



# Field measurement of shear wave velocity of soils

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## ABSTRACT

In this study, S-wave velocity survey was carried out at a site near a drilled borehole. The test data of a firm silty clay layer at depth from 5.5 to 12.0 m and a very dense silt layer from 19.5 to 24.0 m are analyzed in detail. The small-strain shear modulus  $G_{\max}$  and Young's modulus  $E_{\max}$  are obtained from the test data. The empirical correlations in the literature are close to the test data of the firm silty clay, but generally underestimate the values of  $G_{\max}$  and  $E_{\max}$  of the very dense silt. A new correlation for  $G_{\max}$  of the very dense silt is proposed in this study.

## RÉSUMÉ

Dans cette étude, une enquête de vélocité à ondes-S a été exécuté à un site près d'un trou à forage. Des données d'une couche d'argile et de limon ferme à une profondeur de 5,5 à 12,0 m ainsi qu'une couche de limon très dense de 19,5 à 24,0 m ont été analysées en détail. À partir de ces données, le module de cisaillement  $G_{\max}$  et le module de Young  $E_{\max}$  ont été obtenues. Les corrélations empiriques dans la littérature sont en fait très proches aux données de test d'argile et de limon ferme, mais généralement sous-estime les valeurs de  $G_{\max}$  et de  $E_{\max}$  du limon très dense. C'est pour cela qu'une nouvelle corrélation pour le  $G_{\max}$  du limon très dense est proposée dans cette étude.

## 1 INTRODUCTION

Shear wave (S-wave) velocity,  $V_s$ , of soils is a small-strain parameter that is widely used to evaluate the dynamic response of soils, including seismic site response, machine foundation vibration and liquefaction potential of soils. The important factors affecting  $V_s$  is the confining effective pressure and void ratio of soils. For the same type of soil in the field,  $V_s$  increases with depth due to the increase of effective stress level in the soil. The  $V_s$  value of soil increases significantly with the density of soil. Stress history, expressed in terms of over-consolidation ratio (OCR), affects the  $V_s$  value, through void ratio and stress level. In addition, geologic aging of soil is also of importance, through the creation of 'bonds' between soil particles. Natural soils are often somewhat "cemented", due to the effect of geologic aging. The relatively weak 'bonds' between soil particles due to aging, may be insignificant for the large-strain shear failure strength, but will significantly increase the small-strain properties such as  $V_s$  and liquefaction potential. A small amount of cementation in sand can significantly increase the  $V_s$  value (Clark et al., 1993).

S-wave velocity can be used to evaluate the liquefaction resistance of sands. This is due to the fact that both  $V_s$  and liquefaction potential of soils are closely related to soil density, confining pressure, stress history and aging. In the past decades, studies have been carried out to relate  $V_s$  to liquefaction resistance of soils (De Alba et al., 1984; Andrus and Stokoe, 2000). In the National Building Code of Canada (2005), shear wave velocity is considered to be a primary parameter that can be used for seismic site classification.

The elastic shear modulus ( $G$ ) and the bulk modulus ( $K$ ) of soil can be expressed as

$$G = \frac{E}{2(1+\nu)} \quad [1]$$

$$K = \frac{E}{3(1-2\nu)} \quad [2]$$

where  $E$  is the Young's modulus, and  $\nu$  is the Poisson's ratio.

In general, the shear modulus of soils decreases with increasing strain level (Hardin and Drnevich, 1972). When the strain level is below the range of  $10^{-5}$  to  $10^{-6}$ , the shear modulus will be close to a constant and will no longer increase with decreasing strain level. In this case, the shear modulus is called the small-strain shear modulus  $G_{\max}$ . The strain level in soils due to shear waves in dynamic in-situ tests such as seismic cross-hole method is generally smaller than  $10^{-5}$  (Gazetas, 1991). Therefore, the shear modulus obtained from shear wave velocity data is considered to be the small-strain shear modulus  $G_{\max}$ .

The shear modulus is related to shear wave velocity as

$$G_{\max} = \rho V_s^2 \quad [3]$$

where  $G_{\max}$  is small-strain shear modulus in Pa,  $\rho$  is the soil density in  $\text{kg/m}^3$ , and  $V_s$  is the shear wave velocity in m/s.

