D.J. 1996. A fifty cent rheometer for yield stress measurements. *Journal of Rheology*, 40(6): 1179–1189.

Pirouz, B. and Williams, M.P.A. 2007. Prediction of Non-Segregating Thickened Tailings Beach Slopes – A New Method. *Proceedings of the tenth international seminar on paste and thickened tailings*, Perth, Australia, March 13^{th}-15^{th} 2007, pp. 315 – 327.

Pullum, L., Graham. L, Rudman, M., Hamilton, R. 2006. High concentration suspension pumping. Minerals Engineering, 19: 471-477.

Shuttleworth, J.A,. Thomson, B.J., and Wates, J.A. 2005. Surface disposal at Bulyanhulu- practical lessons learned. *6^{th} International Conference on Paste and Thickened tailings*, Santiago, Chile, April 20^{th}-22^{nd}.

Simms P., Grabinsky M.W., and Zhan, G. 2007. Modelling evaporation of paste tailings from the Bulyanhulu mine. Canadian *Geotechnical Journal*, 44: 1417-1432.

Simms, P. 2007. On the Relation between Laboratory Flume Tests and Deposition Angles of High Density Tailings. *Proceedings of the tenth international seminar on paste and thickened tailings,* Perth, Australia, March 13^{th}-15^{th}, pp. 329 – 335.

Sofra, F. and Boger, D.V. 2002. Environmental rheology for waste minimisation in the minerals industry. *Chemical Engineering Journal*, 86(3): 319-330.

Theriault, J., Frostiak, J., and Welch, D. 2003. Surface disposal of paste tailings at the Bulyanhulu gold mine, Tanzania. *In Proceedings of Sudbury 2003, Mining and the Environment*, Sudbury, Ontario, Canada, May 26^{th}-28^{th} 2003. Editors G. Spiers, P. Beckett, H. Conroy, pp. 265-269.

Yuhi, M. and Mei C.C. 2004. Slow spreading of fluid mud over a conical surface. *Journal of Fluid Mechanics*, 519:337-358.