The compression wave (P-wave) velocity,  $V_p$ , can be derived from the S-wave velocity ( $V_s$ ) and the Poisson's ratio ( $\nu$ ) as

$$V_p = V_s \sqrt{\frac{2(1-\nu)}{1-2\nu}} \quad [4]$$

Equation 4 should only be used for soils which are not near saturation. The P-waves in fully saturated soils are essentially transmitted solely in the water phase at a

velocity equal to or slightly greater than 1500 m/s. For fully saturated soils, the Poisson's ratio is close to 0.5, making Equation 4 meaningless.

In this study, S-wave velocity survey was carried out at a site near a borehole drilled to a depth of 25 m. The initial purpose of the survey was to provide shear wave velocity profile for the seismic site classification. In this paper, the test results were further analyzed to investigate the effects of soil type, stress level and other soil parameters on the S-wave velocity and elastic moduli.

## 2 FIELD TEST PROGRAM

In practice, the in-situ shear wave velocity can be determined using the cross-hole method, the down-hole method and the seismic cone technique. The cross-hole method is probably the best method for measuring field S-wave velocity variation with depth. Using this method, S-waves are generated in a seismic hole and are detected by a transducer in a neighbouring hole at the same depth to measure the travel time of the S-waves between the seismic holes. At least two and preferably three seismic holes are required, which are typically located at 3 to 5 m apart.

The down-hole method is a more cost effective alternative to the cross-hole method. It requires only one seismic hole in which transducers can be placed at various depths. The waves are generated at the surface and the travel time of the waves from the source to the transducers can be measured. Compared with the cross-hole method, the results from the down-hole method may be less reliable and the interpretation of the test data is more complicated.

The seismic cone technique combines the down-hole method with cone penetration testing (Robertson et al., 1985). A transducer is installed inside the penetrometer and down-hole measurements of S-wave velocity are performed during brief pauses of the cone penetration testing. This method is not suitable for some type of soils such as very dense deposits, or glacial tills in which large cobbles and boulders are common.

In this study, S-wave velocity measurements were performed to a depth of 24 m at a site using the cross-hole method. The soil and groundwater conditions were obtained from an adjacent borehole in which standard penetration testing and in-situ shear vane testing were performed.

### 2.1 Soil and Groundwater Conditions

The soils explored in the adjacent borehole consisted of a layer of loose to compact fill to a depth of 3.5 m, stiff to very stiff clayey silt till from 3.5 to 5.5 m, a firm silty clay layer from 5.5 to 12.0 m, a layer of compact to dense sandy silt to silty sand from 12.0 to 19.5 m, and very dense silt below 19.5 m. More details of the firm silty clay and the very dense silt deposits are presented in Section 3.

The groundwater table measured in the piezometer well in the borehole was at a depth of 5.0 m.

### 2.2 Preparation of Seismic Holes

The proper installation and grouting of the seismic holes are essential for the quality of S-wave velocity measurements. Delayed arrival times and attenuated signal amplitudes can be caused by poor coupling between the casing and the soils. Horizontal spacing between the seismic holes is also an important factor affecting the test results. For large horizontal spacing, the measured S-wave velocity in a soil layer can be affected by the refraction and traveling of the waves in the underlying stronger soil layer. If the holes are spaced too closely, the errors in the measured S-wave velocity resulting from installation quality, soil disturbance and travel time measurement, will become more significant.

In this study, two seismic holes at about 3.4 m in horizontal spacing at surface were drilled to a depth of 25m. The holes were grouted with a bentonite-cement-water mixture consisting of 1 part of bentonite, 1 part of Portland cement and 6.3 parts of water in weight. PVC casings were installed in both seismic holes. The PVC casings were suitable for inclinometer deviation survey. The horizontal distance between the two seismic holes was surveyed at 0.5 m intervals. The centre-to-centre horizontal distance between the two seismic holes varied from 3.4 m at the surface to 4.8 m at a depth of 24 m.

### 2.3 S-Wave Velocity Measurement

Using the cross-hole method, measurements of S-wave velocity were conducted to a depth of 24 m at intervals of 1.0 m.

In the seismic cross-hole surveys, a geophone probe was placed in a seismic hole and the travel time was measured for a seismic wave generated in an adjacent hole. The receivers consisted of 3 geophones, one vertical and two horizontal. The two horizontal geophones were perpendicular to each other. This ensures that at least one geophone is orientated to detect the incoming waves. Generally, the vertical geophone best detects the shear waves.

The record trigger and the geophones are connected and recorded by a seismograph. Within the seismic holes, the seismic hammer is held in place by a hydraulic plate and the geophones are held by a spring-loaded arm. The first shot record is carried out at the bottom of the holes. Additional shots are then taken at 1 m intervals up the holes. At each interval, there are two separate records from a downward stroke and an upward stroke of the hammer. These two records have opposite initial inflections, helping the interpretation of the shear wave arrival time.

The arrival times of the S-waves are recorded for each shot. Using the S-wave arrival times and the distance between the holes, the S-wave velocities are obtained.

## 3 TEST RESULTS AND ANALYSES

From the measured data, the processed S-wave velocities with depth are presented in Figure 1. The shear-wave velocities range from 208 to 475 m/s. Variations in the S-wave velocities correlate well with

changes in soil conditions explored in the adjacent borehole. The average values of the shear wave velocity ( $V_s$ ) of each soil layer are obtained, as listed on Table 1.

The soils at depths from 12.0 to 19.5 m consisted of interbedded layers of sandy silt and fine sand. Due to the various soil types, it is difficult to interpret the S-wave data in these soil layers. In the following sections, analyses are focused on the firm silty clay and the very dense silt deposits.

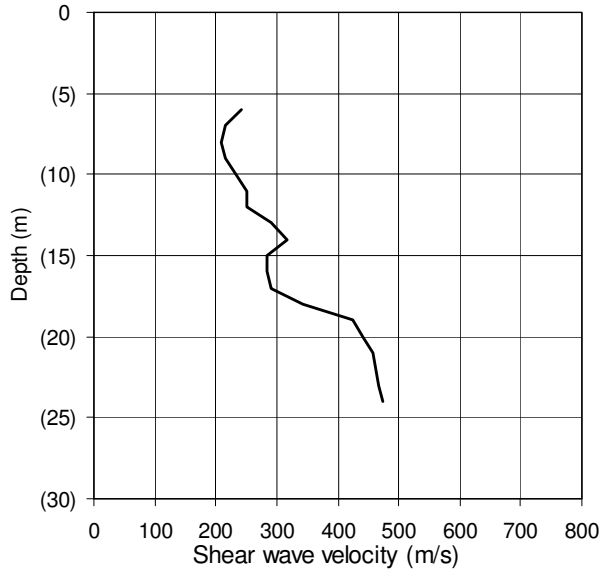


Figure 1. Test Results of S-wave velocities with depth

Table 1. Average S-wave velocity of soil layers

Soil Type	Depth (m)	Average $V_s$ (m/s)
Firm silty clay	5.5 to 12.0	230
Sandy soils	12.0 to 19.5	320
Very dense silt	19.5 to 24.0	460

### 3.1 Firm Silty Clay

The silty clay was at depths from 5.5 to 12.0 m. The average value of the measured undrained shear strength ( $C_u$ ) from the field vane tests is 43 kPa. The water content of silty clay is about 23%. The liquid limit (LL) is 30% and the plastic index (PI) is 13. The silty clay is classified as inorganic clay of low plasticity (CL). The estimated void ratio is  $e = 0.62$ .

According to Ladd (1991), the undrained shear strength ( $C_u$ ) of the silty clay can be approximately related to the over-consolidation ratio (OCR) as

$$\frac{C_u}{\sigma'_v} = 0.25(OCR)^{0.8} \quad [5]$$

where  $\sigma'_v$  is the effective vertical stress in the soil. Using Equation 5 and  $C_u = 43$  kPa, the estimated average over-

consolidation ratio of the silty clay is  $OCR = 1.3$ , and the pre-consolidation stress is  $P_c = 180$  kPa.

The average value of the S-wave velocity of the firm silty clay is  $V_s = 230$  m/s. Using Equation 3 and a soil density of  $\rho = 1900$  kg/m<sup>3</sup>, the small-strain shear modulus of the silty clay can be obtained as  $G_{max} = 101$  MPa from the measured  $V_s$ .

As  $C_u = 43$  kPa, the shear modulus to undrained strength ratio is  $G_{max}/C_u = 2300$  for the firm silty clay, which is within the general range of  $G_{max}/C_u$  values from about 1000 to 2500 (Gazetas, 1991).

A general empirical correlation for  $G_{max}$  of both granular and cohesive soils has been proposed by Hardin (1978), expressed as

$$G_{max} = 625 \frac{(OCR)^\mu}{0.3 + 0.7e^2} \sqrt{p_a \sigma'_m} \quad [6]$$

where  $\sigma'_m$  is the mean confining effective pressure,  $p_a$  is the atmospheric pressure in the same units as  $\sigma'_m$  and  $G_{max}$ , and  $\mu$  is a parameter increasing with the plasticity index  $PI$  as shown in Figure 2.

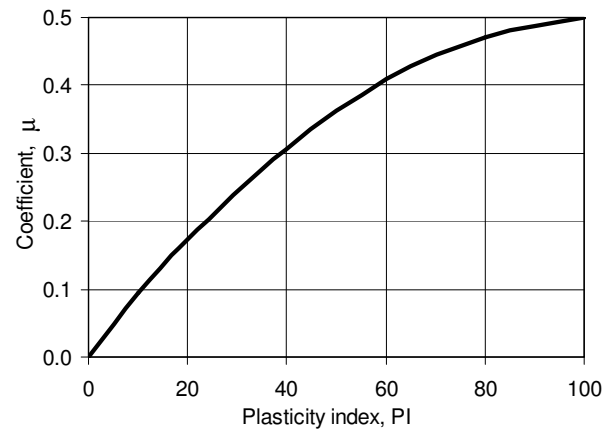


Figure 2. Coefficient  $\mu$  with plasticity Index  $PI$  (after Gazetas, 1991)

The mean confining effective pressure is derived using

$$\sigma'_m = \frac{1 + 2K_o}{3} \sigma'_v \quad [7]$$

where  $\sigma'_v$  is the vertical effective pressure in the soil, and  $K_o$  is the at rest earth pressure coefficient. According to Mayne and Kulhawy (1982),  $K_o$  can be estimated using

$$K_o = (1 - \sin \phi)(OCR)^{\sin \phi} \quad [8]$$

in which  $\phi$  is the friction angle of soil. Using the estimated value of  $\phi = 27^\circ$ , and  $OCR = 1.3$ , the calculated value of the at rest earth pressure coefficient is  $K_o = 0.61$  for the firm silty clay.

The calculated  $G_{\max}$  values of the silty clay obtained from Equation 6 range from 102 MPa at a depth of 6.0 m to 124 MPa at 12.0 m. As shown in Figure 3, the  $G_{\max}$  values calculated from Equation 6 are close to those obtained from the measured S-wave velocity. Equation 6 is considered suitable for estimating the small-strain shear modulus  $G_{\max}$  of the silty clay soil.

Using Equation 3 and a Poisson's ratio of  $\nu = 0.5$  for the saturated silty clay, the small-strain Young's modulus ( $E_{\max}$ ) can be obtained from the small-strain shear modulus  $G_{\max}$  values from the test data. The calculated  $E_{\max}$  values with depth are shown in Figure 4.

The  $E_{\max}$  values of the silty clay ranges from 247 to 356 MPa, with an average value of  $E_{\max} = 305$  MPa. As the undrained shear strength  $C_u = 43$  kPa, the average  $E_{\max}$  to  $C_u$  ratio of the silty clay is

$$\frac{E_{\max}}{C_u} \approx 7100 \quad [9]$$

The  $E_{\max}/C_u$  value is very high, as  $E_{\max}$  is obtained from small-strain conditions in the order of  $10^{-5}$  or less. The small-strain Young's modulus  $E_{\max}$  should not be used directly in conventional geotechnical applications such as settlement analysis of footings.

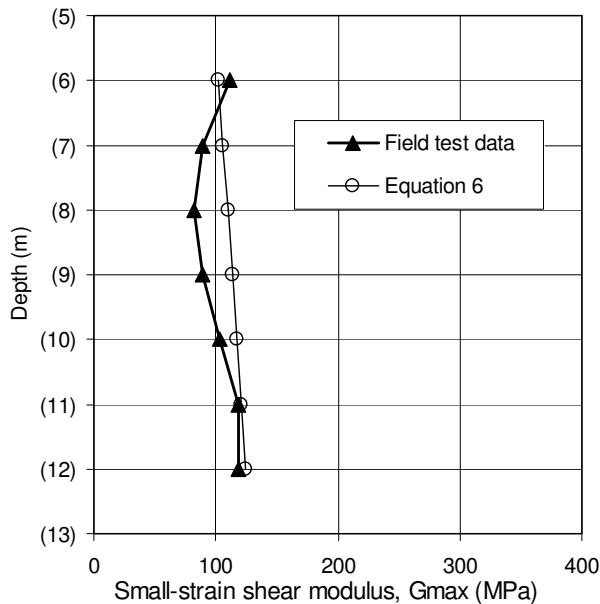


Figure 3. Comparison of  $G_{\max}$  values from  $V_s$  test results and empirical correlation of firm silty clay

The immediate settlement of a footing on a saturated clay soil can be calculated using

$$S_i = C_s q B \left( \frac{1 - \nu^2}{E_u} \right) \quad [10]$$

where  $C_s$  is the shape and rigidity factor,  $q$  is the uniform (average) pressure of the footing,  $\nu$  is the Poisson's ratio,

and  $B$  is the characteristic dimension of the footing. For a circular footing,  $B$  represents its diameter.

The equivalent undrained Young's modulus ( $E_u$ ) can be taken as  $E_u = 600C_u$  for soils with plasticity index  $PI < 30$  and over-consolidation ratio  $OCR < 3$  (Holtz, 1991). This  $E_u/C_u$  value of 600 is relatively low, as  $E_u$  is a parameter at large-strain conditions (generally in the order of  $10^2$  to  $10^3$ ). The small-strain modulus  $E_{\max}$  of the firm silty clay is more than 10 times the large-strain modulus  $E_u$ .

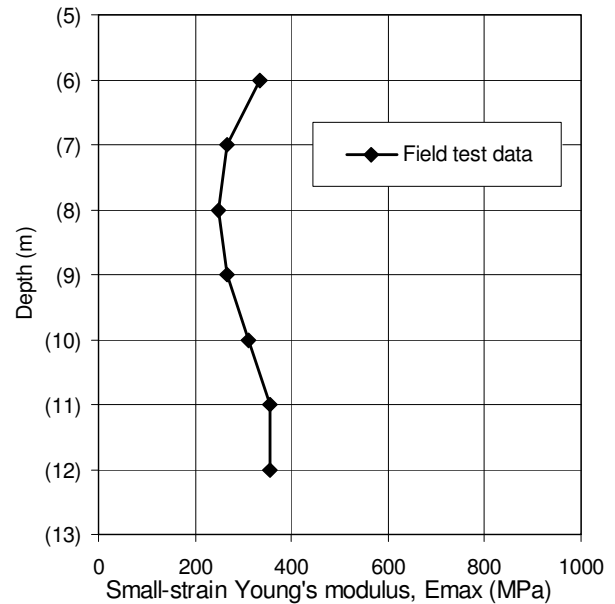


Figure 4. Small-strain Young's modulus  $E_{\max}$  of firm silty clay

### 3.2 Very Dense Silt

The very dense silt was at depths from 19.5 to 24.0 m. According to the results of grain size analyses, the deposit contains about 83% of silt, 10% of clay, and 7% of sand. The liquid limit (LL) is 19.5% and the plastic index (PI) is 3.5. The deposit is classified as inorganic silt of low compressibility (ML). The silt was very dense, with an average SPT 'N' value of 94. The water content is about 15%. The estimated void ratio is  $e = 0.40$ .

As shown in Figure 1, the measured S-wave velocity of the very dense silt at depth from 19.5 to 24.0 m ranges from 440 to 475 m/s, increasing slightly with depth. Using Equation 3, the calculated values of the small-strain shear modulus  $G_{\max}$  vary from 420 to 485 MPa, as shown in Figure 5.

For the silt which is essentially cohesionless, Equation 6 of Hardin (1978) becomes

$$G_{\max} = 625 \frac{\sqrt{p_a \sigma'_m}}{0.3 + 0.7 e^2} \quad [6a]$$

Seed et al. (1986) have proposed an empirical correlation between  $G_{\max}$  and the Standard Penetration Test (SPT) resistance, expressed as

$$G_{\max} = 4500(N_1)^{1/3} \sqrt{\sigma'_m} \quad [11]$$

where the small-strain shear modulus  $G_{\max}$  and the mean confining effective stress  $\sigma'_m$  are in kPa. The corrected SPT resistance is

$$N_1 = N \left( \frac{p_a}{\sigma'_v} \right)^{1/2} \left( \frac{ER}{60} \right) \quad [12]$$

in which,  $N$  is the SPT resistance (blows per 0.3m),  $\sigma'_v$  is the vertical effective stress,  $p_a$  is the atmospheric pressure, and  $(ER/60)$  is the rod energy ratio normalized to 60%.

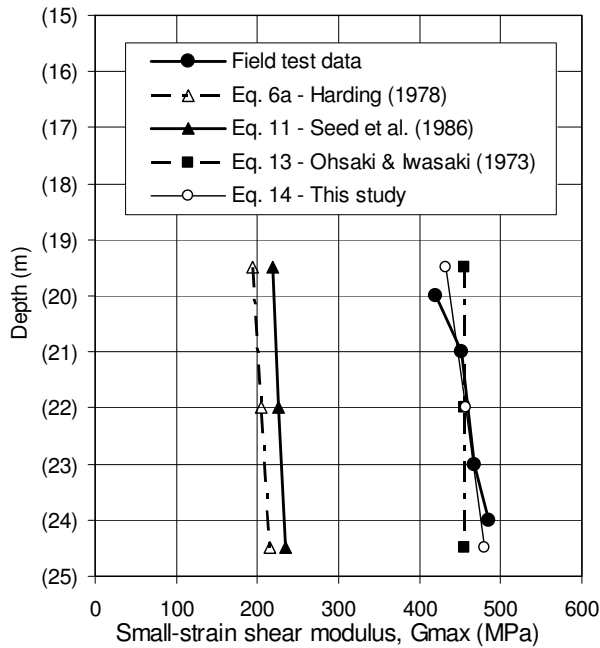


Figure 5. Small-strain shear modulus  $G_{\max}$  of very dense silt

Another correlation that has been frequently quoted in the literature is the one proposed by Ohsaki and Iwasaki (1973), in the form of

$$G_{\max} = 12000N^{0.8} \quad [13]$$

where  $G_{\max}$  is in kPa.

The  $G_{\max}$  values obtained from Equation 6a, Equation 11 and Equation 13 are shown in Figure 5. The  $G_{\max}$  values obtained from Equation 6a of Harding (1978) and Equation 11 of Seed et al. (1986) are about a half of those obtained from the  $V_s$  test results. The  $G_{\max}$  from

Equation 13 of Ohsaki and Iwasaki (1973) is close to the average  $G_{\max}$  value obtained from the  $V_s$  test results.

Considering the general forms of Equation 6 and Equation 6a of Hardin (1978) for cohesionless soil, a new correlation is derived from the  $V_s$  test results of the very dense silt, expressed as

$$G_{\max} = 1400 \frac{\sqrt{p_a \sigma'_m}}{0.3 + 0.7e^2} \quad [14]$$

The  $G_{\max}$  values from Equation 14 are also plotted in Figure 5, which fits well with the test results.

It should be mentioned that Equation 14 is derived from the test results of the very dense silt at depths ranging from 20 to 24 m. It should be cautious to use Equation 14 in practice, as this correlation has been derived from a limited number of test data. Engineering judgement is required to use Equation 14 for a specific site.

#### 4 SUMMARY AND CONCLUSIONS

In this study, S-wave velocity survey was carried out at a site near a drilled borehole. The test data of a firm silty clay layer at depths from 5.5 to 12.0 m and a very dense silt layer at 19.5 to 24.0 m are analyzed in detail. The silty clay had an undrained shear strength of 43 kPa and the average SPT 'N' value of the very dense silt was 94. The test results and the analysis findings are summarized as follows:

1. The average S-wave velocity is 230 m/s for the firm silty clay and 460 m/s for the very dense silt.
2. For the silty clay, the values of the small-strain shear modulus  $G_{\max}$  from the test data are close to those obtained from the empirical correlation of Harding.
3. The small-strain Young's modulus  $E_{\max}$  of the silty clay is more than 10 times the equivalent undrained Young's modulus  $E_u$  at large-strain conditions.
4. For the very dense silt, the values of the small-strain shear modulus  $G_{\max}$  from the test data are about two times those obtained from the empirical correlations of Harding and Seed et al.
5. The new correlation (Equation 14) can be used to estimate the value of  $G_{\max}$  of the very dense silt.

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